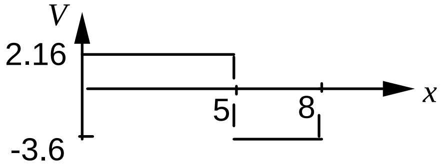
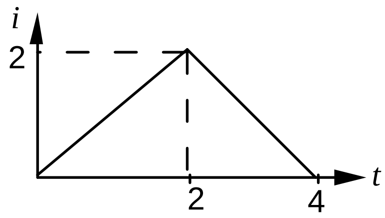
Complete solutions to Exercise 6(a)

1. $\frac{dy}{dx}$ is the gradient so we have (a) 3 (b) -1 c) $-\pi$ (d) 1000

2. The graph of V is the gradient of the lines $2.16x$ for $0 \le x \le 5$ and $28.8 - 3.6x$ for $5 \le x \le 8$, that is 2.16 and -3.6 respectively.



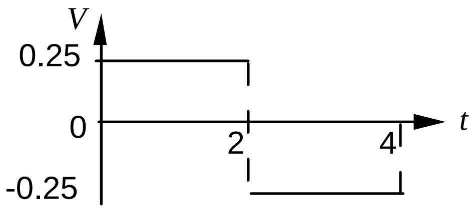
3.



The gradient for t between 0 and 2 is 1, so $\frac{d}{dt}(t) = 1$. The gradient for t between 2 and 4 is -1, so $\frac{d}{dt}(-t) = -1$.

When $0 \le t \le 2$, $V = 0.25\frac{d}{dt}(t) = 0.25 \times 1 = 0.25$

When $2 < t \le 4$, $V = 0.25\frac{d}{dt}(-t) = 0.25 \times (-1) = -0.25$



4. Since $i = 10mA$, a constant value, $\frac{di}{dt} = 0$. Hence $V = 0$.

5. (a) By putting $f(x) = x$ into (6.1) we have

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{(x+h)-h}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}}$$

$$= \lim_{h\to 0} 1 = 1$$

(b) Substituting $f(x) = 5x$ into (6.1) gives:

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{5(x+h)-5x}{h}$$

$$= \lim_{h\to 0}\frac{5\cancel{h}}{\cancel{h}} = 5$$

(6.1) $$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

(c) Substituting $f(x) = x^2 + x$ into (6.1) gives:

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{\left[(x+h)^2 + (x+h)\right] - \left(x^2 + x\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left[x^2 + 2hx + h^2 + x + h\right] - x^2 - x}{h}$$

$$= \lim_{h\to 0}\frac{2hx + h^2 + h}{h}$$

$$= \lim_{h\to 0}\frac{\not{h}(2x + h + 1)}{\not{h}}$$

$$= \lim_{h\to 0}(2x + 1 + h) = 2x + 1$$

(d) Substituting $f(x) = \frac{1}{x}$ into (6.1) gives:

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0}\frac{x - (x+h)}{h(x+h)x} \quad \left[\begin{array}{c}\text{Multiplying Numerator and}\\ \text{Denominator by } x(x+h)\end{array}\right]$$

$$= \lim_{h\to 0}\frac{-h}{h(x+h)x}$$

$$= -\lim_{h\to 0}\frac{1}{(x+h)x}$$

$$= -\frac{1}{x\cdot x}$$

$$= -\frac{1}{x^2}$$

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6. The Maple solutions are:

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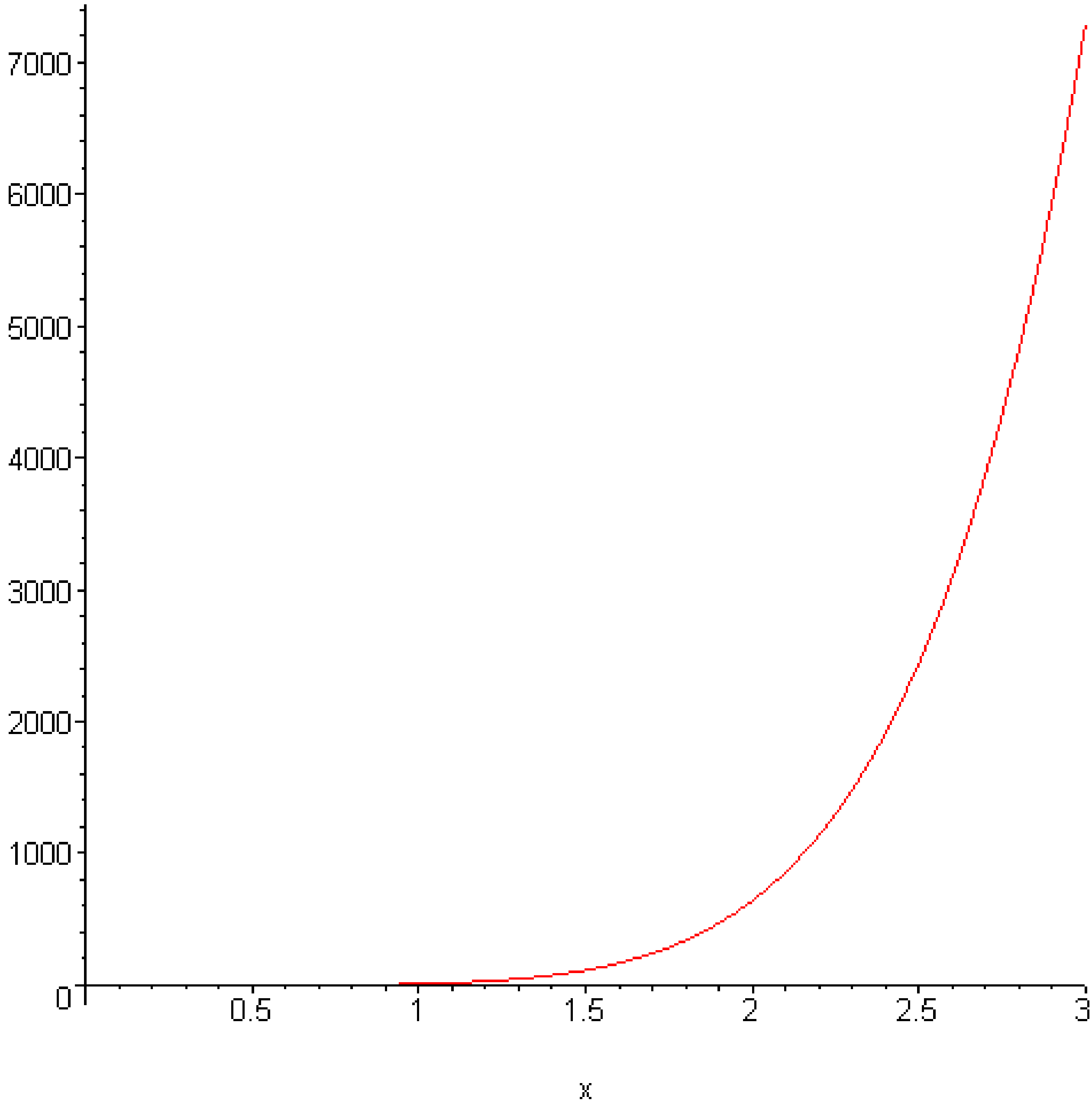
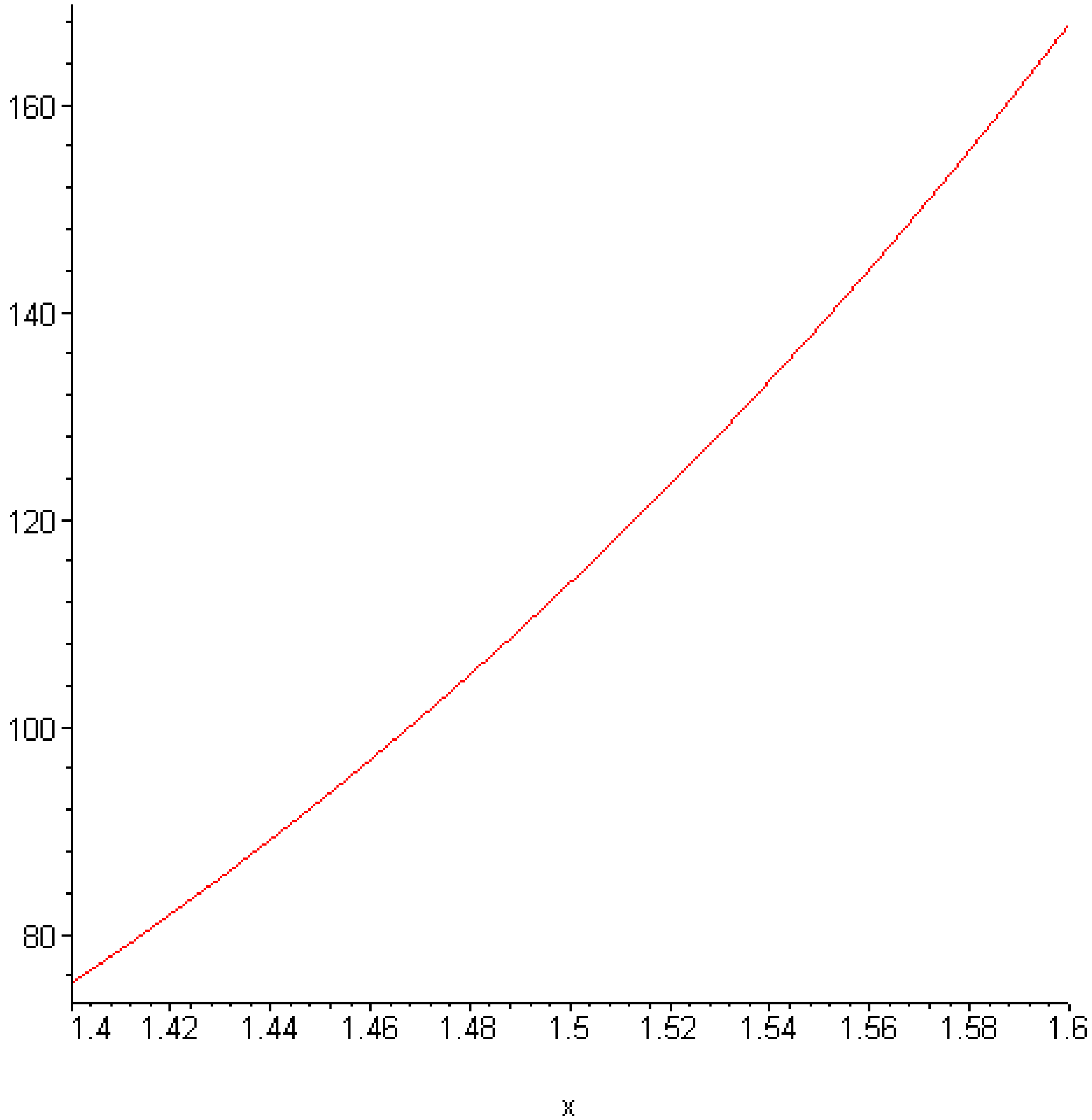
> plot(10*x^6,x=0..3);
```

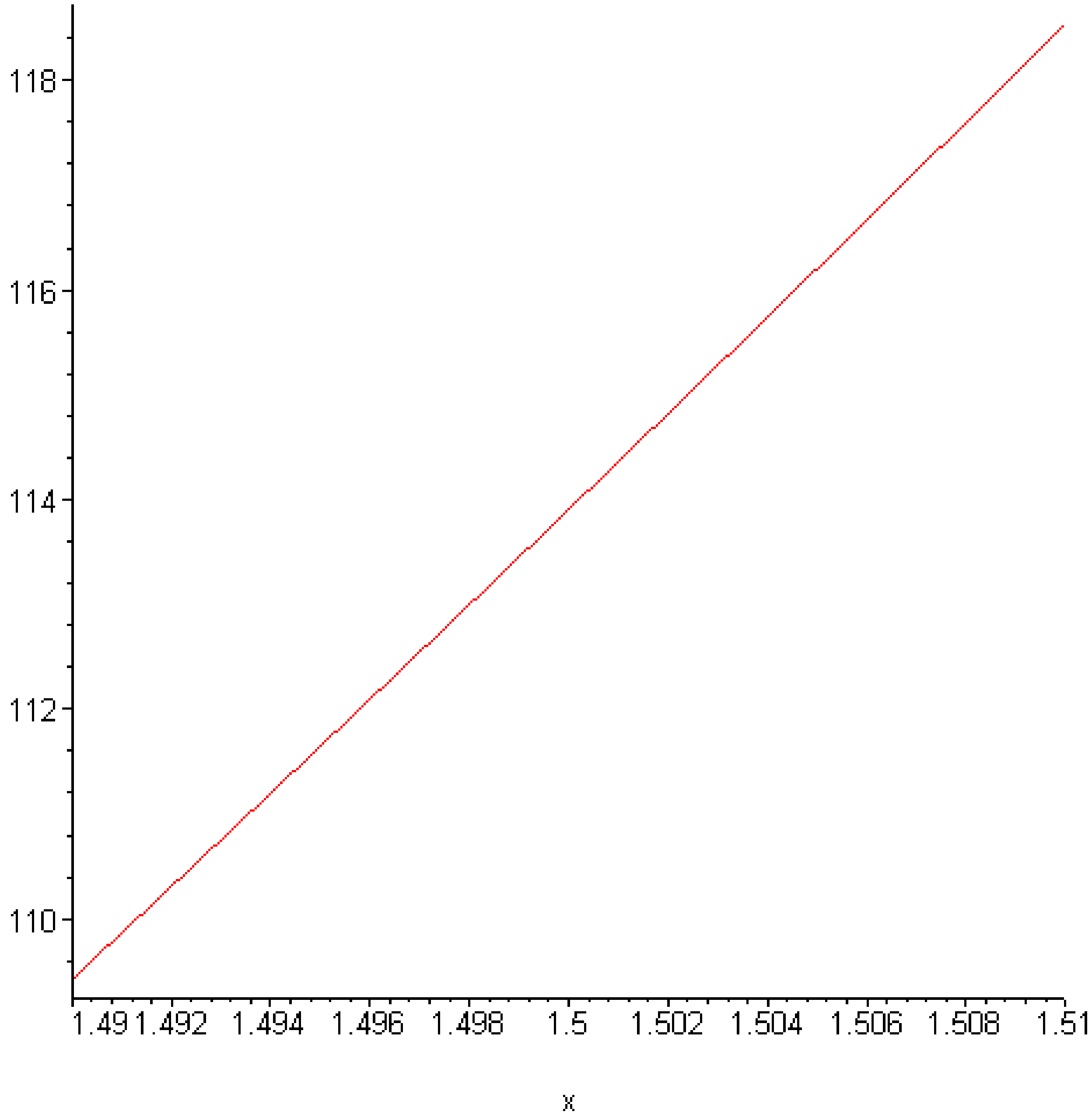
(6.1) $$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$



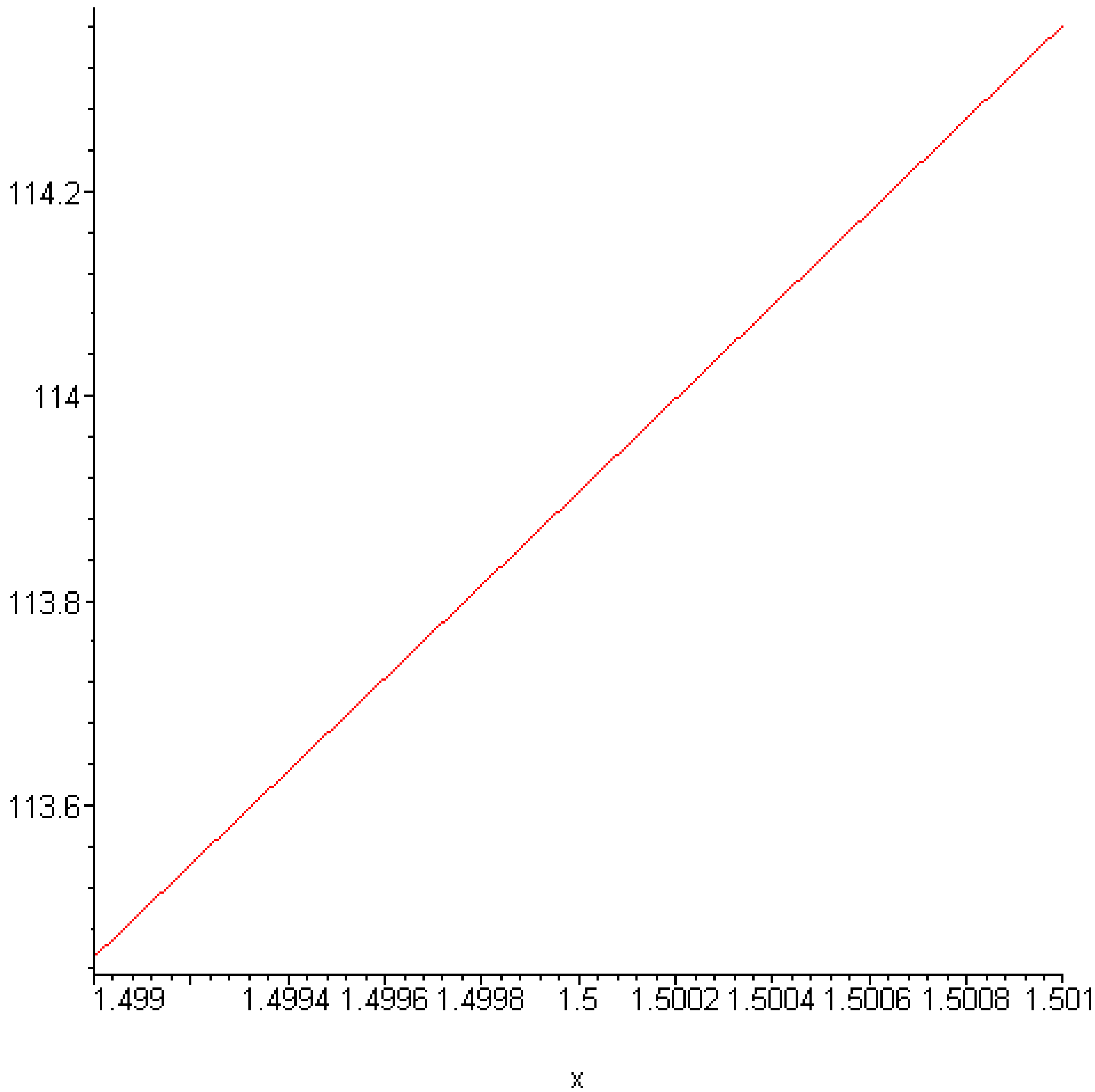

```
> plot(10*x^6,x=1.4..1.6);
```




```
> plot(10*x^6,x=1.49...1.51);
```




```
> plot(10*x^6,x=1.499..1.501);
```




```
> simplify((10*(1.5+h)^6-10*1.5^6)/h);
```

$$455.6250000 + 759.3750000\,h + 675.\,h^2 + 337.5000000\,h^3 + 90.\,h^4 + 10.\,h^5$$

```
> grad:=limit(%,h=0);
```

$$grad := 455.6250000$$