**Chapter 6 – Rules of deductive inference**
**Answers to select "Getting familiar with…" exercises.**

**Getting familiar with… basic rules of inference.**

**1.**
1. A
2. (A ⊃ B)　　/.: B
_____
**3. B　1, 2 modus ponens**

**3.**
1. (P & Q)
2. (R & S)　　/.: P
_____
**3. P　1 simplification**

**5.**
1. ((R v S) & Q)
2. (~Q v S)
3. T　　　　/.: (Q & T)
_____
**4. Q　1 simplification**
**5. (Q & T)　3, 4 conjunction**

**7.**
1. ((A ⊃ B) ⊃ (C v D))
2. ~ (C ⊃ D)　　/.: ~(A ⊃ B)
_____
**3. ~(A ⊃ B)　1, 2 modus tollens**

**9.**
1. ((P ⊃ Q) & (S ⊃ R))
2. (~ Q & ~ R)　　　/.: (~P & ~S)
_____
**3. (P ⊃ Q)　1 simplification**
**4. ~ Q　2 simplification**
**5. ~P　3, 4 modus tollens**
**6. (S ⊃ R)　1 simplification**
**7. ~R　2 simplification**
**8. ~S　5,6 modus tollens**
**9. (~P & ~S)　5, 8 conjunction**

**11.**
1. ((P &Q) & W)
2. R          /.: W
_____
**3. W  1 simplification**


**13.**
1. A
2. (B v C)
3. ((A & (B v C)) ⊃ D)     /.: D
_____
**4. (A & (B v C))  1, 3 conjunction**
**5. D  3, 4 modus ponens**


**15.**
1. ((P v Q) ⊃ (W & ~Y))
2. (~Q & W)
3. (X ⊃ Y)
4. (P v Q)       /.: (~X & ~Q)
_____
**5. (~W & ~Y)    1, 4  modus ponens**
**6. ~Y     5 simplification**
**7. ~X     3, 6 modus tollens**
**8. ~Q    2 simplification**
**9. (~X & ~Q)  7, 8 conjunction**


**17.**
1. ~P
2. (S ⊃ R)
3. (R ⊃ Q)
4. (Q ⊃ P)        /.: ~S
_____
**5. ~Q     1, 4 modus tollens**
**6. ~R     3, 5 modus tollens**
**7. ~S     2, 6 modus tollens**


**19.**
1. ~(B v D)
2. (A ⊃ (B v D))
3. (H ⊃ ((E & F) & G))
4. H        /.: (~A & E)
_____
**5. ~A                 1, 2 modus tollens**
**6. ((E & F) & G)  3, 4 modus ponens**
**7. (E & F)           6 simplification**
**8. E                  7 simplification**
**9. (~A & E)         5, 8 conjunction**

**Getting familiar with… more rules of inference.**

**a.**
**1.**
1. ((A v B) ⊃ C)
2. (F & D)
3. (C ⊃ (E v H)) /.: ((A v B) ⊃ (E v H))
___
**4. ((A v B) ⊃ (E v H))  1, 3 hypothetical syllogism**


**3.**
1. (~P v (D v Z))
2. (~(D v Z) v B)
3. ~ B      /.: ~P
___
**4. ~(D v Z)    2, 3 hypothetical syllogism**
**5. ~P          1, 4 hypothetical syllogism**


**5.**
1. ((P v Q) v (~R v ~S))
2. ~(P v Q)
3. ~~ S          /.: ~R
___
**4. (~R v ~S)    1, 2 hypothetical syllogism**
**5. ~R            3, 4 hypothetical syllogism**


**7.**
1. (((P v Q) & (R & S)) & (T v U))
2. (A & B)        /.: (B v P)
___
**3. B            2 simplification**
**4. (B v P)    3 addition**


**9.**
1. A
2. ((A v B) ⊃ ~C)
3. (~ C ⊃ F)        /.: ((A v B) ⊃ F)
___
**4. ((A v B) ⊃ F)        2, 3 hypothetical syllogism**

**b.**
**11.**
1. (~S ⊃ Q)
2. (R ⊃ ~T)
3. (~S v R)          /.: (Q v ~T)
---
**4. (Q v ~T)  1-3 constructive dilemma**


**13.**
1. ((H ⊃ B) & (O ⊃ C))
2. (Q ⊃ (H v O))
3. Q          / ... (B v C)
---
**4. (H v O)   2, 3 modus ponens**

**5. (H ⊃ B)  1 simplification**
**6. (O ⊃ C)  1 simplification**
**7. (B v C)   4-6 constructive dilemma**


**15.**
1. (B ⊃ (A v C))
2. (B & ~A)        /.: C
---
**3. B            2 simplification**

**4. ~A          2 simplification**
**5. (A v C)  1, 3 modus ponens**
**6. C            4, 5 disjunctive syllogism**


**17.**
1. ((A & B) ⊃ ~C)
2. (C v ~D)
3. (A ⊃ B)
4. (E & A)          /.: ~D
---
**5. A            4 simplification**

**6. B            3, 5 modus ponens**
**7. (A & B)  5, 6 conjunction**
**8. ~C          1, 7 modus ponens**
**9. ~D          2, 8  disjunctive syllogism**

**19.**
1. (F ⊃ (G ⊃ ~H))
2. ((F & ~W) ⊃ (G v T))
3. (F & ~T)
4. (W ⊃ T)      /∴ ~H

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| | |
|---|---|
| **5. F** | **3 simplification** |
| **6. ~T** | **3 simplification** |
| **7. ~W** | **4, 6 modus tollens** |
| **8. (F & ~W)** | **5, 7 conjunction** |
| **9. (G v T)** | **2, 8 modus ponens** |
| **10. G** | **6, 9  disjunctive syllogism** |
| **11. (G ⊃ ~H)** | **1, 5 modus ponens** |
| **12. ~H** | **10, 11 modus ponens** |

**Getting familiar with… rules of replacement.**

**1.**
1. ( ~ (P ≡ R) v ~ (Q & S))
2. ~((P ≡ R) & (Q & S))
**DeMorgan's Law**

**3.**
1. ~~ (A & B) v (Q ⊃ R)
2. (A & B) v (Q ⊃ R)
**Double Negation**

**5.**
1. ((P & Q) ⊃ R)
2. ~R ⊃ ~(P & Q)
**Transposition**

**7.**
1. (R & Z)
2. (R & Z) v (R & Z)
**Tautology**

**9.**
1. (Q v (R & S))
2. ((Q v R) & (Q v S))
**Distribution**

**Getting familiar with… proofs.**

**1.**
1. ((A v B) ⊃ C)
2. (F & D)
3. A
4. (C ⊃ (E v H))      /.: ((A v B) ⊃ (E v H))

| | | |
|---|---|---|
| 5. | **(A v B)** | **assumption for conditional proof** |
| 6. | **C** | **1, 5** |
| 7. | **(E v H)** | **4, 6  modus ponens** |

8.  **((A v B) ⊃ (E v H))      5-8 conditional proof**


**3.**
1. (~P v D)
2. (~D & B)
3. ((Z ⊃ P) & A)      /.: (~Z & A)

| | | |
|---|---|---|
| 4. | **~(~Z & A)** | **assumption for indirect proof** |
| 5. | **(~~Z v ~A)** | **4  DeMorgan's Law** |
| 6. | **(Z v ~A)** | **5 double negation** |
| 7. | **A** | **3 simplification** |
| 8. | **~~A** | **7 double negation** |
| 9. | **Z** | **6, 8 disjunctive syllogism** |
| 10. | **(Z ⊃ P)** | **3 simplification** |
| 11. | **P** | **9, 10 modus ponens** |
| 12. | **~~P** | **11 double negation** |
| 13. | **D** | **1, 12 disjunctive syllogism** |
| 14. | **~D** | **2 simplification** |
| 15. | **(D & ~D)** | **13, 14 conjunction** |

16.  **(~Z & A)    4-15 indirect proof**

**5.**

1. ((A v B) v (~C v ~D))
2. ~(A v B)
3. ~~ D              /.: ~C

| | | |
|---|---|---|
| 4. | C | assumption for indirect proof |
| 5. | (~C v ~D) | 1, 2 disjunctive syllogism |
| 6. | ~~C | 4 double negation |
| 7. | ~D | 5, 6 disjunctive syllogism |
| 8. | D | 3 double negation |
| 9. | (D & ~D) | 7, 8 conjunction |
| 10. | ~C | 4-9 indirect proof |

**7.**

1. (((M v N) & (O & P)) & (Q v R))
2. (A & B)          /.: (P & B)

| | | |
|---|---|---|
| 3. | ~(P & B) | assumption for indirect proof |
| 4. | (~P v ~B) | 3 DeMorgan's Law |
| 5. | B | 2 simplification |
| 6. | ~~B | 5 double negation |
| 7. | ~P | 4, 6 disjunctive syllogism |
| 8. | ((M v N) & (O & P)) | 1 simplification |
| 9. | (O & P) | 8 simplification |
| 10. | P | 9 simplification |
| 11. | (P & ~P) | 7, 10 conjunction |
| 12. | (P & B) | 3-11 indirect proof |

**9.**

1. A
2. ((A v B) ⊃ ~C)
3. (D ⊃ ~~ C)         /.: ~D

| | | |
|---|---|---|
| 4. | D | assumption for indirect proof |
| 5. | ~~C | 3, 4 modus ponens |
| 6. | ~(A v B) | 2, 5 modus tollens |
| 7. | (~A & ~B) | 6 DeMorgan's Law |
| 8. | ~A | 7 simplification |
| 9. | (A & ~A) | 1, 8 conjunction |
| 10. | ~D | 4-9 indirect proof |

**11.**
1. B
2. ((B v D) ⊃ ~H)
3. (H v F)          /.: (C ⊃ F)

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| | | |
|---|---|---|
| 4. | C | **assumption for conditional proof** |
| 5. | (B v D) | **1 addition** |
| 6. | ~H | **2, 5 modus ponens** |
| 7. | F | **3, 6 disjunctive syllogism** |

8. **(C ⊃ F)     4-7 conditional proof**


**13.**
1. ((M ⊃ O) v S)
2. ((~S & N) & M)
3. M
4. (P & ~O)          /.: (S v B)

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| | | |
|---|---|---|
| 5. | ~(S v B) | **assumption for indirect proof** |
| 6. | ~O | **4 simplification** |
| 7. | (~S & N) | **2 simplification** |
| 8. | ~S | **7 simplification** |
| 9. | (M ⊃ O) | **1, 8 disjunctive syllogism** |
| 10. | O | **3, 9 modus ponens** |
| 11. | (O & ~O) | **6, 10 conjunction** |

12. **(S v B)     5-11 indirect proof**


**15.**
1. (X v Y)
2. ((X & W) ⊃ (Z ⊃ Y))
3. (~Y & W)
4. Z          /.: R

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| | | |
|---|---|---|
| 5. | ~R | **assumption for indirect proof** |
| 6. | ~Y | **3 simplification** |
| 7. | X | **1, 6 disjunctive syllogism** |
| 8. | W | **3 simplification** |
| 9. | (X & W) | **7, 8 conjunction** |
| 10. | (Z ⊃ Y) | **2, 9 modus ponens** |
| 11. | Y | **4, 10 modus ponens** |
| 12. | (Y & ~Y) | **6, 11 conjunction** |

13. **R     5-12 indirect proof**

**15. Alternative ending**

11. ~Z        6, 10 modus tollens
12. (Z & ~Z)  4, 11 conjunction
13. R            5-12 conditional proof


**17.**
1. X
2. ((~S v Y) v Z)
3. (~Z & ~~S)     /.: (W ⊃ (Y v R))

4. | W                  assumption for conditional proof
5. | ~Z                 3 simplification
6. | (~S v Y)           2, 5 disjunctive syllogism
7. | ~~S                3 simplification
8. | Y                  6, 7 disjunctive syllogism
9. | (Y v R)            8 addition
10.   (W ⊃ (Y v R))    4-9 conditional proof


**19.**
1. (A ⊃ (~A v F))
2. ~ F                    /.: ~A

3. | A              assumption for indirect proof
4. | (~A v F)   1, 3 modus ponens
5. | ~A            2, 4 disjunctive syllogism
6. | (A & ~A)    3, 5 conjunction
7.   ~A            3-6 indirect proof


**Getting familiar with… formal fallacies.**

**a.**
**1.**
1. He's the president of the company or I'm a monkey's uncle.
2. Here is the memo announcing that he is president.
3. So, I'm obviously not a monkey's uncle.

**Affirming the disjunct**

**3.**
1. It is either raining or storming.
2. It is certainly raining.
3. Thus, it is not storming.

**Affirming the disjunct**


**5.**
1. If it drops below 0$^o$ C, either the roads will become icy or the water line will freeze.
2. It is -5$^o$ C (below 0$^o$).
3. So, either the roads will become icy or the water line will freeze.
4. The roads are icy.
5. Therefore, the water line is probably not frozen.

**Affirming the disjunct**


**b.**
**1. Affirming the disjunct**
1. Either the Bulls will win or the Suns will.
**2. The Bulls will win.**
**3. So, the Suns will not win.**

If you're worried that this inference is a good one because only one team can win, remember that if only one team could win, then the teams are playing each other and the "or" is exclusive. But we cannot assume the "or" is exclusive unless it is explicitly stated. In this case, the team could be playing different teams or in different tournaments.

**3. Affirming the consequent**
1. If the road is wet, your tires won't have as much traction.
**2. You won't have as much traction.**
**3. Therefore, the road is wet.**

Why is this fallacious? Because the road's being wet is one reason you won't have good traction, but it is certainly not the only reason. There may be ice on the road, or your tires may be bald.

**5. Denying the antecedent**
1. If the bar is close, it will be safe to drive home.
**2. The bar is not close.**
**3. So, it won't be safe to drive home.**

This is fallacious because, even thought the bar's proximity is one reason it will be safe to drive home, there may be others. For instance, the driver is not drunk, the roads are not slippery, it is not late at night, etc.

**7. Affirming the consequent**
1. We will win only if we strengthen our defense.
**2. We have strengthened our defense.**
**3. Therefore, we will win.**

Strengthening the defense may be a *necessary* condition for winning, but it is certainly not *sufficient*. You still have to play the game, your team members have to be in good shape, you have to outplay the other team, etc.

**9. Affirming the disjunct**
1. They will break up unless she is honest.
**2. She is honest.**
**3. Therefore, they will not break up.**

Notice how the inclusive "or" helps us here. They could break up for a number of reasons. If she is not honest, they will break up. That would be the valid conclusion of a disjunctive syllogism. But if she is honest, who knows what will happen? They may break up anyway, perhaps she was too honest; perhaps what she was honest about is a reason to break up. Maybe being honest increases the chances they will stay together, but it doesn't guarantee it.