Complete solutions to Exercise 1(f)

1. a. $4(x+y+z)$ b. $8x(1+y)$ c . $2(x-2y)$ d. $x(3-2x)$

e. Since x is common in **both** terms therefore we have

$$x^2-xy=x(x-y)$$

f. *What is common in* $16x+4x^2$ *?*

Since $16x=4\times 4x$ therefore

$$16x+4x^2=(4\times 4x)+4x^2=4x(4+x)$$

g. *What is common between the two terms in the expression* $9x^2-27x^3$ *?*

The numbers 9 and 27 have 9 in common and x^2 and x^3 have x^2 in common. Hence

$$9x^2-27x^3=9x^2-(9x^2\times 3x)$$

$$=9x^2(1-3x)$$

2. Taking out the common factor in each term:

(a) $s=t\left(u+\frac{1}{2}at\right)$ (b) $F=\frac{m}{t}(v_2-v_1)$ (c) $F=\rho Av_1v_2-\rho Av_1v_1=\rho Av_1(v_2-v_1)$

3. We have $S=\pi r\left[r+\left(r^2+h^2\right)^{1/2}\right]$

4. Similar to **EXAMPLES 24** and **25**.

(a) $(x+5)(x+2)$ (b) $(x+4)(x+1)$ (c) $(x-4)(x-1)$ (d) $(x+2)(x-6)$

(e) $(2x-1)(x+1)$ (f) $(x-4)(x+1)$ (g) $(3x+5)(7x-2)$

(h) *How do we factorize* $6x^2+x-12$ *?*

$$6x^2+x-12=(3x-4)(2x+3)$$

5. We have (a) $x^2-2x+1=(x-1)^2$ (b) $x^2+2x+1=(x+1)^2$

(c) *How do we factorize the given quadratic* x^2-36 *?*

x^2-36 is the difference of two squares which means we can use

(1.15) $a^2-b^2=(a-b)(a+b)$

Hence we have $x^2-36=x^2-6^2=(x-6)(x+6)$.

(d) Similarly by applying (1.15) and writing $\left(\sqrt{7}\right)^2=7$ we have

$$x^2-7=x^2-\left(\sqrt{7}\right)^2=\left(x-\sqrt{7}\right)\left(x+\sqrt{7}\right)$$

(e) This expression $4x^2+12x+9$ is more difficult to factorize:

$$4x^2+12x+9=(2x+3)(2x+3)=(2x+3)^2$$

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6. Use (1.15) for both cases:

(a) $Z^2-R^2=(Z-R)(Z+R)$ (b) $\omega^2L^2-\frac{1}{\omega^2C^2}=\left(\omega L-\frac{1}{\omega C}\right)\left(\omega L+\frac{1}{\omega C}\right)$

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7. $F=\frac{2(VV_s-V^2)}{V_s^{\,2}-V^2}=\frac{2V(V_s-V)}{\underbrace{(V_s-V)(V_s+V)}_{\text{by }(1.15)}}=\frac{2V}{V_s+V}$ $\left[(1.15)\quad a^2-b^2=(a-b)(a+b)\right]$

(Cancelling the common term $(\mathrm{V_s}-\mathrm{V})$ in the numerator and denominator).

8. (a) Factorizing

$$\frac{3wLx^2}{6EI}-\frac{wx^3}{6EI}=\frac{3wLx^2}{6EI}-\frac{wx^2x}{6EI}$$

$$=\underbrace{\frac{wx^2}{6EI}}_{\text{taking out the common factor}}(3L-x)$$

(b) In a similar manner

$$\frac{wLx^3}{4EI}-\frac{3wx^4}{8EI}=\frac{2wLx^3}{8EI}-\frac{3wx^3x}{8EI}$$

$$=\frac{wx^3}{8EI}(2L-3x)$$

(c) Also

$$\frac{wx^4}{24EI}-\frac{wLx^3}{12EI}+\frac{wL^2x^2}{24EI}=\frac{wx^2x^2}{24EI}-\frac{2wLx^2x}{24EI}+\frac{wL^2x^2}{24EI}$$

$$=\frac{wx^2}{24EI}(x^2)-\frac{wx^2}{24EI}(2Lx)+\frac{wx^2}{24EI}L^2$$

$$=\frac{wx^2}{24EI}\left(x^2-2Lx+L^2\right)$$

How do we factorize the bracket term, $x^2-2Lx+L^2$*?*

We can use, (1.14), $a^2-2ab+b^2=(a-b)^2$

$$x^2-2Lx+L^2=(x-L)^2$$

So we have

$$\frac{wx^4}{24EI}-\frac{wLx^3}{12EI}+\frac{wL^2x^2}{24EI}=\frac{wx^2}{24EI}(x-L)^2$$

9. Very similar to **EXAMPLE 26**.

10. We have x and w which is common in every term of the numerator and $4EI$ common on the denominator (4 goes into 8, 12 and 24). Hence

$$y=\frac{wx}{4EI}\left(\frac{x^2}{3}\right)-\frac{wx}{4EI}\left(\frac{lx}{2}\right)+\frac{wx}{4EI}\left(\frac{l^2}{6}\right)$$

$$=\frac{wx}{4EI}\left(\frac{x^2}{3}-\frac{lx}{2}+\frac{l^2}{6}\right) \qquad (\dagger)$$

How do we handle the terms inside the bracket $\frac{x^2}{3}-\frac{lx}{2}+\frac{l^2}{6}$?

We need to determine the Lowest Common Multiple of 2, 3 and 6. Clearly it is 6. So

$$\frac{x^2}{3}-\frac{lx}{2}+\frac{l^2}{6}=\frac{2x^2}{6}-\frac{3lx}{6}+\frac{l^2}{6}$$

$$=\frac{2x^2-3lx+l^2}{6}$$

Substituting this into (†) gives

$$y = \frac{wx}{4EI}\left(\frac{2x^2 - 3lx + l^2}{6}\right)$$

$$= \frac{wx}{24EI}\left(2x^2 - 3lx + l^2\right)$$

We need to factorize the terms in the bracket, $2x^2 - 3lx + l^2 = (2x-l)(x-l)$

Hence our result: $y = \frac{wx}{24EI}(2x-l)(x-l)$

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11. (a) We have $\mathrm{N} = \dfrac{\mathrm{Z}_0 + \frac{1}{2}\mathrm{Z}_1}{\mathrm{Z}_0 - \frac{1}{2}\mathrm{Z}_1}$, multiply the numerator and denominator by 2:

$$N = \frac{2Z_0 + Z_1}{2Z_0 - Z_1}$$

Multiplying both sides by $2Z_0 - Z_1$ and expanding gives

$$\left(2Z_0 - Z_1\right)N = 2Z_0 + Z_1$$

$$2Z_0N - Z_1N = 2Z_0 + Z_1$$

Collecting the Z_0 terms on the Left Hand Side and Z_1 terms on the Right Hand Side:

$$2Z_0N - 2Z_0 = Z_1N + Z_1$$

Factorizing: $$2Z_0\left(N-1\right) = Z_1\left(N+1\right)$$

Thus $$Z_1 = \frac{2Z_0\left(N-1\right)}{N+1} = 2Z_0\left(\frac{N-1}{N+1}\right)$$

(b) We have

$$Z_1\left(N-1\right)^2 + 2Z_0\left(N^2-1\right) = Z_1\left(N+1\right)^2$$

$$2Z_0\left(N^2-1\right) = Z_1\underbrace{\left[\left(N+1\right)^2 - \left(N-1\right)^2\right]}_{=4N}$$

$$= 4NZ_1$$

Hence $Z_1 = \dfrac{2Z_0\left(N^2-1\right)}{4N} = \dfrac{Z_0\left(N^2-1\right)}{2N} = Z_0\left(\dfrac{N^2-1}{2N}\right)$