Complete solutions to Exercise 14(a)

1. Integrate each function twice and substitute the given conditions to obtain the following:

(a) $s=-4.9t^2$ (b) $s=1.63t^3$ (c) $s=ut+\frac{1}{2}at^2$

2. (a) Characteristic equation is given by

$$m^2+5m+6=0$$
$$(m+3)(m+2)=0$$
$$m_1=-3,\ m_2=-2$$

Since we have distinct roots so by (14.4) we have

$$y=Ae^{-3x}+Be^{-2x}$$

(b) Characteristic equation is $m^2+4m+4=0$

$$(m+2)^2=0,\ \text{hence } m=-2 \quad \text{[Equal Roots]}$$

Substituting $m=-2$ into, (14.5), $y=(A+Bx)e^{mx}$ gives

$$y=(A+Bx)e^{-2x}$$

(c) Characteristic equation is $m^2-2m+4=0$.
Using the quadratic formula (1.16) with $a=1$, $b=-2$ and $c=4$ gives

$$m=\frac{2\pm\sqrt{4-16}}{2}=1\pm\frac{1}{2}\sqrt{-12}=1\pm j\frac{\sqrt{12}}{2}$$
$$m=1\pm j\frac{\sqrt{4\times 3}}{2}=1\pm j\frac{\sqrt{4}\times\sqrt{3}}{2}=1\pm j\frac{2\sqrt{3}}{2}$$
$$m=1\pm j\sqrt{3}$$

Putting $\alpha=1$ and $\beta=\sqrt{3}$ into (14.6) gives $y=e^x\left[A\cos\left(\sqrt{3}x\right)+B\sin\left(\sqrt{3}x\right)\right]$

3. We will just state the characteristic equation and then the solution.
(a) We have

$$m^2+3m+2=0$$
$$(m+2)(m+1)=0$$
$$m_1=-2,\ m_2=-1 \quad \text{[Distinct Roots]}$$

By (14.4) the general solution is $y=Ae^{-2x}+Be^{-x}$

(1.16) $m=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

(14.4) If m_1 and m_2 then $y=Ae^{m_1x}+Be^{m_2x}$

(14.5) Equal roots m then $y=(A+Bx)e^{mx}$

(14.6) If $m=\alpha\pm j\beta$ then $y=e^{\alpha x}\left[A\cos(\beta x)+B\sin(\beta x)\right]$

(b) The characteristic equation is given by

$$m^2 - 7m + 6 = 0$$
$$(m-6)(m-1) = 0$$
$$m_1 = 6,\ m_2 = 1 \qquad [\text{Distinct Roots}]$$

By (14.4) the general solution is $y = Ae^{6x} + Be^{x}$

(c) We have

$$m^2 - 6m + 9 = 0$$
$$(m-3)^2 = 0 \text{ gives } m_1 = m_2 = 3 \qquad [\text{Equal Roots}]$$
$$y = (A + Bx)e^{3x}$$

(d) We have

$$m^2 - 4m + 5 = 0$$
$$m = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm j \qquad [\text{Complex Roots}]$$

Using (14.6) with $\alpha = 2$ and $\beta = 1$ gives $y = e^{2x}[A\cos(x) + B\sin(x)]$.

(e) We have

$$m^2 - 8m + 16 = 0$$
$$(m-4)^2 = 0 \text{ gives equal roots } m = 4$$

By (14.5) we have the general solution $y = (A + Bx)e^{4x}$

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4. (a) Characteristic equation is

$$m^2 + 6m + 9 = 0$$
$$(m+3)^2 = 0,\ m = -3$$

Since we have equal roots, $m = -3$, so by (14.5)

$$y = (A + Bx)e^{-3x}$$

(b) Characteristic equation is

$$10m^2 + 50m + 250 = 0$$
$$m^2 + 5m + 25 = 0 \qquad [\text{Dividing by 10}]$$

Putting $a = 1$, $b = 5$ and $c = 25$ into (1.16) gives

$$m = \frac{-5 \pm \sqrt{25 - (4 \times 25)}}{2} = -\frac{5}{2} \pm \frac{\sqrt{25(1-4)}}{2}$$
$$= -\frac{5}{2} \pm \frac{\sqrt{25}\sqrt{-3}}{2} = -\frac{5}{2} \pm j\frac{5\sqrt{3}}{2} = m \qquad [\text{Complex Roots}]$$

(1.16) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(14.4) If m_1 and m_2 then $y = Ae^{m_1 x} + Be^{m_2 x}$

(14.6) If $m = \alpha \pm j\beta$ then $y = e^{\alpha x}\left[A\cos(\beta x) + B\sin(\beta x)\right]$

(14.8) If $r^2 + k^2 = 0$ then $y = A\cos(kx) + B\sin(kx)$

Putting $\alpha = -\frac{5}{2}$ and $\beta = \frac{5\sqrt{3}}{2}$ into (14.6) gives

$$y = e^{-\frac{5}{2}x}\left[A\cos\left(\frac{5\sqrt{3}}{2}x\right) + B\sin\left(\frac{5\sqrt{3}}{2}x\right)\right]$$

(c) Characteristic equation is

$$-m^2 - 3m + 8 = 0$$

$$m^2 + 3m - 8 = 0 \qquad [\text{Multiplying by } -1]$$

Putting $a = 1$, $b = 3$ and $c = -8$ into (1.16)

$$m = \frac{-3 \pm \sqrt{9+32}}{2} = -\frac{3}{2} \pm \frac{\sqrt{41}}{2}$$

$$m_1 = \frac{-3+\sqrt{41}}{2}, \quad m_2 = \frac{-3-\sqrt{41}}{2}$$

By (14.4) $y = Ae^{m_1 x} + Be^{m_2 x}$ where m_1 and m_2 are as above .

(1.16) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(14.4) If m_1 and m_2 then $y = Ae^{m_1 x} + Be^{m_2 x}$