Complete solutions to Exercise 1(h)

1. (a) Similar to **EXAMPLE 30**, $x=1,\ y=1$.

(b) We add the two given simultaneous equations:

$$\begin{array}{rr} & x+y=2 \\ + & x-y=0 \\ \hline & 2x+0=2 \end{array}$$

We have $2x=2$ which gives $x=1$. *How do we find the value of the other unknown, y?*

Substitute $x=1$ into the first equation $x+y=2$:

$$1+y=2 \text{ which gives } y=1$$

Hence $x=1$ and $y=1$ is the solution.

(c) We can label each linear equation

$$2x-3y=5 \qquad (*)$$

$$x-y=2 \qquad (**)$$

How do we eliminate one of the unknowns?

Eliminate x by multiplying equation (**) by 2 and subtracting from equation (*):

$$\begin{array}{rrl} & 2x-3y=5 & (*) \\ - & 2x-2y=4 & [\text{Multiplying } (**) \text{ by } 2] \\ \hline & 0-y=1 & \end{array}$$

Hence $y=-1$. *How do we determine x?*

By substituting $y=-1$ into the given equation (**):

$$x-(-1)=2$$

$$x+1=2 \text{ which gives } x=1$$

The solution is $x=1$ and $y=-1$.

(d) We label each equation:

$$2x-3y=35 \qquad (\$)$$

$$x-y=2 \qquad (\$\$)$$

We need to eliminate one of the unknowns. *Which one?*

To make life easier it is better to eliminate x. *How?*

Multiply equation ($$) by 2 and subtract from equation ($):

$$\begin{array}{rrl} & 2x-3y=35 & (\$) \\ - & 2x-2y=4 & [\text{Multiplying } (\$\$) \text{ by } 2] \\ \hline & 0-y=31 & \end{array}$$

We have $y=-31$. *How do we find x?*

Substitute $y=-31$ into the given equation ($$):

$$x-(-31)=2$$

$$x+31=2 \text{ which gives } x=-29$$

The solution is $x=-29$ and $y=-31$.

(e) We label each equation:

$$5x-7y=2 \qquad (\dagger)$$

$$9x-3y=6 \qquad (\dagger\dagger)$$

We need to eliminate one of the unknowns. *Which one?*
Eliminate *y*. *How?*
Multiply equation (†) by 3 and multiply (††) by 7:

$$15x - 21y = 6 \qquad \left[\text{Multiplying } (\dagger) \text{ by } 3\right]$$

$$63x - 21y = 42 \qquad \left[\text{Multiplying } (\dagger\dagger) \text{ by } 7\right]$$

To eliminate *y* we subtract these equations

$$\begin{array}{rr} & 63x - 21y = 42 \\ - & 15x - 21y = 6 \\ \hline & 48x - 0 = 36 \end{array} \qquad \left[\text{Subtracting}\right]$$

From $48x = 36$ we have $x = \frac{36}{48} = \frac{3}{4}$. *How do we find y?*

Substitute $x = \frac{3}{4}$ into the given equation (†):

$$5\left(\frac{3}{4}\right) - 7y = 2$$

$$\frac{15}{4} - 7y = 2$$

$$7y = \frac{15}{4} - 2 = \frac{7}{4}$$

How do find y from $7y = \frac{7}{4}$ *?*

Dividing both sides by 7:

$$y = \frac{\cancel{7}}{4(\cancel{7})} = \frac{1}{4}$$

The solution is $x = \frac{3}{4}$ and $y = \frac{1}{4}$.

(f) We have the equations

$$\pi x - 5y = 2 \qquad (*)$$

$$\pi x - y = 1 \qquad (**)$$

Subtracting these equations we have

$$\begin{array}{rr} & \pi x - 5y = 2 \\ - & \pi x - y = 1 \\ \hline & 0 - 4y = 1 \end{array}$$

From the last line $-4y = 1$ we have $y = -\frac{1}{4}$. *How do we determine x?*

Substitute $y = -\frac{1}{4}$ into (**):

$$\pi x - \left(-\frac{1}{4}\right) = 1$$

$$\pi x + \frac{1}{4} = 1 \qquad \pi x = \frac{3}{4} \quad \text{which gives } x = \frac{3}{4\pi}$$

The solution is $x = \frac{3}{4\pi}$ and $y = -\frac{1}{4}$.

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2. Putting $\mathrm{E} = 53, \mathrm{W} = 120$ into $E = aW + b$ gives

$$53 = 120a + b \qquad (\dagger)$$

and $\mathrm{E} = 45.5, \mathrm{W} = 70$ into $E = aW + b$ gives

$$45.5 = 70\mathrm{a} + \mathrm{b} \qquad (\dagger\dagger)$$

Subtract (††) from (†)

$$7.5 = 50a$$

Hence $$a = \frac{7.5}{50} = 0.15$$

Substituting $\mathrm{a} = 0.15$ into (†) gives

$$(120 \times 0.15) + b = 53$$

$$18 + b = 53$$

$$b = 35 \text{ N}$$

Putting $a = 0.15$ and $b = 35$ into $E = aW + b$ gives $\mathrm{E} = 0.15\mathrm{W} + 35$

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3. Putting $t = 2$, $s = 33$ into $ut + \frac{1}{2}at^2 = s$ gives

$$2u + \left(\frac{1}{2}a2^2\right) = 33$$

$$2u + 2a = 33 \qquad (\dagger)$$

Putting $\mathrm{t} = 3$, $\mathrm{s} = 64.5$ into $ut + \frac{1}{2}at^2 = s$ gives

$$3u + \frac{3^2}{2}a = 64.5$$

$$3u + 4.5a = 64.5 \qquad (\dagger\dagger)$$

Multiply (†) by 3

$$6u + 6a = 99 \qquad (*)$$

Multiply (††) by 2

$$6u + 9a = 129 \qquad (**)$$

(**) – (*) gives

$$3a = 30$$

$$a = 10$$

Substituting $\mathrm{a} = 10$ into (†)

$$2\mathrm{u} + 20 = 33$$

$$2\mathrm{u} = 13$$

$$\mathrm{u} = 6.5$$

$$a = 10 \ \mathrm{m/s^2} \text{ and } u = 6.5 \ \mathrm{m/s}$$

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4. Opening up the brackets gives

$$25I_1 - 25I_2 + 56I_1 = 2.225$$

$$17I_2 - 3I_1 + 3I_2 = 1.31$$

Simplifying

$$81I_1 - 25I_2 = 2.225$$
$$-3I_1 + 20I_2 = 1.31$$

Solving these gives $I_1 = 50$ mA and $I_2 = 73$ mA

5. We have

$$\ell_0(1+55\alpha) = 20.11 \qquad (\dagger)$$
$$\ell_0(1+120\alpha) = 20.24 \qquad (\dagger\dagger)$$

$(\dagger)$ divided by $(\dagger\dagger)$ gives

$$\frac{\ell_0(1+55\alpha)}{\ell_0(1+120\alpha)} = \frac{20.11}{20.24}$$
$$20.24(1+55\alpha) = 20.11(1+120\alpha)$$
$$20.24 + 1113.2\alpha = 20.11 + 2413.2\alpha$$
$$20.24 - 20.11 = 2413.2\alpha - 1113.2\alpha$$
$$0.13 = 1300\alpha$$
$$\alpha = \frac{0.13}{1300} = 1\times10^{-4}$$

Substituting $\alpha = 1\times10^{-4}$ into $(\dagger\dagger)$ results in

$$\ell_0\left(1+\left(120\times1\times10^{-4}\right)\right) = 20.24$$
$$1.012\ell_0 = 20.24$$
$$\ell_0 = 20$$

Hence $\ell_0 = 20m$ and $\alpha = 1\times10^{-4}/{}^\circ C$

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6. From the first formula we have $\frac{1}{R_1} = \left(1.2\times10^{-3}\right) - \frac{1}{R_2}$ $(\dagger)$

Substituting this into

$$\frac{5}{R_1} + \frac{8}{R_2} = 5\left(\frac{1}{R_1}\right) + \frac{8}{R_2} = 6.6\times10^{-3}$$

gives

$$5\left(\left(1.2\times10^{-3}\right) - \frac{1}{R_2}\right) + \frac{8}{R_2} = 6.6\times10^{-3}$$
$$\left(6\times10^{-3}\right) + \frac{3}{R_2} = \left(6.6\times10^{-3}\right)$$
$$\frac{3}{R_2} = \left(6.6\times10^{-3}\right) - \left(6\times10^{-3}\right) = 6\times10^{-4}$$
$$R_2 = \frac{3}{6\times10^{-4}} = 5000$$

Substituting R_2 into $(\dagger)$ gives

$$\frac{1}{R_1} = \left(1.2\times10^{-3}\right) - \frac{1}{5000} = 0.001$$
$$R_1 = \frac{1}{0.001} = 1000$$
$$R_1 = 1k\Omega \text{ and } R_2 = 5k\Omega$$

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7. Using TABLE 1 and considering dimensions

$$MLT^{-2} = \left(ML^{-3}\right)^a \left(L^2\right)^b \left(LT^{-1}\right)^c$$

$$\underbrace{=}_{\substack{\text{using rules}\\ \text{of indices}}} M^a L^{-3a} L^{2b} L^c T^{-c}$$

$$MLT^{-2} = M^a \underbrace{L^{-3a+2b+c}}_{\text{by }(1.5)} T^{-c}$$

Equating powers of M, L and T respectively gives

$$1 = a$$

$$1 = -3a + 2b + c \qquad (*)$$

$$-2 = -c \text{ so } c = 2$$

Substituting $a = 1$, $c = 2$ into (*)

$$-3 + 2b + 2 = 1$$

$$b = 1$$

Substituting $a = 1$, $b = 1$ and $c = 2$ into $F = K\rho^a A^b v^c$ gives

$$F = K\rho A v^2$$