Complete solutions to Exercise 1(g)

1. (a) $x = 1/2$

(b) We need to factorize $x^2 + 5x + 6 = 0$. *How?*

$$x^2 + 5x + 6 = (x+3)(x+2) = 0$$

Therefore $x = -2,\ x = -3$.

(c) Factorizing $x^2 - 10x + 21$ gives

$$x^2 - 10x + 21 = (x-3)(x-7) = 0$$

This means that $x = 3,\ x = 7$.

(d) Factorizing the given quadratic $6x^2 - 13x - 5$ gives

$$6x^2 - 13x - 5 = (2x-5)(3x+1) = 0$$

We have

$$(3x+1) = 0 \text{ or } (2x-5) = 0$$

$$3x = -1 \text{ or } 2x = 5$$

$$x = -\frac{1}{3} \text{ or } x = \frac{5}{2}$$

(e) In a similar manner we have $5x^2 + 14x - 3 = (5x-1)(x+3) = 0$. Solving this

$$(5x-1)(x+3) = 0 \text{ implies that } x = \frac{1}{5},\ x = -3$$

(f) *How do we factorize* $x^2 - 1$*?*

Use the difference of two squares (1.15) $a^2 - b^2 = (a-b)(a+b)$:

$$x^2 - 1 = (x-1)(x+1) = 0$$

Therefore $x = 1,\ \ x = -1$.

(g) Similarly we have

$$x^2 - 2x + 1 = (x-1)^2 = 0$$

This means $x = 1$.

(h) The factorization in this case is more difficult:

$$4x^2 + 8x + 4 = (2x+2)(2x+2) = (2x+2)^2 = 0$$

How do we solve the equation $(2x+2)^2 = 0$ *?*

$$(2x+2)^2 = 0 \text{ implies } 2x+2 = 0 \text{ which gives } x = -1.$$

Our solution is $x = -1$.

(i) *How do we solve the given quadratic equation* $3x^2 + 9x + 6 = 0$ *?*

By taking the elementary step of dividing this equation by 3 we have

$$x^2 + 3x + 2 = 0$$

It is easier to factorize $x^2 + 3x + 2 = 0$:

$$x^2 + 3x + 2 = (x+1)(x+2) = 0$$

Therefore $x = -1,\ \ x = -2$.

(j) Again life is easier if we divide the given quadratic $-2x^2 + 6x - 4 = 0$ by -2:

$$x^2 - 3x + 2 = 0$$

How do we solve this quadratic $x^2 - 3x + 2 = 0$ *?*

Factorizing gives $x^2 - 3x + 2 = (x-1)(x-2) = 0$ which implies $x = 1,\ x = 2$.

2. Substituting $v = 35,\ a = 3.2$ and $s = 187$ into $u^2 + 2as = v^2$ gives

$$u^2 + (2 \times 3.2 \times 187) = 35^2$$

$$u^2 = 35^2 - (2 \times 3.2 \times 187) = 28.2$$

$$u = \sqrt{28.2} = 5.31 m/s \quad (2 \text{ d.p.})$$

3. Factorizing : $2x^2 - 3xL + L^2 = (2x - L)(x - L)$

$$(2x - L)(x - L) = 0$$

gives $2x - L = 0$ or $x - L = 0$. Thus

$$x = \frac{L}{2} \text{ or } x = L.$$

4. We have $4.3t^2 + 1.9t = 50$. Rearranging

$$4.3t^2 + 1.9t - 50 = 0$$

Using (1.16) $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 4.3,\ b = 1.9$ and $c = -50$ gives

$$t = \frac{-1.9 \pm \sqrt{1.9^2 - (4 \times 4.3 \times (-50))}}{2 \times 4.3}$$

$$= \frac{-1.9 \pm \sqrt{3.61 + 860}}{8.6}$$

$$= \frac{-1.9 \pm 29.387}{8.6}$$

$$t = \frac{-1.9 + 29.387}{8.6} \quad \text{or} \quad \frac{-1.9 - 29.387}{8.6}$$

$$t = 3.20s \quad \text{or} \quad t = -3.64s$$

Since $t \geq 0$ so $t = 3.20s$ (2 d.p.).

5. Let w be the width then $w = \ell - 5$ and

$$\ell(\ell - 5) = 84$$

$$\ell^2 - 5\ell - 84 = 0$$

$$(\ell - 12)(\ell + 7) = 0$$

$$\ell = 12 \quad \text{or} \quad \ell = -7$$

Dimensions are $\ell = 12$m and $w = 12 - 5 = 7$m.

6. Putting $M = 0$ gives $-20x^2 - 500x + 3000 = 0$

Dividing by -20: $x^2 + 25x - 150 = 0$

Factorizing

$$(x - 5)(x + 30) = 0$$

$$x = 5 \text{ or } x = -30$$

Hence $x = 5$ gives $M = 0$.

7. $$M = \frac{15}{8}x - \frac{29}{4}\left(x-\frac{1}{2}\right)^2 = \frac{15}{8}x - \frac{58}{8}\left(x-\frac{1}{2}\right)^2$$

$$= \frac{1}{8}\left[15x - 58\underbrace{(x^2 - x + 0.25)}_{\text{by } (1.14)}\right]$$

$$= \frac{1}{8}\left[15x - 58x^2 + 58x - 14.5\right]$$

$$= \frac{1}{8}\left[73x - 58x^2 - 14.5\right]$$

With $M = 0$ we have

$$-58x^2 + 73x - 14.5 = 0$$

Multiplying by -1 gives $58x^2 - 73x + 14.5 = 0$

Putting $a = 58$, $b = -73$ and $c = 14.5$ into (1.16) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives

$$x = \frac{73 \pm \sqrt{(-73)^2 - (4 \times 58 \times 14.5)}}{2 \times 58}$$

$$= 1.011 \qquad \text{or} \qquad 0.247$$

$x = 1.01m$ (2 d.p.) or $x = 0.25m$ (2 d.p.)

8. Putting $h = 0$ gives $-4.9t^2 + 55t + 12 = 0$. So solving produces $t = 11.44s$

9. We have $h = 0$ because h is the height above ground, so

$$-\frac{1}{2}gt^2 + ut + h_0 = 0$$

Multiplying by -1: $\frac{1}{2}gt^2 - ut - h_0 = 0$

Putting $a = \frac{1}{2}g$, $b = -u$ and $c = -h_0$ into (1.16) gives

$$t = \frac{u \pm \sqrt{(-u)^2 - \left(4 \times \frac{1}{2}g \times (-h_0)\right)}}{2 \times \frac{1}{2}g}$$

$$t = \frac{u \pm \sqrt{u^2 + 2gh_0}}{g}$$

(1.14) $(a-b)^2 = a^2 - 2ab + b^2$

(1.16) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

10. We have

$$\frac{-T}{2wL^{\frac{3}{2}}} = \frac{4kL^{\frac{5}{2}} - 3DL^{\frac{1}{2}} - 3kL^{\frac{5}{2}}}{2L^3}$$

$$= \frac{kL^{\frac{5}{2}} - 3DL^{\frac{1}{2}}}{2L^3}$$

$$\underbrace{=}_{\substack{\text{dividing numerator}\\ \text{and denominator by}\\ L^3}} \frac{kL^{-\frac{1}{2}} - 3DL^{-\frac{5}{2}}}{2}$$

$$\frac{kL^{-\frac{1}{2}} - 3DL^{-\frac{5}{2}}}{2} + \frac{T}{2wL^{\frac{3}{2}}} = 0$$

Multiplying by $2L^{\frac{5}{2}}$ gives a quadratic

$$kL^2 - 3D + \frac{T}{w}L = 0$$

$$kL^2 + \frac{T}{w}L - 3D = 0$$

How do we solve this quadratic?

Use (1.16) with $a = k$, $b = \frac{T}{w}$ and $c = -3D$:

$$L = \frac{-\frac{T}{w} \pm \sqrt{\frac{T^2}{w^2} + (4k \times 3D)}}{2k}$$

$$= \frac{-\frac{T}{w} \pm \sqrt{\frac{T^2 + (4k \times 3D)w^2}{w^2}}}{2k}$$

$$= \frac{-\frac{T}{w} \pm \frac{1}{w}\sqrt{T^2 + 12kDw^2}}{2k}$$

$$L = \frac{-T \pm \sqrt{T^2 + 12kDw^2}}{2kw}$$

(1.16) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$