Complete solutions to Exercise 9(d)

1. We first find the velocity by integrating:

$$v = \int -2.5\cos(2\pi t)\,dt$$
$$= -\frac{2.5\sin(2\pi t)}{2\pi} + C$$

Substituting $t = 0,\ v = 0$ gives $C = 0$. To find x we integrate v:

$$x = \int -\frac{2.5\sin(2\pi t)}{2\pi} = \frac{2.5\cos(2\pi t)}{4\pi^2} + D$$

Substituting $t = 0,\ x = 0$:

$$0 = \frac{2.5}{4\pi^2} + D \text{ which gives } D = -\frac{2.5}{4\pi^2}$$

Hence

$$x = \frac{2.5\cos(2\pi t)}{4\pi^2} - \frac{2.5}{4\pi^2} = \frac{2.5}{4\pi^2}\left[\cos(2\pi t) - 1\right]$$

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2. Integrating

$$v_x = \int t\,dt = \frac{t^2}{2} + C$$

Using $t = 0,\ v_x = 2$ gives

$$2 = 0 + C,\ C = 2$$
$$v_x = \frac{t^2}{2} + 2$$

How do we find x?
Integrate again:

$$x = \int\left(\frac{t^2}{2} + 2\right)dt$$
$$= \frac{t^3}{6} + 2t + D$$

Using $t = 0,\ x = 3$ gives $D = 3$. Hence

$$x = \frac{t^3}{6} + 2t + 3$$

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3. Differentiating the velocity gives the acceleration, a.

$$a = \frac{dv}{dt} = 1.2t + 1$$

To find the acceleration at $t = 2$ we substitute this into $a = 1.2t + 1$

$$a = (1.2 \times 2) + 1 = 3.4 \text{ m/s}^2$$

Displacement

$$s = \int_0^2 \left(0.6t^2 + t\right)dt$$
$$= \left[\frac{0.6t^3}{3} + \frac{t^2}{2}\right]_0^2$$
$$= \frac{0.6 \times 2^3}{3} + \frac{2^2}{2} = 3.6 \text{ m}$$

4. By (9.11) we have

$$v = \int 4t^3 dt$$

$$v = t^4 + C$$

When $t = 0,\ v = 3$ gives $C = 3$.

$$v = t^4 + 3$$

Substituting $t = 1.5$

$$v = 1.5^4 + 3 = 8.06 \text{ m/s} \quad \text{(2 d.p.)}$$

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5. The acceleration, $a = 2.9$. By (9.11) we have

$$v = \int 2.9 dt = 2.9t + C$$

When $t = 0,\ v = 0$ gives $C = 0$, so

$$v = 2.9t$$

The height, h, can be obtained by integrating v:

$$h = \int 2.9t dt = \frac{2.9t^2}{2} + D$$

$$h = 1.45t^2 + D$$

At $t = 0,\ h = 0$ gives $D = 0$. Hence

$$h = 1.45t^2$$

Substituting $t = 2 \times 60 = 120$ into h:

$$h = 1.45 \times 120^2 = 20.88 \times 10^3 m = 20.88 \text{ km} \quad \text{(2 d.p.)}$$

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6. The distance covered is given by the area. Use Simpson's rule (9.4)

$$\text{Distance } \approx \frac{25}{3}\left[0 + 2(45 + 64 + 77) + 4(33 + 55 + 72 + 80) + 82\right]$$

$$= 11.783 = 11.8 \text{ km (3 s.f.)}$$

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7. Integrating a with respect to t gives the velocity v

$$v = \int a dt = at + C$$

Substituting $t = 0,\ v = u$

$$u = 0 + C \text{ gives } C = u$$

Hence $v = u + at$. To find s we integrate $v = u + at$

$$s = \int (u + at) dt$$

$$= ut + \frac{1}{2}at^2 + D \qquad (*)$$

Putting $t = 0,\ s = 0$ into (*) gives $D = 0$, hence $s = ut + \frac{1}{2}at^2$.

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8. The acceleration of the stone is the constant acceleration due to gravity

$$a = 9.81$$

$$v = \int 9.81 dt = 9.81t + C$$

(9.4) $$\int_a^b f(x) dx \approx \frac{h}{3}\left[y_0 + 2(y_2 + y_4 + \ldots) + 4(y_1 + y_3 + \ldots) + y_n\right]$$

(9.11) $$v = \int a dt$$

At $t = 0,\ v = 0$ which gives $C = 0$

$$v = 9.81t$$

Integrating again gives the height, h:

$$h = \int 9.81t\,dt = 4.905t^2 + D$$

At $t = 0,\ h = 0$ gives $D = 0$

$$h = 4.905t^2 \qquad (\dagger)$$

Since the height of the building is $20m$ we substitute $h = 20$ into $(\dagger)$

$$4.905t^2 = 20$$

$$t = \sqrt{\frac{20}{4.905}} = 2.02s \quad \text{(2 d.p.)}$$

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9. Integrating a with respect to t gives velocity, v

$$v = \int \left[-\cos(\omega t) + \sin(\omega t)\right] dt$$

$$v = -\frac{\sin(\omega t)}{\omega} - \frac{\cos(\omega t)}{\omega} + C$$

Substituting $t = 0,\ v = 0$

$$0 = -\frac{\sin(0)}{\omega} - \frac{\cos(0)}{\omega} + C$$

$$= -\frac{1}{\omega} + C \quad \text{which gives} \quad C = \frac{1}{\omega}$$

Hence

$$v = \frac{1}{\omega}\left[1 - \sin(\omega t) - \cos(\omega t)\right]$$

Integrating this gives

$$x = \frac{1}{\omega}\left[t + \frac{\cos(\omega t)}{\omega} - \frac{\sin(\omega t)}{\omega}\right] + D \qquad (\dagger)$$

Putting $t = 0,\ x = 0$

$$0 = \frac{1}{\omega}\left[0 + \frac{\cos(0)}{\omega} - \frac{\sin(0)}{\omega}\right] + D$$

$$= \frac{1}{\omega}\left[0 + \frac{1}{\omega} - 0\right] + D$$

$$0 = \frac{1}{\omega^2} + D \quad \text{which gives} \quad D = -\frac{1}{\omega^2}$$

Substituting $D = -\frac{1}{\omega^2}$ into $(\dagger)$

$$x = \frac{1}{\omega}\left[t + \frac{\cos(\omega t)}{\omega} - \frac{\sin(\omega t)}{\omega}\right] - \frac{1}{\omega^2}$$

$$= \frac{1}{\omega^2}\left[\omega t + \cos(\omega t) - \sin(\omega t) - 1\right]$$

Multiplying both sides by ω^2 gives the required result:

$$\omega^2 x = \omega t + \cos(\omega t) - \sin(\omega t) - 1$$