Complete solutions to Exercise 9(e)

1. First we obtain P and then we use (9.12):

$$P = \frac{1500}{V^{1.35}} = 1500V^{-1.35}$$

Substituting $P = 1500V^{-1.35}$, $V_1 = 0.01$ and $V_2 = 0.1$ into (9.12):

$$W = \int_{0.01}^{0.1} 1500V^{-1.35} dV$$

$$= 1500\left[\frac{V^{-0.35}}{-0.35}\right]_{0.01}^{0.1}$$

$$= -\frac{1500}{0.35}\left[0.1^{-0.35} - 0.01^{-0.35}\right] = 11.885 \text{ kJ}$$

2. We have

$$P = \frac{1789}{V^{1.55}} = 1789V^{-1.55}$$

Substituting $V_1 = 0.6$, $V_2 = 0.023$ and $P = 1789V^{-1.55}$ into (9.12):

$$W = \int_{0.6}^{0.023} 1789V^{-1.55} dV$$

$$= 1789\left[\frac{V^{-0.55}}{-0.55}\right]_{0.6}^{0.023}$$

$$= -\frac{1789}{0.55}\left[0.023^{-0.55} - 0.6^{-0.55}\right] = -21.592 \text{ kJ}$$

(The negative sign indicates compression).

3. We have $P = 1675V^{-1.6}$, $V_1 = 0.15$ and $V_2 = 0.037$. Applying (9.12):

$$W = 1675\int_{0.15}^{0.037} V^{-1.6} dV$$

$$= 1675\left[\frac{V^{-0.6}}{-0.6}\right]_{0.15}^{0.037}$$

$$= -\frac{1675}{0.6}\left[0.037^{-0.6} - 0.15^{-0.6}\right] = -11.467 \text{ kJ}$$

4. Substituting $P = \frac{C}{V}$ into (9.12):

$$W = \int_{V_1}^{V_2} \frac{C}{V} dV \underbrace{=}_{\text{by (8.2)}} C\left[\ln(V)\right]_{V_1}^{V_2}$$

$$= C\left[\ln(V_2) - \ln(V_1)\right] \underbrace{=}_{\text{by (5.12)}} C\ln\left(\frac{V_2}{V_1}\right)$$

(5.12) $$\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)$$

(8.2) $$\int \frac{du}{u} = \ln|u|$$

(9.12) $$W = \int_{V_1}^{V_2} P dV$$

5. Putting $M = P(l - x)$ into (9.13)

$$\frac{d^2 y}{dx^2} = \frac{P}{EI}(l - x)$$

Integrating

$$\frac{dy}{dx} = \frac{P}{EI}\left(lx - \frac{x^2}{2}\right) + C$$

Substituting $x = 0,\ \frac{dy}{dx} = 0$

$$0 = \frac{P}{EI}\left(l(0) - \frac{0^2}{2}\right) + C \text{ gives } C = 0$$

We have

$$\frac{dy}{dx} = \frac{P}{EI}\left(lx - \frac{x^2}{2}\right)$$

Integrating again

$$y = \frac{P}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + D$$

Putting $x = 0,\ y = 0$

$$0 = \frac{P}{EI}(0) + D \text{ gives } D = 0$$

Thus

$$y = \frac{P}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) = \frac{P}{6EI}\left(3lx^2 - x^3\right)$$

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6. Using (9.13) with $M = -\frac{wx^2}{2}$ we have

$$\frac{d^2 y}{dx^2} = \frac{1}{EI}\left(-\frac{wx^2}{2}\right)$$

Integrating

$$\frac{dy}{dx} = -\frac{w}{EI}\left(\frac{x^3}{6}\right) + C \qquad (\dagger)$$

Substituting $x = L,\ \frac{dy}{dx} = 0$

$$0 = -\frac{w}{EI}\left(\frac{L^3}{6}\right) + C \text{ gives } C = \frac{wL^3}{6EI}$$

Putting this value of C into $(\dagger)$ gives

$$\frac{dy}{dx} = \frac{w}{6EI}\left(L^3 - x^3\right)$$

$$y = \frac{w}{6EI}\left[L^3 x - \frac{x^4}{4}\right] + D \qquad (*)$$

(9.13) $$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

Substituting $x = L,\ y = 0$

$$0 = \frac{w}{6EI}\left[L^4 - \frac{L^4}{4}\right] + D$$

$$D = -\frac{w}{6EI}\frac{3L^4}{4} = -\frac{3wL^4}{24EI}$$

Substituting this into (*)

$$y = \frac{w}{6EI}\left(L^3x - \frac{x^4}{4}\right) - \frac{3wL^4}{24EI}$$

$$= \frac{w}{24EI}\left(4L^3x - x^4 - 3L^4\right)$$

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7. Substituting $M = -\frac{w}{2}(L-x)^2$ into (9.13)

$$\frac{d^2y}{dx^2} = -\frac{w}{2EI}(L-x)^2$$

Integrating gives

$$\frac{dy}{dx} = \frac{w}{2EI}\frac{(L-x)^3}{3} + C$$

$$= \frac{w}{6EI}(L-x)^3 + C \qquad (\dagger)$$

Using the condition $x = 0,\ \frac{dy}{dx} = 0$:

$$0 = \frac{w}{6EI}(L-0)^3 + C \text{ gives } C = -\frac{wL^3}{6EI}$$

Putting this value of C into (†)

$$\frac{dy}{dx} = \frac{w}{6EI}(L-x)^3 - \frac{wL^3}{6EI}$$

$$= \frac{w}{6EI}\left[(L-x)^3 - L^3\right]$$

How do we find y?
Integrate again

$$y = \frac{w}{6EI}\left[-\frac{(L-x)^4}{4} - L^3x\right] + D \qquad (\dagger\dagger)$$

Substituting $x = 0,\ y = 0$:

$$0 = \frac{w}{6EI}\left(-\frac{L^4}{4}\right) + D \text{ gives } D = \frac{wL^4}{24EI}$$

Putting $D = \frac{wL^4}{24EI}$ into (††):

(9.13) $$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$y=\frac{w}{6EI}\left[-\frac{(L-x)^4}{4}-L^3x\right]+\frac{wL^4}{24EI}$$

$$=\frac{w}{24EI}\left[-(L-x)^4-4L^3x\right]+\frac{wL^4}{24EI}$$

$$y=\frac{w}{24EI}\left[L^4-4L^3x-(L-x)^4\right]$$

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8. Putting $M=\frac{w}{6}\left(x^3-L^2x\right)$ into (9.13)

$$\frac{d^2y}{dx^2}=\frac{w}{6EI}\left(x^3-L^2x\right)$$

Integrating gives

$$\frac{dy}{dx}=\frac{w}{6EI}\left[\frac{x^4}{4}-\frac{L^2x^2}{2}\right]+C$$

Integrating again

$$y=\frac{w}{6EI}\left[\frac{x^5}{20}-\frac{L^2x^3}{6}\right]+Cx+D \qquad (*)$$

Using the condition $x=0,\ y=0$

$$0=\frac{w}{6EI}[0]+C(0)+D \text{ gives } D=0$$

Substituting the other condition $x=L,\ y=0$ into (*)

$$0=\frac{w}{6EI}\left[\frac{L^5}{20}-\frac{L^5}{6}\right]+CL$$

$$=\frac{w}{6EI}\left[-\frac{7L^5}{60}\right]+CL$$

$$C=\frac{w}{6EIL}\left[\frac{7L^5}{60}\right]=\frac{7wL^4}{360EI}$$

Substituting $C=\frac{7wL^4}{360EI}$ and $D=0$ into (*)

$$y=\frac{w}{6EI}\left[\frac{x^5}{20}-\frac{L^2x^3}{6}\right]+\frac{7wL^4x}{360EI}$$

$$=\frac{w}{360EI}\left[3x^5-10L^2x^3\right]+\frac{7wL^4x}{360EI}$$

$$y=\frac{w}{360EI}\left[3x^5-10L^2x^3+7L^4x\right]$$

(9.13) $$\frac{d^2y}{dx^2}=\frac{M}{EI}$$