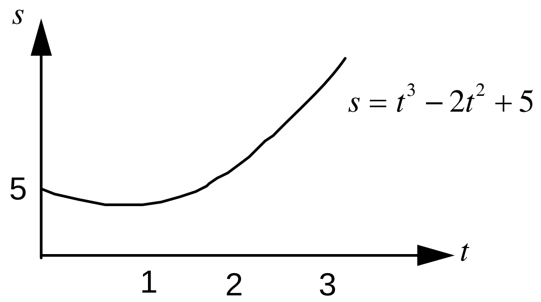
Complete solutions to Exercise 2(e)

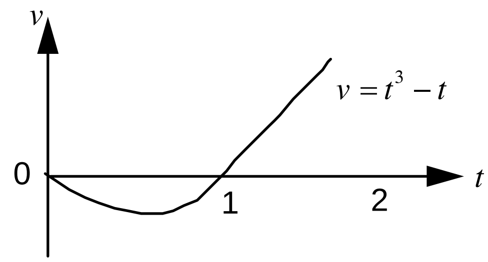
1. We establish a table of values, $s = t^3 - 2t^2 + 5$:

t	$s = t^3 - 2t^2 + 5$
0	$0^3 - (2 \times 0^2) + 5 = 5$
1	$1^3 - (2 \times 1^2) + 5 = 4$
2	$2^3 - (2 \times 2^2) + 5 = 5$
3	$3^3 - (2 \times 3^2) + 5 = 14$

Connecting these points gives the curve:

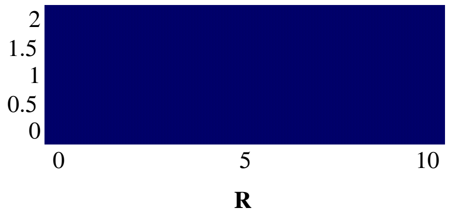


2. Similarly establishing a table of values for $t = 0, 0.5, 1, 1.5$ and 2 gives $v = 0, -0.375, 0, 1.875$ and 6 respectively. Connecting these points gives:



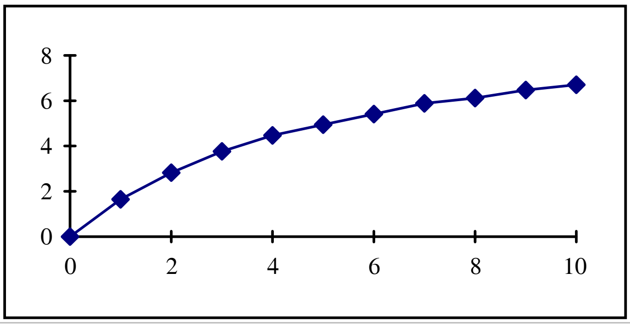
3. By putting $R = 0, 1, 2, \ldots, 9, 10$ into $I = \dfrac{10}{5+R}$ we have:

R	$I = \dfrac{10}{5+R}$
0	$I = \dfrac{10}{(5+0)} = \dfrac{10}{5} = 2$
1	$I = \dfrac{10}{(5+1)} = \dfrac{10}{6} = 1.67$
2	$10/7 = 1.43$
3	$10/8 = 1.25$
4	$10/9 = 1.11$
5	$10/10 = 1$
6	$10/11 = 0.91$
7	$10/12 = 0.83$
8	$10/13 = 0.77$
9	$10/14 = 0.71$
10	$10/15 = 0.67$



$$I = \frac{10}{5+R}$$

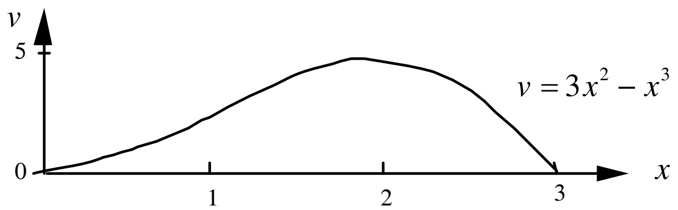
4. Similarly we have:



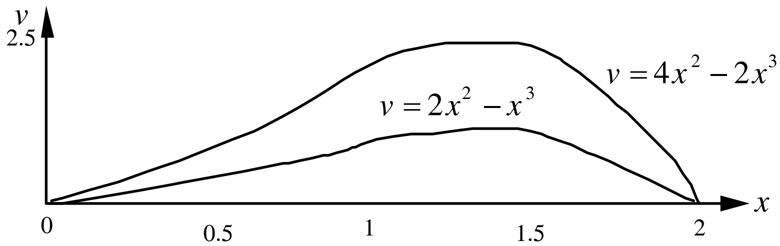
$$V = \frac{10R}{5+R}$$

5. By substituting given x values into $v = 3x^2 - x^3$ gives:

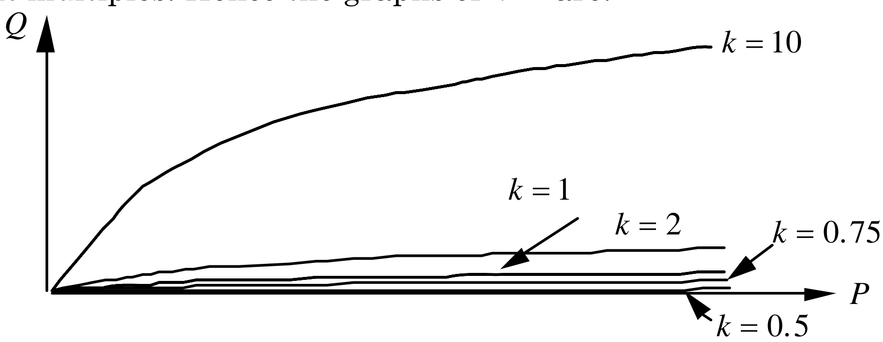
x	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$v = 3x^2 - x^3$	0	0.625	2	3.375	4	3.125	0



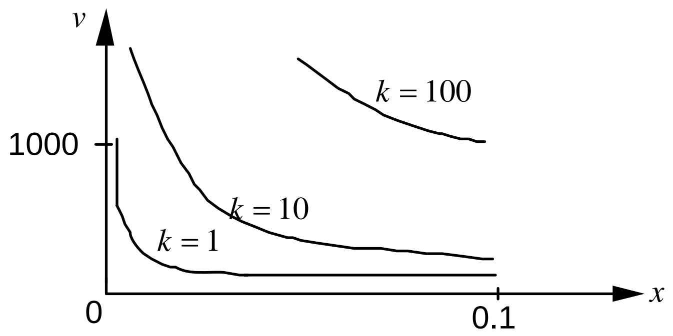
6. Similar graphs to question 5:



7. Since $P^{\frac{1}{2}}$ is $\sqrt{P}$ we have the $\sqrt{x}$ graph of Fig 27 (Chapter 2) with different multiples. Hence the graphs of $kP^{\frac{1}{2}}$ are:

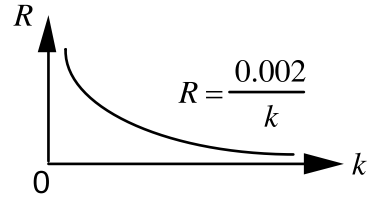


8. The graphs of $v = k/x$:

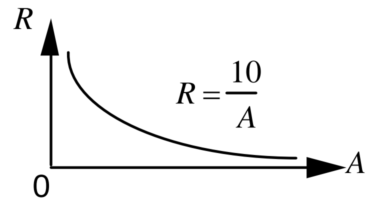


The larger k gives larger velocity, v, but still decreases with distance x.

9. We have $R = \frac{0.01}{5k} = \frac{0.002}{k}$, similar graph to $\frac{1}{x}$. As k increases, the thermal resistance, R, decreases and as k goes to infinity R goes to zero.



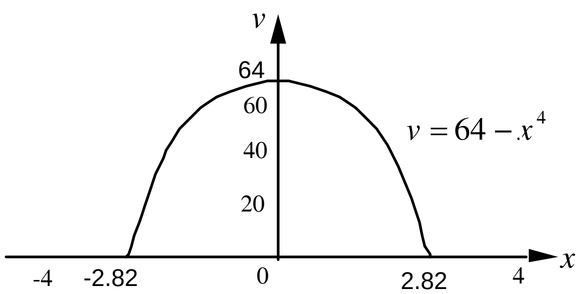
10. We have $R = \frac{0.05}{(5\times10^{-3})A} = \frac{10}{A}$. Similar to the graph of $\frac{1}{x}$. As A increases, R decreases.



11. The graph, $v = 64 - x^4$, is like x^4 but inverted and shifted up by 64 units. It crosses the x axis at $v = 0$:

$$64 - x^4 = 0$$
$$x^4 = 64$$
$$x = 2.82 \text{ or } -2.82$$

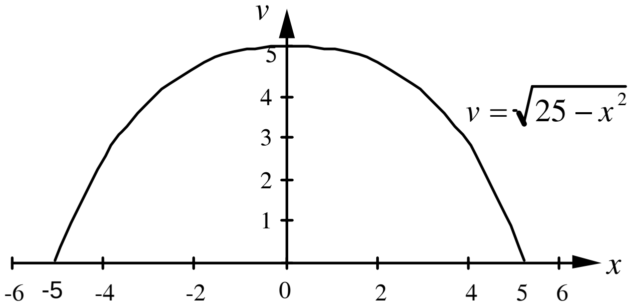
At $x = 0$, $v = 64 - 0^4 = 64$. Collecting these together gives the graph:



12. The graph, $v = \sqrt{25 - x^2}$, crosses the x axis at

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$x^2 = 25$$
$$x = 5 \text{ or } -5$$

At $x = 0$, $v = \sqrt{25} = 5$. Thus the graph between -5 and 5 for v is given by:



13. (Maple). Write each of the following and then press ENTER after each line:

(a) $\text{plot}((400*R)/(10+R)\text{^}2, R = 0..20);$

(b) $\text{plot}((500*R)/(10*(10\text{^}3)+R)\text{^}2, R = 0..20*(10\text{^}3));$

(c) $\text{plot}((2500*R)/(1500+R)\text{^}2, R = 0..3000);$

From the resulting graphs:
For (a) note that maximum P value occurs between 5 and 15, so we plot (a) again for $R = 5..15$. By using the mouse, we can see that the maximum P occurs when $R = 10$.
Similarly for (b) plot between $5*10^3$ and $15*10^3$, the graph shows maximum P occurs at $R = 10*10^3$.
For (c), maximum P occurs at $R = 1500$.
You should notice that if $P = \dfrac{aR}{(b+R)^2}$ then maximum P occurs when $R = b$.

14. You can use MAPLE or any other computer algebra package to plot the graphs.
For (a) at $R_L = 10$, $V = 30$ volt .
For (b) at $R_L = 3 \times 10^3$, $V = 7.5$ volt .
For (c) at $R_L = 15 \times 10^3$, $V = 5$ volt .
Notice that $V = \dfrac{E}{2}$, (half the e.m.f. voltage).
If $R_L = R$ then $V = \dfrac{ER_L}{R + R_L}$ becomes:

$$V = \frac{ER_L}{R_L + R_L} = \frac{ER_L}{2R_L} = \frac{E}{2} \quad \text{(cancelling the } R_L\text{'s)}$$