Complete solutions to Exercise 4(a)

1. Using a calculator we have 0.57, 0.68 and 0.38

2. We use TABLE 1 for this question:

(i) $\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$ (ii) $\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}+1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

(iii) $\frac{\sqrt{3}}{2} + \sqrt{3} = \sqrt{3}\left(\frac{1}{2}+1\right) = \frac{3\sqrt{3}}{2}$ (iv) $\frac{\left(1/\sqrt{2}\right)}{1 \cdot \left(1/\sqrt{2}\right)} = 1$

(v) $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{\left(\sqrt{2}\right)^2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{3}}{2}$

(vi) $\left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$

3. The $4m$ is opposite angle $25°$, and we need to find the hypotenuse AC. Using (4.7) gives:

$$\text{length AC} = \frac{4}{\sin(25°)} = 9.46m \quad \text{(2 d.p.)}$$

Let X be the midpoint of AB in Fig 13. Then we can find AX by using Pythagoras's theorem:

$$\text{AX} = \sqrt{9.46^2 - 4^2} = 8.57m \quad \text{(2 d.p.)}$$

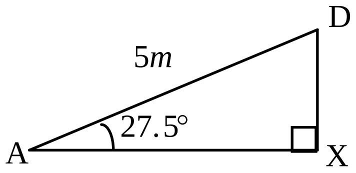
Span AB$= 2 \times 8.57 = 17.15m$.

4. We have

$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{opp/hyp}{adj/hyp} \underbrace{=}_{\text{cancelling the } hyp's} \frac{opp}{adj} \underbrace{=}_{\text{by (4.3)}} \tan(\theta)$$

5. By (4.3) we have $\tan(\theta) = \frac{3}{6}$, gives $\theta = \tan^{-1}\left(\frac{3}{6}\right) = 26.57°$.

6. Consider triangle AXD, angle DAX is half of $55°$ which is $27.5°$.

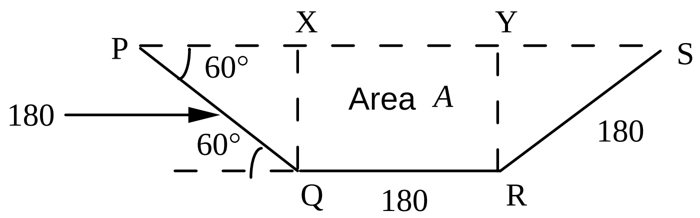


DX is opposite to $27.5°$,
so using (4.4):

$DX = 5 \sin 27.5° = 2.3087$

length $h = 2 \times \text{DX} = 2 \times 2.3087 = 4.62m$

7. We have:



The area of triangle RYS is the same as the area of triangle PXQ, thus

Area $A = (2\times$ Area of triangle PXQ$)+$ (Area of rectangle QXYR)

Length PX is adjacent to the angle P$=60°$, so by adj $=$ hyp $\times \cos(\theta)$ we have

$$PX = 180 \times \cos(60°) = 90 \text{ mm}$$

(4.3)	$\tan(\theta) = opp/adj$
(4.4)	$opp = hyp \times \sin(\theta)$
(4.7)	$hyp = opp/\sin(\theta)$

Length QX is opposite to the angle P=$60°$, so by (4.4),

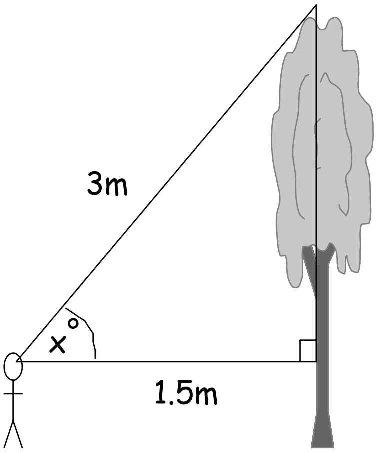
$$QX = 180 \times \sin(60°) \underbrace{=}_{\text{by table 1}} \frac{180 \times \sqrt{3}}{2} = 90\sqrt{3} \text{ mm}$$

Area of triangle PXQ=$\frac{1}{2}\left(90 \times 90\sqrt{3}\right)$, area of rectangle QXYR=$180 \times 90\sqrt{3}$

Total area $A = \left(2 \times \frac{1}{2} \times 90 \times 90\sqrt{3}\right) + \left(180 \times 90\sqrt{3}\right) = 42088.83 = 42.1 \times 10^{3} \text{ mm}^{2}$

The final answer is correct to 3 s.f.

8. (a) We are given that:



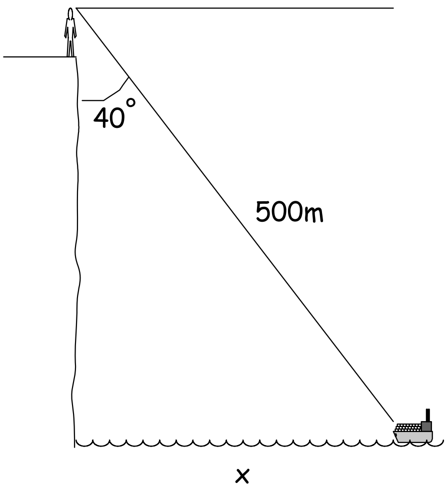
By applying the cosine we have

$$\cos(x) = \frac{1.5}{3} = 0.5$$

Taking inverse cosine gives

$$x = \cos^{-1}(0.5) = 60°$$

(b)



How can we find x?

The length x is opposite the given angle and we have the hypotenuse length of 500m

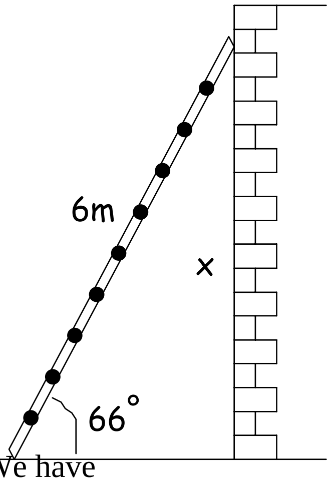
Hence

$$\sin(40°) = \frac{x}{500}$$

Transposing gives

$$x = 500\sin(40°) = 321.39\text{ m}$$

(c) We are given the diagram:



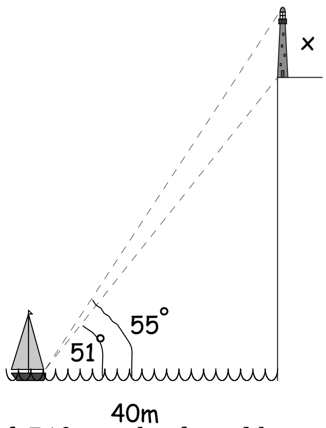
How can we find x?
This is the same as part (b). We have

$$\sin(66°) = \frac{x}{6}$$

Transposing gives

$$x = 6\sin(66°) = 5.48\text{ m}$$

(d) This problem is slightly more challenging. We have



The height h_1 at an angle of $51°$ can be found by using

$$\tan(51°) = \frac{h_1}{40}$$

Transposing this gives

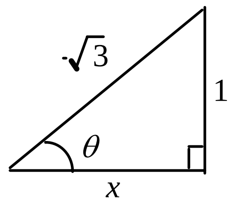
$$h_1 = 40\tan(51°) = 49.4$$

Similarly

$$h_2 = 40\tan(55°) = 57.13$$

Hence $x = h_2 - h_1 = 57.13 - 49.4 = 7.73\text{ m}$.

9. We place all the known numbers in a triangle and evaluate the unknown side, x, by using Pythagoras. Then we apply (4.1)-(4.3):
(i) We have the diagram:



From this we deduce the following:

$$x = \sqrt{\left(\sqrt{3}\right)^2 - 1^2} = \sqrt{2}$$

$$\cos(\theta) \underbrace{=}_{\text{by (4.2)}} \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$

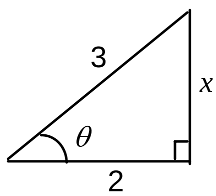
$$\tan(\theta) \underbrace{=}_{\text{by (4.3)}} \frac{1}{\sqrt{2}}$$

$$\operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\sqrt{2/3}} = \sqrt{\frac{3}{2}}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

(ii) We have



Evaluating the trigonometric ratios we have

$$x = \sqrt{3^2 - 2^2} = \sqrt{5}$$

$$\sin(\theta) \underbrace{=}_{\text{by (4.1)}} \frac{\sqrt{5}}{3} \qquad \tan(\theta) \underbrace{=}_{\text{by (4.3)}} \frac{\sqrt{5}}{2}$$

$$\operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)} = \frac{3}{\sqrt{5}}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{2/3} = \frac{3}{2}$$

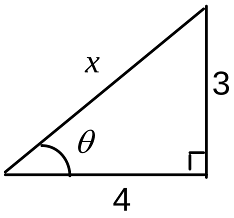
$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{\sqrt{5}/2} = \frac{2}{\sqrt{5}}$$

(4.1) $\sin(\theta) = opp/hyp$

(4.2) $\cos(\theta) = adj/hyp$

(4.3) $\tan(\theta) = opp/adj$

(iii) Similarly we have



Using this triangle:

$$x = \sqrt{3^2 + 4^2} = 5$$

$$\sin(\theta) \underbrace{=}_{\text{by (4.1)}} \frac{3}{5} \qquad \cos(\theta) \underbrace{=}_{\text{by (4.2)}} \frac{4}{5}$$

$$\operatorname{cosec}(\theta) = \frac{5}{3}, \ \sec(\theta) = \frac{5}{4} \ \text{ and } \ \cot(\theta) = \frac{4}{3}$$

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10. Using a calculator and rounding to 3 d.p. we have

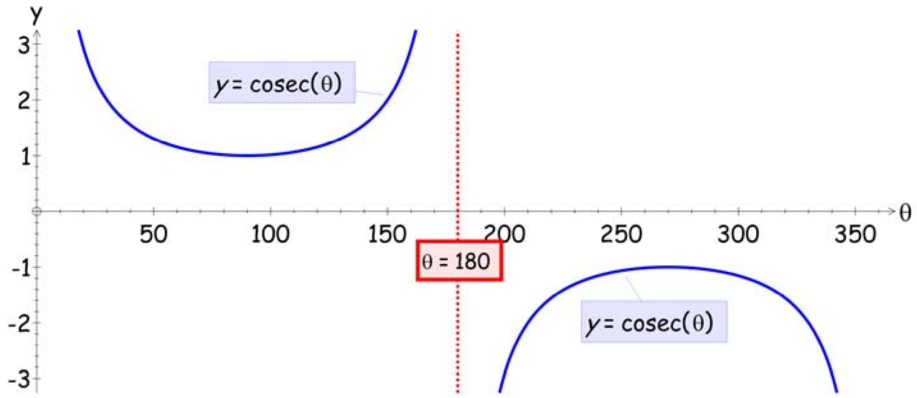
$$\sec(36.4°) = \frac{1}{\cos(36.4°)} = 1.242$$

$$\cot(29.17°) = \frac{1}{\tan(29.17°)} = 1.791$$

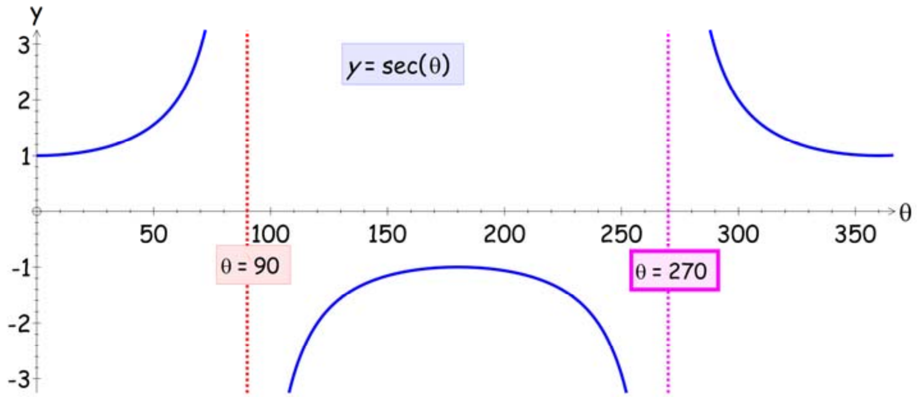
$$\operatorname{cosec}(44.44°) = \frac{1}{\sin(44.44°)} = 1.428$$

11. We have the following graphs:

(a)



(b)



(c) The graph of $y = \cot(\theta)$ is