Complete solutions to Exercise 9(b)

(All the computer algebra solutions are given as MAPLE commands).

1. We use Simpson's rule with $n = 10$. We have

$$\int_0^{\pi/2} \sin(t)dt \approx \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8) + y_{10}\right] \quad (*)$$

What is the value of the width h?

Since there are 10 equal intervals between 0 to $\pi/2$ so we have

$$h = \frac{\pi/2 - 0}{10} = \frac{\pi}{20} = 9°$$

It is easier to evaluate $\sin(t)$ with degrees rather than radians. We can establish a table of values for $y = \sin(t)$ at $t = 0°,\ 9°,\ 18°, \ldots, 90°$. Hence

t	0°	9°	18°	27°	36°	45°	54°	63°	72°	81°	90°
$\sin(t)$	0	0.156	0.309	0.454	0.588	0.707	0.809	0.891	0.951	0.988	1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

Putting all these values into (*) gives

$$\int_0^{\pi/2} \sin(t)dt \approx \frac{\pi/20}{3}\left[\begin{array}{r} 0 + 4(0.156 + 0.454 + 0.707 + 0.891 + 0.988) \\ +2(0.309 + 0.588 + 0.809 + 0.951) + 1 \end{array}\right]$$

$$= \frac{\pi}{60}[19.098] = 0.999969$$

For exact value:

$$\int_0^{\pi/2} \sin(t)dt = -\left[\cos(t)\right]_0^{\pi/2}$$

$$= -\left[\cos\left(\frac{\pi}{2}\right) - \cos(0)\right] = -[0-1] = 1$$

$$\%\ error = \frac{1 - 0.999969}{1} \times 100 = 0.0031\%$$

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2. Using Simpson:

$$\text{(9.4)} \qquad \int_a^b y dx = \frac{h}{3}\left\{y_0 + 4(y_1 + y_3 + y_5 + \ldots) + 2(y_2 + y_4 + y_6 + \ldots) + y_n\right\}$$

What are the values of a, b and h?

The lower limit $a = 0$, so $b = \frac{\pi}{3}$ and the uniform width h

$$h = \frac{\pi/3}{6} = \frac{\pi}{18}$$

because we require 6 equal intervals. What else do we need to find? $y_0,\ y_1, \ldots, y_6$. We can evaluate these and place them in a table:

x	$\sqrt{\sin(x)}$	
0	$\sqrt{\sin(0)} = 0$	y_0
$\frac{\pi}{18}$	$\sqrt{\sin\left(\frac{\pi}{18}\right)} = 0.4166$	y_1
$\frac{2\pi}{18}$	$\sqrt{\sin\left(\frac{2\pi}{18}\right)} = 0.5848$	y_2
$\frac{3\pi}{18}$	$\sqrt{\sin\left(\frac{3\pi}{18}\right)} = 0.7071$	y_3
$\frac{4\pi}{18}$	$\sqrt{\sin\left(\frac{4\pi}{18}\right)} = 0.8016$	y_4
$\frac{5\pi}{18}$	$\sqrt{\sin\left(\frac{5\pi}{18}\right)} = 0.8752$	y_5
$\frac{6\pi}{18} = \frac{\pi}{3}$	$\sqrt{\sin\left(\frac{\pi}{3}\right)} = 0.9306$	y_6

Putting $n = 6$ into (9.4) gives:

$$\int_0^{\pi/3} \sqrt{\sin(x)}dx \approx \frac{\pi/18}{3}\left\{y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right\}$$

$$= \frac{\pi}{54}\left\{0 + 4(0.4166 + 0.7071 + 0.8752) + 2(0.5848 + 0.8016) + 0.9306\right\}$$

$$= 0.681$$

(9.4) $$\int_a^b y dx \approx \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots) + y_n\right]$$

3. We use Simpson's rule with $n=10$, $h=0.1$ and $y_0=230.0$, $y_1=202.6,\ldots,\ y_{10}=160.5$:

$$\int_2^3 F\,dt \approx \frac{0.1}{3}\left[y_0+4\left(y_1+y_3+y_5+y_7+y_9\right)+2\left(y_2+y_4+y_6+y_8\right)+y_{10}\right]$$

$$=\frac{0.1}{3}\left[\begin{array}{l}230.0+4(202.6+178.7+168.0+163.4+161.1)\\ \qquad +2(188.3+172.4+165.6+162.0)+160.5\end{array}\right]$$

$$=175.41\text{ N s}$$

4. Similarly we apply Simpson's rule (9.4) with $n=10$, $h=0.5$ and $y's$ can be read from the second row of the table:

$$F=\int_0^4 w\,dx \approx \frac{0.5}{3}\left[0+4\left(1.7+5.6+7.3+3.2\right)+2\left(2.3+8.8+4.9\right)+2.5\right]$$

$$=17.62\text{ N}$$

5. In this case we have to first find i^2:

t	0	0.1	0.2	0.3	0.4	0.5	0.6
i	0	0.25	0.31	0.43	0.37	0.20	0
i^2	0	0.0625	0.0961	0.1849	0.1369	0.04	0

Applying Simpson's rule with $n=6$, $h=0.1$ and $y's$ are i^2 in the third row of the above table:

$$\int_0^{0.6} i^2\,dt \approx \frac{0.1}{3}\left[0+4\left(0.0625+0.1849+0.04\right)+2\left(0.0961+0.1369\right)+0\right]=0.054$$

$$i_{R.M.S.} \approx \sqrt{\frac{1}{0.6}\left(0.054\right)}=0.3\text{ A}$$

6. Using Simpson's rule:

$$W=\int_{0.01}^{0.09} P\,dV \approx \frac{0.01}{3}\left[\begin{array}{l}560.73+4(237.86+98.67+59.24+42.31)\\ \qquad +2(140.33+73.66+48.73)+35.20\end{array}\right]$$

$$=9.58\text{ J}$$

7. Applying Simpson's rule:

$$\text{cross-section area} \approx \frac{2.5}{3}\left\{\begin{array}{l}0.3+4[1.78+2.96+5.30+6.11+1.76]\\ \qquad +2[2.05+4.01+6.78+4.35]+0.4\end{array}\right\}$$

$$=88.93$$

Volume of flow/second $\approx 88.93\times 1.67=149\ \text{m}^3/\text{s}$.