Complete solutions to Exercise 9(a)

1. We apply the trapezium rule, (9.1), with $h = 0.5,\ a = 0,\ b = 3$ and $y_0 = 3.2,\ y_1 = 5.6,\ y_2 = 7.0,\ y_3 = 7.7,\ y_4 = 8.4,\ y_5 = 9.9$ and $y_6 = 11.6$:

$$\text{Impulse of force } = \int_0^3 F dt \approx \frac{0.5}{2}\left[3.2 + 2\left(5.6 + 7.0 + 7.7 + 8.4 + 9.9\right) + 11.6\right]$$
$$= 23 \text{ N s}$$

---

2. (a) We can first establish a table and then slot the values into (9.1):

x	0	0.25	0.5	0.75	1.0
e^{-x^2}	1	0.939	0.779	0.570	0.368

Applying (9.1) with uniform width $h = 0.25$ and the y values read from the second row:

$$\int_0^1 e^{-x^2} dx \approx \frac{0.25}{2}\left[1 + 2\left(0.939 + 0.779 + 0.570\right) + 0.368\right] = 0.743$$

(b) Since we have 4 equal intervals, so $h = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}$. As (a) we form a table of values for $\sqrt{\cos(x)}$:

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
$\sqrt{\cos(x)}$	1	0.961	0.841	0.619	0

Using (9.1):

$$\int_0^{\pi/2} \sqrt{\cos(x)} dx \approx \frac{\pi/8}{2}\left[1 + 2\left(0.961 + 0.841 + 0.619\right) + 0\right] = 1.147$$

---

3. (i) (a) The uniform width $h = 0.25$, we have:

x	0	0.25	0.5	0.75	1.0
x^3	0	0.01 6	0.125	0.422	1

Using (9.1):

$$\int_0^1 x^3 dx \approx \frac{0.25}{2}\left[0 + 2\left(0.016 + 0.125 + 0.422\right) + 1\right] = 0.266$$

(b) Similarly with $h = 0.125$ we only have to evaluate x^3 for $x = 0.125,\ 0.375,\ 0.625$ and 0.875 because the others have been evaluated in the above table:

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1.000
x^3	0	0.002	0.016	0.053	0.125	0.244	0.422	0.670	1

Applying (9.1):

$$\int_0^1 x^3 dx \approx \frac{0.125}{2}\left[0 + 2\left(0.002 + 0.016 + 0.053 + 0.125 + 0.244 + 0.422 + 0.670\right) + 1\right]$$
$$= 0.254$$

(8.1) $\int x^n dx = x^{n+1}/n + 1$

(9.1) $\int_a^b y dx \approx \frac{h}{2}\left[y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right]$

(ii) Exact value is found by using (8.1):

$$\int_0^1 x^3 dx = \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4} = 0.25$$

(iii) (a) $\quad \% \text{ error} = \dfrac{0.25 - 0.266}{0.25} \times 100 = -6.4\%$ (using 4 intervals)

(b) $\quad \% \text{ error} = \dfrac{0.25 - 0.254}{0.25} \times 100 = -1.6\%$ (using 8 intervals)

(iv) As the number of interval increases so the accuracy of the estimation increases.

---

4. Applying (9.1) with $h = 1$ and $y_0 = 2.1,\ y_1 = 9.56,\ y_2 = 11.36,\ y_3 = 12.08,$ $y_4 = 12.98$ and $y_5 = 13.76$:

$$\text{Distance} = \int_0^5 v dt \approx \frac{1}{2}\left[2.1 + 2(9.56 + 11.36 + 12.08 + 12.98) + 13.76\right]$$
$$= 53.91 \text{ m}$$

---

5. Cross - sectional area is found by using the trapezium rule (9.1):

$$\text{Cross} - \text{sectional area} \approx \frac{1.5}{2}\left[0 + 2\begin{pmatrix}1.04 + 1.65 + 3.10 + 4.66 + 4.12 \\ +3.21 + 2.33 + 1.78 + 0.76\end{pmatrix} + 0\right]$$
$$= 33.975 \text{ m}^2$$

Volume/sec $\approx 33.975 \times 2.05 = 69.65 \text{ m}^3/\text{s}$.

---

6. Using (9.1) with $h = 0.1$ and the y values given by the second row of the table:

$$\int_0^{0.6} v dt \approx \frac{0.1}{2}\left[4 + 2(3.92 + 3.86 + 3.77 + 3.61 + 3.52) + 3.41\right]$$
$$= 2.24 \ Vs$$

$$\bar{v} = \frac{1}{0.6}\int_0^{0.6} v dt = \frac{2.24}{0.6} = 3.73 \text{ V}$$

(9.1) $$\int_a^b y dx \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n\right]$$