3. FURTHER PROOFS

1. Proof of the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Fig 3A The lens formula (see Textbook p153)

In Fig 3A, the ratio $\frac{\text{image height}}{\text{object height}} = \frac{v}{u}$

The ratio is also equal to $\frac{v - f}{f}$ (similar triangles)

$$\therefore \frac{v}{u} = \frac{v - f}{f}$$

$$vf = vu - uf$$

$$\therefore vf + uf = vu$$

Dividing each term by uvf therefore gives the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

2. Proof that the moment of a horizontal rule of weight *W* and length *L* about one end = ½ *WL*

Fig 3B Centre of gravity of a rule (see Fig 1.19, Textbook p18)

Consider the rule divided along its length into n equal elements of length δx, where n is a large even number.

The weight of each element = W / n.

The moment of the first element from the pivot = $\frac{W}{n} \times 0.5\,\delta x = \frac{W\,\delta x}{2n}$

The moment of the second element from the pivot = $\frac{W}{n}(0.5\,\delta x + \delta x) = 3\frac{W\,\delta x}{2\,n}$

The moment of the r^{th} element from the pivot = $(2r - 1)\frac{W\,\delta x}{2n}$

$\therefore$ The total moment about the pivot = $\{1 + 3 + \ldots\ldots\ldots + (2n-1)\}\frac{W\,\delta x}{2n}$

It can be shown that if n is an even number, $\{1 + 3 + \dots\dots + (2n-1)\} = n^2$

Hence the total moment about the pivot $= n^2 \dfrac{W\delta x}{2n} = W\dfrac{n\delta x}{2} = W\dfrac{L}{2}$

For other regular shapes, the total moment is determined using integration which is a more generalised mathematical method of adding the contributions of each element.