1. DATA ANALYSIS

Uncertainties in calculations

Rule 1 When two measurements are added or subtracted, the uncertainty in the result is always equal to the sum of the uncertainties of the two measurements.

Measurement 1 ; $d_1 \pm \Delta d_1$, where Δd_1 is the uncertainty of the measurement,

Measurement 2 ; $d_2 \pm \Delta d_2$, where Δd_2 is the uncertainty of the measurement,

a) The **difference** between the two measurements $= (d_1 \pm \Delta d_1) - (d_2 \pm \Delta d_2)$

$$= (d_1 - d_2) \pm (\Delta d_1 + \Delta d_2)$$

This is because the difference can range from

a maximum value of $(d_1 + \Delta d_1) - (d_2 - \Delta d_2) = (d_1 - d_2) + (\Delta d_1 + \Delta d_2)$, to

a minimum value of $(d_1 - \Delta d_1) - (d_2 + \Delta d_2) = (d_1 - d_2) - (\Delta d_1 + \Delta d_2)$,

The uncertainty is therefore $\Delta d_1 + \Delta d_2$

b) The **sum** of the two measurements $= (d_1 \pm \Delta d_1) + (d_2 \pm \Delta d_2)$

$$= (d_1 + d_2) \pm (\Delta d_1 + \Delta d_2)$$

This is because the sum can range from

a maximum value of $(d_1 + \Delta d_1) + (d_2 + \Delta d_2) = (d_1 + d_2) + (\Delta d_1 + \Delta d_2)$, to

a minimum value of $(d_1 - \Delta d_1) - (d_2 - \Delta d_2) = (d_1 + d_2) - (\Delta d_1 + \Delta d_2)$,

The uncertainty is therefore $\Delta d_1 + \Delta d_2$

Rule 2 When two measurements are multiplied or divided , the <u>percentage</u> uncertainty in the result is always equal to the sum of the <u>percentage</u> uncertainties of the two measurements.

Measurement 1; $d_1 \pm \Delta d_1 = d_1 \left(1 \pm \frac{\Delta d_1}{d_1}\right)$, where $\frac{\Delta d_1}{d_1}$ (× 100%) is the percentage uncertainty of the measurement ,

Measurement 2 ; $d_2 \pm \Delta d_2 = d_2 \left(1 \pm \frac{\Delta d_2}{d_2}\right)$, where $\frac{\Delta d_2}{d_2}$ (× 100) is the percentage uncertainty of the measurement,

a) The **product** of the two measurements $= d_1\left(1 \pm \frac{\Delta d_1}{d_1}\right) \times d_2\left(1 \pm \frac{\Delta d_2}{d_2}\right)$

$$= d_1 d_2 \left(1 \pm \frac{\Delta d_1}{d_1}\right) \times \left(1 \pm \frac{\Delta d_2}{d_2}\right)$$

$$= d_1 d_2 \left(1 \pm \frac{\Delta d_1}{d_1} \pm \frac{\Delta d_2}{d_2} + \frac{\Delta d_1}{d_1}\frac{\Delta d_2}{d_2}\right)$$

$= d_1 d_2 \left(1 \pm \frac{\Delta d_1}{d_1} \pm \frac{\Delta d_2}{d_2}\right)$ as the last term in the bracket above is negligible compared with the other terms .

∴ The percentage uncertainty in the product $d_1 d_2$ is $\frac{\Delta d_1}{d_1} + \frac{\Delta d_2}{d_2}$ (x 100 %)

a) The **quotient** of the two measurements $= \dfrac{d_1\left(1 \pm \frac{\Delta d_1}{d_1}\right)}{d_2\left(1 \pm \frac{\Delta d_2}{d_2}\right)}$

$$= \frac{d_1}{d_2} \times \frac{\left(1 \pm \frac{\Delta d_1}{d_1}\right)}{\left(1 \pm \frac{\Delta d_2}{d_2}\right)}$$

To progress further with this analysis , we need to use the mathematical approximation

$$\frac{1}{(1 + y)} = (1 - y) \quad \text{for} \quad y << 1$$

This is valid provided $y << 1$ because $(1 + y)(1 - y) = 1 + y - y + y^2 = 1$ (as y^2 is negligible)

$$\therefore \quad \frac{\left(1 \pm \frac{\Delta d_1}{d_1}\right)}{\left(1 \pm \frac{\Delta d_2}{d_2}\right)} = \left(1 \pm \frac{\Delta d_1}{d_1}\right) \times \left(1 \pm \frac{\Delta d_2}{d_2}\right)$$

$$= \frac{d_1}{d_2}\left(1 \pm \frac{\Delta d_1}{d_1} \pm \frac{\Delta d_2}{d_2} \pm \frac{\Delta d_1}{d_1}\frac{\Delta d_2}{d_2}\right)$$

$$= \frac{d_1}{d_2}\left(1 \pm \frac{\Delta d_1}{d_1} \pm \frac{\Delta d_2}{d_2}\right) \quad \text{as the last}$$

term in the bracket above is negligible compared with the other terms .

∴ The percentage uncertainty in the quotient $\frac{d_1}{d_2}$ is $\frac{\Delta d_1}{d_1} + \frac{\Delta d_2}{d_2}$ (× 100 %)

Further note

Where a measured quantity y is raised to a power n ,

the percentage uncertainty in y^n = n times the percentage uncertainty of y.

This follows because y^n = the product of y with itself n times so the percentage uncertainty of y must be added to itself n times to give the percentage uncertainty in y^n.

Examples

1. The diameter of a wire was measured at 0.28 ± 0.01 mm. Calculate

(a) the percentage uncertainty in the diameter of the wire ,

(b) the area of cross-section of the wire,

(c) the percentage uncertainty in the area of cross section of the wire.

Solution

(a) % uncertainty in diameter d = $\frac{0.01}{0.28} \times 100$ = 3.6 %

(b) The area of cross-section $A = \frac{\pi d^2}{4} = \frac{\pi (0.28)^2}{4}\ mm^2 = 6.16 \times 10^{-2}\ mm^2$

(c) % uncertainty in A = 2 × % uncertainty in d because $A = \frac{\pi d^2}{4}$

∴ % uncertainty in A = 7.2 %

2. The mass and length of a certain length of the above wire was measured as below ;-

Mass = 0.496 ± 0.002 grams , Length = 1000 ± 2 mm

Calculate

(a) the density of the wire in kg m^{-3}

(b) the percentage uncertainty of the density

(c) the uncertainty of the density

Hence state the calculated density value including the uncertainty of this value.

Solution

(a) Area of cross-section , $A = 6.16 \times 10^{-2} \times 10^{-6}$ m2

∴ Volume, V = length × area of cross-section = $1.00 \times 6.12 \times 10^{-8}$ m^3

Mass , $m = 0.496 \times 10^{-3}$ kg

∴ Density = $\frac{\text{Mass}}{\text{Volume}} = \frac{0.496 \times 10^{-3}\ \text{kg}}{6.16 \times 10^{-8}\ \text{m}^3} = 8050\ \text{kg m}^{-3}$

(b) % uncertainty in length $L = \frac{2}{1000} \times 100 = 0.2\ \%$

Because the volume V = length L × area of cross-section A,

% uncertainty in volume V = % uncertainty in length + % uncertainty in A = 7.4%

% uncertainty in mass $m = \frac{0.002}{0.496} \times 100 = 0.4\ \%$

∴ % uncertainty in density = % uncertainty in m + % uncertainty in V = 7.8%

(c) Uncertainty of the density = 7.8 % of 8050 kg m^{-3} = 628 kg m^{-3}.

∴ Density = 8050 ± 630 kg m^{-3}