Complete solutions to Exercise 5(c)

1. We use our calculator:

(a) $G=\ln\left(\frac{0.1}{0.01}\right)=2.30\,nep$ (b) $G=\ln\left(\frac{10}{0.1}\right)=4.61\ nep$

(c) $G=\ln\left(\frac{5}{15}\right)=-1.10\,nep$

2. (a) $G=\frac{1}{2}\ln\left(\frac{1}{0.2}\right)=0.80\,nep$ (b) $G=\frac{1}{2}\ln(20)=1.50\,nep$

3. Remember $1mV=1\times10^{-3}V$.

$$G=20\log\left(\frac{1}{1\times10^{-3}}\right)=20\log\left(10^{3}\right)\underbrace{=}_{\text{by }(5.19)}(3\times20)\underbrace{\log(10)}_{=1}=60\,\text{dB}$$

4. We use $G=10\log\left(P_{out}/P_{in}\right)$ with $G=5$ and $P_{in}=0.1$:

$$10\ \log\left(\frac{P_{out}}{0.1}\right)=5$$

$$\log\left(\frac{P_{out}}{0.1}\right)=\frac{5}{10}=0.5$$

How do remove the $\log$ function?
Taking exponentials to the base 10:

$$\frac{P_{out}}{0.1}=10^{0.5}$$

$$P_{out}=0.1\times10^{0.5}=0.316W$$

5. Since 10 is a common factor of the Right Hand Side we have:

$$G=10\left[\log(p_1)+\log(p_2)+\log(p_3)\right]$$

$$\underbrace{=}_{\text{by }(5.17)}10\log(p_1\times p_2\times p_3)$$

6. Substituting $p_{out}=I_{out}^2R$ and $p_{in}=I_{in}^2R$ into $G=10\log\left(\frac{p_{out}}{p_{in}}\right)$ gives:

$$G=10\log\left(\frac{I_{out}^2R}{I_{in}^2R}\right)$$

$$=10\log\left[\left(\frac{I_{out}}{I_{in}}\right)^2\right]\quad(\text{Cancelling } R's)$$

$$G\underbrace{=}_{\text{by }(5.19)}20\log\left(\frac{I_{out}}{I_{in}}\right)$$

(5.17) $\log(A)+\log(B)=\log(A\times B)$

(5.19) $\log\left(A^n\right)=n\log(A)$

7. We have:

$$20\log\left(\frac{I_{out}}{I_{in}}\right)=20$$

$$\log\left(\frac{I_{out}}{I_{in}}\right)=1 \ \text{(Dividing by 20)}$$

$$\left(\frac{I_{out}}{I_{in}}\right)=10^1=10$$

$$I_{out}=10\times I_{in} \ \left(\text{Multiplying by } I_{in}\right)$$

$$=10\times1\times10^{-3} \qquad \left(\text{because } I_{in}=1\times10^{-3}\right)$$

$$I_{out}=0.01A$$

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8. By substituting $p_{out}=\frac{V_{out}^2}{R}$ and $p_{in}=\frac{V_{in}^2}{R}$ into $G=10\log\left(\frac{p_{out}}{p_{in}}\right)$ gives:

$$G=10\log\left(\frac{V_{out}^2/R}{V_{in}^2/R}\right)$$

$$=10\log\left(\frac{V_{out}^2}{V_{in}^2}\right)$$

$$=10\log\left[\left(\frac{V_{out}}{V_{in}}\right)^2\right]\underbrace{=}_{\text{by (5.19)}}20\log\left(\frac{V_{out}}{V_{in}}\right)$$

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9. Let $p=\frac{p_o}{p_i}$ then we have:

(a) $10\log(p)=3$ gives $\log(p)=\frac{3}{10}=0.3$ (Dividing by 10)

Taking exponential to the base 10: $p=10^{0.3}=2$

(b) $10\log(p)=10$. Thus $\log(p)=1$:

$$p=10^1=10$$

(c) Similarly $10\log(p)=20$, $\log(p)=2$:

$$p=10^2=100$$

To find the voltage ratio, $\frac{V_o}{V_i}=V$, we only have to take the square root of the p values, because $V=\sqrt{p}$ from $\log(p)=\log(V^2)$ or $V^2=p$:

(a) $V^2=2$ gives $V=\sqrt{2}$

(b) $V^2=10$ gives $V=\sqrt{10}$

(c) $V^2=100$ gives $V=10$

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10. We have

$$9-10e^{-t}=0$$

$$9=10e^{-t} \ \text{(Rearranging)}$$

$$e^{-t}=0.9 \ \text{(Dividing by 10)}$$

(5.19) $$\log\left(A^n\right)=n\log(A)$$

Taking natural logs, $\ln$, of both sides gives:

$$\ln\left(e^{-t}\right)=\ln(0.9)$$

$$-t\underbrace{\ln(e)}_{=1}=\ln(0.9)$$

$$-t=\ln(0.9)$$

$$t=-\ln(0.9)=0.105\sec s$$

11. We have:

$$9\left(1-e^{-0.1t}\right)=3$$

$$1-e^{-0.1t}=\frac{3}{9}=\frac{1}{3}\ \left(\text{Dividing by } 9\right)$$

$$1-\frac{1}{3}=e^{-0.1t}\ \left(\text{Rearranging}\right)$$

$$e^{-0.1t}=\frac{2}{3}$$

Taking natural logs, $\ln$, of both sides gives $\ln\left(e^{-0.1t}\right)=\ln\left(\frac{2}{3}\right)$, so by (5.13) we have:

$$-0.1t\underbrace{\ln(e)}_{=1}=\ln\left(\frac{2}{3}\right)=-0.405$$

$$t=\frac{-0.405}{-0.1}=4.05\sec s$$

12. Applying the change of base rule, (5.23), in each case and rounding to 3 d.p. we have

(a) $\log_2(7.6)=\dfrac{\ln(7.6)}{\ln(2)}=2.926$

(b) $\log_{7.6}(2)=\dfrac{\ln(2)}{\ln(7.6)}=0.342$

(c) $\log_7(6.3)=\dfrac{\ln(6.3)}{\ln(7)}=0.946$

(d) $\log_{6.3}(7)=\dfrac{\ln(7)}{\ln(6.3)}=1.057$

13. (a) $\ln\left[\ln(1000)\right]=\ln(6.907)=1.93$

(b) $\ln\left[\ln\left(10^{10}\right)\right]=\ln[23.026]=3.14$

(c) $\ln\left[\ln\left(10^{50}\right)\right]=\ln[115.129]=4.75$

The $\ln(\ln)$ function makes a large number much smaller.

14. By **EXAMPLE 9** we have $\log\left(PV^n\right)=\log(C)$. Taking exponentials to the base 10 gives:

$$PV^n=C$$

15. We have

$$e^{x\ln(a)}=\left[e^{\ln(a)}\right]^x\underbrace{=}_{\text{by }(5.16)}[a]^x=a^x$$

(5.13) $\ln\left(A^n\right)=n\ln(A)$

(5.23) $\log_a(N)=\dfrac{\log_b(N)}{\log_b(a)}$

16. By taking exponential of both sides we obtain

$$e^{\ln(1+y)} = e^{x+x^2/2}$$

$$\underbrace{1+y}_{\text{by }(5.16)} = \underbrace{e^x e^{x^2/2}}_{\text{by }(5.1)}$$

$$y = e^{x^2/2}e^x - 1$$

17. (a) *How do we solve* $27^x = 3$ *?*

Note that $27 = 3^3$. Substituting this $27 = 3^3$ we have

$$\left(3^3\right)^x = 3$$

$$3^{3x} = 3 \qquad \left[\text{Using } \left(a^m\right)^n = a^{mn}\right]$$

This means that $3x = 1$ which implies that $x = \frac{1}{3}$.

(b) We need to solve $5\ln\left(x^2 - 9\right) = 3$. Dividing by 5 yields

$$\ln\left(x^2 - 9\right) = 0.6$$

Taking exponential of both sides gives

$$x^2 - 9 = e^{0.6} = 1.822$$

Adding 9 to both sides gives

$$x^2 = 9 + 1.822 = 10.822$$

$$x = \pm\sqrt{10.822} = \pm 3.29$$

(c) *How do we solve* $\log_9\left(3^{x-1}\right) = x$ *?*

By using the definition of logs we have

$$3^{x-1} = 9^x$$

$$3^{x-1} = \left(3^2\right)^x = 3^{2x}$$

We have $x - 1 = 2x$ implies that $x = -1$.

(d) Taking natural logs of $3^x = 18$:

$$\ln\left(3^x\right) = \ln(18)$$

$$x\ln(3) = \ln(18) \quad \text{implies that} \quad x = \frac{\ln(18)}{\ln(3)} = 2.63$$

(5.1) $a^{m+n} = a^m a^n$

(5.16) $e^{\ln(a)} = a$

18. We have

$$\theta_1 - \theta_2 = -\frac{Q}{2\pi kL}\left[\ln(r_1) - \ln(r_2)\right]$$

$$\underbrace{=}_{\text{taking the negative sign inside the brackets}} \frac{Q}{2\pi kL}\left[-\ln(r_1) + \ln(r_2)\right]$$

$$= \frac{Q}{2\pi kL}\left[\ln(r_2) - \ln(r_1)\right]$$

$$= \frac{Q}{2\pi kL}\underbrace{\ln\left(\frac{r_2}{r_1}\right)}_{\text{by (5.12)}}$$

Rearranging gives

$$\frac{2\pi kL(\theta_1 - \theta_2)}{\ln\left(\frac{r_2}{r_1}\right)} = Q$$

19. We have by rearranging:

$$e^{11600v/\eta T} = \frac{i}{I_s} + 1$$

Taking ln' s of both sides gives:

$$\frac{11600v}{\eta T} = \ln\left(\frac{i}{I_s} + 1\right)$$

$$v = \frac{\eta T}{11600}\ln\left(\frac{i}{I_s} + 1\right) \quad \left(\text{Multiplying by } \frac{\eta T}{11600}\right)$$

Substituting $\eta = 2$, $i = 5 \times 10^{-3}$, $I_s = 0.1 \times 10^{-6}$ and $T = 330$ gives

$$v = \frac{2 \times 330}{11600} \times \ln\left(\left[\frac{5 \times 10^{-3}}{0.1 \times 10^{-6}}\right] + 1\right)$$

$$= 0.62\,volts$$

(5.12) $$\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)$$