Reading More Logic

In *Doing Philosophy* we set out what logic is used for in philosophy, explore some of the basic ideas behind logic and stress the importance of being able to read the simple logical statements that can be found in philosophy texts at all levels. We have not attempted to teach you logic, just to give you the general meaning of the symbols you might encounter, so that they are not overwhelming and do not block your studies. If you want to learn formal logic there are links to helpful websites in the Resources section of this site.

In this section we explore aspects of predicate logic (logic about properties and qualities) and quantification, something you are likely to encounter in slightly more advanced philosophical texts. Then some examples from real philosophical texts are given *without answers*. These are examples you can look at in class.

A quick check

Before turning to predicate logic, review your understanding of the purpose of logic and the basic understanding of symbols introduced in *Doing Philosophy.*

In order to check your understanding look at the following example of a logical statement, which represents a straightforward argument.

$(a\&b) \leftrightarrow c$, $(a\&b) \rightarrow (d\&e)$, c therefore $(d\&e)$

What does this argument say?
Answer:
Take this in parts. The word ‘therefore’ indicates that there is an argument intended here with ‘($d\&e$)’ as the conclusion and the other statements, separated

by commas, as the premises. It could be written as follows to make this a bit clearer:

1. $(a \& b) \leftrightarrow c$
2. $(a \& b) \rightarrow (d \& e)$
3. $\dfrac{c}{\vdash (d \& e)}$

Now take each line at a time. Refer to the definitions for each of the symbols given in the book and look at the components in brackets first. The first line, for example, says that the statements *a* and *b* together are equivalent to the statement *c*. That means that if both *a* and *b* are true then *c* is true. (What happens to *c* if either *a* or *b* are false?)

Look at line 2. It says that *if a and b, then d and e follows*. In other words, if *a* and *b* are both true then it cannot be the case that *d* and *e* are both false.

Can you see how the rest of the argument works? We can take the equivalence expressed in line 1 and see that it follows that:

$c \rightarrow (d \& e)$, which means that *if c, then d and e*

Given that line 3 now gives us *c* can you see how the conclusion follows? Even if this is not clear, can you read each of the components of the argument and understand the role the symbols and letters have? If not, go back and check the text in *Doing Philosophy*.

Predicate logic

Predicates are the parts of simple statements (sentences or propositions) that assign properties or qualities to the subjects of the statement. The following table shows some examples.

Sentence	Subject	Predicate
Adam is a runner	Adam	is a runner
This is a hammer	This	is a hammer
X is blue	*X*	is blue

We could say of a particular golf ball that it is white; the sentence ‘this golf ball is white’ says the subject ‘this golf ball’ has the predicate ‘is white’. The subject part of the sentence refers to the particular golf ball, the predicate to the property of being white.

In this example we could replace ‘this golf ball’ with *g* in just the way we have seen for other individual things, and say:

g is white

Now it might be obvious that being a golf ball is also a property. That is, we could also make ‘is a golf ball’ a predicate. In this case we are saying that a particular *unqualified* thing has the property of being a golf ball, so the subject of the sentence expressing this is just a place holder for something. If this sounds a bit hard to capture, think about it this way: imagine we pick up a banana and ask if the statement ‘this thing has the property of being a golf ball’ is true. The answer is negative: the statement is false. But we have been thinking about this banana as a thing in general terms to ask the question.

The way properties, as predicates in statements, are usually expressed is by using capital letters in front of the letters standing for the things with those properties. So, from our example about golf balls, if we use *F* for ‘being white’ and *x* for any object, we would read:

***Fx* as meaning ‘thing *x* is white’** (sometimes this is shown as ***F*(*x*)**)

And if we have two statements, ‘x is a golf ball and x is white’ and we let *G* stand for ‘is a golf ball’ we can use the logical symbols for joining statements to say:

Gx & *Fx*

And from this we can imagine we might make statements of all sorts of forms, combing statements about properties using the logical connections we have already encountered. And this is indeed what philosophers do to further clarify the forms of their arguments.

What do each of the following say?

$Gx \mathbin{\&} \neg Fx$

$Gx \rightarrow Fx$

$\neg Gx \vee \neg Fx$

One last point on predicates is that we have already seen in the book how some properties express relationships between things: '. . . is taller than . . .' and '. . . is older than . . .' express relationships between two terms in a statement, '. . . lies in between . . . and . . .' expresses a relationship that has three terms in a complete statement.

If we wanted to say that *x* is taller than *y* where we express '. . . is taller than . . .' by *H* the convention is to put the variable letters in order after the predicate letter, thus

Hxy or ***H*(*x*,*y*)**

Similarly, if we express '. . . lies in between . . . and . . .' by *J*, then to say that *x* lies in between *y* and *z* is shown as:

Jxyz or ***J*(*x*,*y*,*z*)**

Quantification

One last component of simple logical arguments you are likely to encounter covers the way that quantities are expressed in symbols.

The conventional symbols are ∀ for 'All' and ∃ for 'Some'. In this context 'some' means one or more. They work in the following way.

Take the statement:

$\forall x\ (Fx \rightarrow Gx)$

This says ‘for all x, if x is F then x is G’ which means that for anything we pick in the world, if it has the property expressed by F, it will have the property picked out by G. For example, if we take F to mean ‘is a poet’ and G to mean ‘is Canadian’ the statement above would claim ‘All poets are Canadian’.

Now consider:

$$\exists x\ (Fx\ \&\ Gx)$$

This says ‘some (there is at least one) x such that x is F and x is G’. If we take F and G to mean the same as above, then the statement is claiming ‘Some (there is at least one) poets are Canadian’.

Quantification at the beginning of a statement can include more than one variable. For example,

$$\forall x \exists y (Fxy)$$

This says ‘for all x, and for at least one y, x stands in relation F to y’. For example, if F means ‘is happier than’ (and we assume the variables stand for people) the statement claims that there is (at least) someone happier than everyone else.

If we want to say there is *just one* of anything, this can be expressed using the symbols we have encountered so far (it is a standard logic exercise to work out how to do this), but it also often represented simply by $\exists!$:

$$\exists! x (Fx)$$

Which means ‘there is exactly one x with property F’.

It is beyond our aims to explore these ideas much further, but it is important to note the way brackets are used with quantification because it can affect how you read logical symbols. The quantified terms are placed in front of the bracketed terms over which they have what is called *scope*. This means they can only be taken to be working for the terms in the brackets that follow them. A new quantifier and new brackets means that the variable could be standing for something else.

Conventions and summary

Finally, a note on conventions. Philosophers tend to stick to certain conventions when using letters and symbolic notation, and understanding those conventions can give important clues to what is meant (although in *Doing Philosophy* we have used a variety of conventions to give you practice in recognizing that these forms are not always used). While there are always exceptions, and alternatives exist for most symbols, the following provides a useful guide as a summary:

1. There are logical symbols for truth-functional operations: $\neg$, &, $\vee$, $\rightarrow$, $\leftrightarrow$ . . .
2. Brackets are used to indicate scope and structure.
3. The quantifiers are: $\forall$ and $\exists$.
4. Constants standing in for specific named things are *usually* designated by lower-case letters from the beginning of the alphabet: *a*, *b*, *c*. . . .
5. Variables standing in for any object are *usually* designated by lower-case letters from the end of the alphabet: *x*, *y*, *z*. . . .
6. Statements, sentence or propositions are *often* represented by lower-case letters from the back third of the alphabet: *p*, *q*, *r*. . . .
7. Predicates are *usually* represented by upper-case letters: *F*, *G*, *H*. . . .

Examples from philosophical texts

The following are examples of the use of logic and the logical display of arguments adapted from real philosophical texts, some in words, some in symbols, some a mixture of both. Read each and then have a go at answering the questions.

We have not provided answers so that you can look at these examples in tutorials and seminars.

The aim is for you to familiarize yourself in reading arguments expressed in this way so that you feel confident in tackling them when you encounter them. It is important that you can *read* the formal setting out of the argument, not that you can test the arguments (although it is good practice if you know how to!).

1. $\exists x$ (x is the king of France), $\forall x$ (if x is the king of France, then x is benevolent). So x is benevolent.
2. (each line is a separate example)
   $p \vee q, \neg q \vdash p$
   $p \rightarrow q$ therefore $\neg q \rightarrow \neg p$
3. (each line is a separate example)
   $\forall x\ (Fx)$ therefore Fa
   $\forall x(Fx) \rightarrow \forall \mathrm{x}(\mathrm{Gx})$ therefore $\forall x(Fx \rightarrow Gx)$

Note: this last one is interesting because it is in fact *invalid*. Can you work out why? This is an argumentation error that can be found in arguments in the press and is worth looking out for. The error lies in the fact the first *x* is expressed and quantified as having scope over only the first predicate, it doesn't follow that the second *x* stands for the same thing. Note any real life instances of this error.

4. $\neg$(is $\rightarrow$ ought) $\rightarrow$ values are not in the world
5. Let *R* be '*x* aims at *y*'
   $\forall x \exists y(\mathrm{Rxy})$ therefore $\forall x \exists! y(Rxy)$