Chapter 5 – Truth tables
Answers to select "Getting familiar with…" exercises.

Getting familiar with… constructing truth tables.

For each of the following, construct the basic structure of a truth table. Ignore the operators for this set. (Save these; you will use them again in Getting familiar with . . . truth tables for operators.)

1. (P v Q)

(P	v	Q)
T		T
T		F
F		T
F		F

3. ((A v B) & A)

((A	v	B)	&	A)
T		T		T
T		F		T
F		T		F
F		F		F

5. ((A ⊃ B) v C)

((A	⊃	B)	v	C)
T		T		T
T		T		F
T		F		T
T		F		F
F		T		T
F		T		F
F		F		T
F		F		F

7. (Q ⊃ ((R v P) & S))

(Q	⊃	((R	v	P)	&	S)
T		T		T		T
T		T		T		F
T		T		F		T
T		T		F		F
T		F		T		T
T		F		T		F
T		F		F		T
T		F		F		F
F		T		T		T
F		T		T		F
F		T		F		T
F		T		F		F
F		F		T		T
F		F		T		F
F		F		F		T
F		F		F		F

9. (~ (C & F) v ~ (F & C))

(~	(C	&	F)	v	~	(F	&	C))
	T		T			T		T
	T		F			F		T
	F		T			T		F
	F		F			F		F

Getting familiar with . . . truth tables for operators.

Refer back to your answers to "Getting familiar with . . . constructing truth tables." For each of those, construct a complete truth table, then construct one for each of the following. Start with the claims enclosed in the most number of parentheses, and move to the next most enclosed until you get to the main operator. Two examples have been provided.

1. (P v Q)

(P	v	Q)
T	T	T
T	T	F
F	T	T
F	F	F

3. ((A v B) & A)

((A	v	B)	&	A)
T	T	T	T	T
T	T	F	T	T
F	T	T	F	F
F	F	F	F	F

5. ((A ⊃ B) v C)

((A	⊃	B)	v	C)
T	T	T	T	T
T	T	T	T	F
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	T	F
F	T	F	T	T
F	T	F	T	F

7. (Q ⊃ ((R v P) & S)) (Note that the major operator is a conjunction.)

(Q	⊃	((R	v	P)	&	S))
T	T	T	T	T	T	T
T	T	T	T	T	F	F
T	T	T	T	F	T	T
T	T	T	T	F	F	F
T	T	F	T	T	T	T
T	T	F	T	T	F	F
T	F	F	F	F	F	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	T	T	T	F	F
F	T	T	T	F	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	T	F	T	T	F	F
F	T	F	F	F	T	T
F	T	F	F	F	F	F

9. (~ (C & F) v ~ (F & C)) (Note that the major operator is a disjunction.)

(~	C	&	F)	v	~	(F	&	C))
F	T	F	T	F	F	T	T	T
F	T	F	F	T	T	F	F	T
T	F	T	T	T	T	T	F	F
T	F	F	F	T	T	F	F	F

1. (A & ~ B)

(A	&	~	B)
T	F	F	T
T	T	T	F
F	F	F	T
F	F	T	F

3. ~ (A ⊃ ~ B)

~	(A	⊃	~	B)
T	T	F	F	T
F	T	T	T	F
F	F	T	F	T
F	F	T	T	F

5. ~ (~ W & ~ P)

~	(~	W	&	~	P)
T	F	T	F	F	T
T	F	T	F	T	F
T	T	F	F	F	T
F	T	F	T	T	F

7. ~ (P & Q)

~	(P	&	Q)
F	T	T	T
T	T	F	F
T	F	F	T
T	F	F	F

9. (A v (B ⊃ C))

(A	v	(B	⊃	C))
T	T	T	T	T
T	T	T	F	F
T	T	F	T	T
T	T	F	T	F
F	T	T	T	T
F	F	T	F	F
F	T	F	T	T
F	T	F	T	F

Getting familiar with . . . using truth tables to test for validity.

For each of the following arguments, construct its truth table and test it for validity.

1. ((P ⊃ Q) & P) /.: Q

((P	⊃	Q)	&	P)	/.:	Q
T	T	T	T	T		T
T	F	F	F	T		F
F	T	T	F	F		T
F	T	F	F	F		F

Valid. There is no row where the premise is true and the conclusion is false.

3. (M ≡ ~ N) ; ~ (N & ~ M) /.: (M ⊃ N)

(M	≡	~	N)	;	~	(N	&	~	M)	/.:	(M	⊃	N)	
T	F	F	T		T	T	F	F	T		T	T	T	
T	T	T	F		T	F	F	F	T		T	F	F	←
F	T	F	T		F	T	T	T	F		F	T	T	
F	F	T	F		T	F	F	T	F		F	T	F	

Invalid. In row 2, both premises are true, and the conclusion is false.

5. (A ⊃ A) /.: A

(A	⊃	A)	/.:	A	
T	T	T		T	
F	T	F		F	←

Invalid. In row 2, the premise is true, but the conclusion is false. How could this be? It is obviously true that *<u>if</u>* A is true, *<u>then</u>* A is true. But *<u>is</u>* A really true? That's what the conclusion says. And that doesn't follow from the conditional.

7. ~ R ; (S ⊃ R) /.: ~ S

~	R	;	(S	⊃	R)	/.:	~	S
F	T		T	T	T		F	T
F	T		F	T	T		T	F
T	F		T	F	F		F	T
T	F		F	T	F		T	F

Valid. There is no row where the premise is true and the conclusion is false.

9. ((A & B) v (C v D)) ; ~ (C v D) /.: A

((A	**&**	**B)**	**v**	**(C**	**v**	**D))**	**;**	**~**	**(C**	**v**	**D)**	**/.:**	**A**
T	T	T	**T**	T	T	T		**F**	T	T	T		**T**
T	T	T	**T**	T	T	F		**F**	T	T	F		**T**
T	T	T	**T**	F	T	T		**F**	F	T	T		**T**
T	T	T	**T**	F	F	F		**T**	F	F	F		**T**
T	F	F	**T**	T	T	T		**F**	T	T	T		**T**
T	F	F	**T**	T	T	F		**F**	T	T	F		**T**
T	F	F	**T**	F	T	T		**F**	F	T	T		**T**
T	F	F	**F**	F	F	F		**T**	F	F	F		**T**
F	F	T	**T**	T	T	T		**F**	T	T	T		**F**
F	F	T	**T**	T	T	F		**F**	T	T	F		**F**
F	F	T	**T**	F	T	T		**F**	F	T	T		**F**
F	F	T	**F**	F	F	F		**T**	F	F	F		**F**
F	F	F	**T**	T	T	T		**F**	T	T	T		**F**
F	F	F	**T**	T	T	F		**F**	T	T	F		**F**
F	F	F	**T**	F	T	T		**F**	F	T	T		**F**
F	F	F	**F**	F	F	F		**T**	F	F	F		**F**

Valid. There is no row where both premises are true and the conclusion is false.

11. (P ⊃ Q) /.: R

(P	⊃	Q)	/.:	R	
T	T	T		T	
T	T	T		F	←
T	F	F		T	
T	F	F		F	
F	T	T		T	
F	T	T		F	←
F	T	F		T	
F	T	F		F	←

Invalid. On at least one row, the premise is true and the conclusion is false.

• Note! There is an error in the text. Numbers 13 and 14 have no conclusion and, therefore, are not arguments. Thus, they cannot be tested for validity. Here, we have supplied conclusions to both, and we have provided the answer to number 13.

13. ((M ≡ ~ N) & ~ (N & ~ M)) /.: ~(N v M)

((M	≡	~	N)	&	~	(N	&	~	M)	/.:	~	(N	v	M)	
T	F	F	T	F	T	T	F	F	T		F	T	T	T	
T	T	T	F	**T**	T	F	F	F	T		**F**	F	T	T	←
F	T	F	T	F	F	T	T	T	F		F	T	T	F	
F	F	T	F	F	T	F	F	T	F		T	F	F	F	

Invalid. In row 2, the premise is true, and the conclusion is false.

14. ((A ≡ ~B) v (B v A)) /.: (A & ~B)

• Note! There is an error in the text. Numbers 15-20 are identical to numbers 5-10. We apologize for this. For further practice, we have supplied six additional exercises here. We have also supplied answers to the odd numbered ones below.

15. (P v ~Q) ; (R ⊃ ~Q) /.: (~P ⊃ R)

16. (A v B) ; (A ⊃ B) /.: (B ⊃ ~A)

17. (~ (Y & O) v W) /.: (Y ⊃ W)

18. (Y ≡ Z) ; (~Y v ~W) ; W /.: Z

19. (E v F) ; (E ⊃ F) ; (C & D) /.: (F ⊃ ~C)

20. (N & ~Q) ; (~R ≡ ~Q) ; R /.: (N ⊃ R)

Select Answers

15. (P v ~Q) ; (R ⊃ ~Q) /.: (~P ⊃ R)

(P	v	~	Q)	;	(R	⊃	~	Q)	/.:	(~	P	⊃	R)	
T	T	F	T		T	F	F	T		F	T	T	T	
T	T	F	T		F	T	F	T		F	T	T	F	
T	T	T	F		T	T	T	F		F	T	T	T	
T	T	T	F		F	T	T	F		F	T	T	F	
F	F	F	T		T	F	F	T		T	F	T	T	
F	F	F	T		F	T	F	T		T	F	F	F	
F	T	T	F		T	T	T	F		T	F	T	T	
F	T	T	F		F	T	T	F		T	F	F	F	←

Invalid.

17. (~ (Y & O) v W) /.: (Y ⊃ W)

(~	(Y	&	O)	v	W)	/.:	(Y	⊃	W)	
F	T	T	T	T	T		T	T	T	
F	T	T	T	F	F		T	F	F	
T	T	F	F	T	T		T	T	T	
T	T	F	F	T	F		T	F	F	←
T	F	F	T	T	T		F	T	T	
T	F	F	T	T	F		F	T	F	
T	F	F	F	T	T		F	T	T	
T	F	F	F	T	F		F	T	F	

Invalid.

19. (E v F) ; (E ⊃ F) ; (C & D) /.: (F ⊃ ~C)

(E	v	F)	;	(E	⊃	F)	;	(C	&	D)	/.:	(F	⊃	~	C)	
T	**T**	T		T	**T**	T		T	**T**	T		T	**F**	F	T	←
T	T	T		T	T	T		T	F	F		T	F	F	T	
T	T	T		T	T	T		F	F	T		T	T	T	F	
T	T	T		T	T	T		F	F	F		T	T	T	F	
T	T	F		T	F	F		T	T	T		F	T	F	T	
T	T	F		T	F	F		T	F	F		F	T	F	T	
T	T	F		T	F	F		F	F	T		F	T	T	F	
T	T	F		T	F	F		F	F	F		F	T	T	F	
F	**T**	T		F	**T**	T		T	**T**	T		T	**F**	F	T	←
F	T	T		F	T	T		T	F	F		T	F	F	T	
F	T	T		F	T	T		F	F	T		T	T	T	F	
F	T	T		F	T	T		F	F	F		T	T	T	F	
F	F	F		F	T	F		T	T	T		F	T	F	T	
F	F	F		F	T	F		T	F	F		F	T	F	T	
F	F	F		F	T	F		F	F	T		F	T	T	F	
F	F	F		F	T	F		F	F	F		F	T	T	F	

Invalid.

Getting familiar with . . . using the short truth table method to test for validity

Test each of the following arguments for validity using the short truth table method.

1. (P v ~Q) ; (R ⊃ ~Q) /.: (~P ⊃ R)
F T T F   F T T F   TF F F

Invalid. In order to make the conclusion false, both P and R must be false. But we are free to set Q's value. If we set Q's value to F, all the premises are true and the conclusion is false.

3. (~(Y & O) v W) /.: (Y ⊃ W)
T T F F T F   T F F

Invalid. In order to make the conclusion false, Y must be true and W must be false. But W is the right disjunct of the premise, so in order for that disjunction to be true, the left disjunct must be true. We can do this if we set O's value to F.

5. (E v F) ; (E ⊃ F) ; (C & D) /.: (F ⊃ ~C)

(E	v	F)	;	(E	⊃	F)	;	(C	&	D)	/.:	(F	⊃	~	C)
T	T	T		T	T	T		T	T	T		T	F	F	T
F	T	T		F	T	T		T	T	T		T	F	F	T

Invalid.

7. (~(A & B) v C) /.: (A ≡ C)

(~	(A	&	B)	v	C)	/.:	(A	≡	C)
T	T	F	F	T	F		T	F	F

Invalid.

9. ((H & I) v L) ; (L & D) ; (D ⊃ ~R) /.: ~L

((H	&	I)	V	L)	;	(L	&	D)	;	(D	⊃	~	R)	/.:	~	L
T/F	T/F	T/F	T	T		T	T	T		T	T	T	F		F	T

Invalid.

11. (L⊃ (P & Q)) ; ~R ; (L & (P & O)) /.: P

(L	⊃	(P	&	Q)	;	~	R	;	(L	&	(P	&	O))	/.:	P
F	T	F	F	T/F		T	F		F	F	F	F	T/F		F

Valid.

13. ((Q v R) ⊃ S) /.: (Q ⊃ S)

((Q	v	R)	⊃	S)	/.:	(Q	⊃	S)
T	T	T/F	F	F		T	F	F

Valid.

15. ((K v L) ⊃ (M v N)) ; ((M v N) ⊃ O) /.: O

((K	v	L)	⊃	(M	v	N))	;	((M	v	N)	⊃	O)	/.:	O
F	F	F	T	F	F	F		F	F	F	T	F		F

Invalid.

17. ((~M & ~N) ⊃ (O ⊃ N)) ; (N ⊃ M) ; ~M /.: ~O

((~	M	&	~	N)	⊃	(O	⊃	N))	;	(N	⊃	M)	;	~	M	/.:	~	O
T	F	T	T	F	F	T	F	F		F	T	F		T	F		F	T

Valid.

19. (A ⊃ C) ; (B ⊃ D) ; (C ⊃ B) ; (S v ~D) ; S /.: ~A

(A	⊃	C)	;	(B	⊃	D)	;	(S	v	~	D)	;	S	/.:	~	A
T	T	T		T/F	T	T/F		T	T	T/F	T/F		T		F	T

Invalid.