4. EXERCISES ON ESSENTIAL MATHEMATICS (Textbook p499)

A. ALGEBRA

A1. Symbols and equations in physics (p3/4)

Q1 Rearrange each of the following equations and hence find the value of y;-

(a) $3y = 15$, (b) $2y + 5 = 9$, (c) $\frac{y}{4} = 6$, (d) $\frac{5y}{3} = 35$, (e) $\frac{2y - 3}{3} = 5$

Q2 Rearrange each of the following equations to make z the subject of the equation;-

(a) $yz = 8$, (b) $3z - 2w = 6$, (c) $\frac{z}{p} = 5$ (d) $\frac{az}{b} = c$

Q3 (a) Rearrange $\rho = \frac{m}{V}$ to make (i) m (ii) V the subject of the equation,

(b) Rearrange $C = 2\pi r$ to make r the subject of the equation,

(c) Rearrange $A = \frac{\pi d^2}{4}$ to make d^2 the subject of the equation

(d) Rearrange $T^2 = \frac{4\pi^2}{g} L$ to make L the subject of the equation.

Click here for answers to A1

A2. Using the dynamics equations (p33)

Q1 (a) Rearrange $v = u + at$ to make (i) a the subject, (ii) t the subject.

(b) Rearrange $s = \frac{(u + v)t}{2}$ to make (i) t the subject , (ii) v the subject .

Q2 (a) Rearrange $s = ut + \frac{1}{2}at^2$ with $u = 0$ to make a the subject.

(b) Rearrange $v^2 = u^2 + 2as$ to make (i) s the subject, (ii) a the subject

Q2 (a) Combine $v = u + at$ and $s = \frac{(u + v)t}{2}$ to eliminate u

(b) Hence show that $s = vt - \frac{1}{2}at^2$

Q3 A rocket launched vertically from the ground accelerates at constant acceleration a for time t_1 before the rocket motors cut off. Show that

(a) its speed when the motors cut off , $v = at_1$

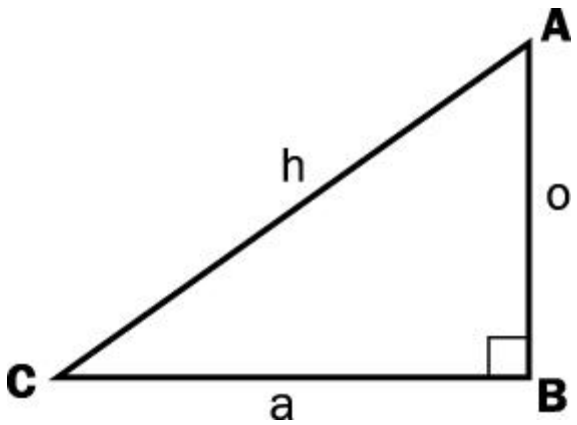
(b) its height at the instant the motors cut off , $h = \frac{1}{2}at_1^2$

(c) its maximum height $H = \frac{1}{2}\left(1 + \frac{a}{g}\right) a t_1^2$

Click here for answers to A2

A3. Pythagoras theorem (p13)

Fig 4A Trigonometry



Q1 Calculate h , given (a) o = 3 , a = 4 (b) o = 5 , a = 12 (c) o = 9 , a = 40

Q2 Calculate o given (a) h = 13 , a = 5 , (b) h = 5 , a = 3 (c) h = 41, a = 9

Q3 Calculate a given (a) h = 10 , o = 8 , (b) h = 39, o = 15 (c) h = 205 , o = 45

Click here for answers to A3

A4. Simultaneous equations (p198)

Q1 Solve the following pairs of equations for y and z ;-

(a) 2y + 3z = 7 , 2y + 4z = 8 (b) y + 2z = 6 , 2y + z = 9

Q2 Solve the following pairs of equations for p and q ;-

(a) 4p - 2q = 16 , 3p + q = 17 (b) 5q - 3p = 11 , p + 2q = 11

Q3 Solve the following equations for E / volts and r / ohms ;-

(a) $E = 2r + 4$, $E = r + 8$ (b) $2E - 4r = 2$, $E - r = 2$

Click here for answers to A4

B. CALCULUS

B1. Rates of change in dynamics (p38,p46)

Q1 An object travels along a straight line without change of direction. Its distance , *s* (in metres) , from a fixed point on the line changes with time *t* (in seconds) in accordance with the equation

$$s = 10t + 6$$

(a) Complete the following table;-

***t* / seconds**	0.00	1.0	2.0	3.0	4.0
***s* / metres**	6.00	16.00			

(b) Calculate the distance moved Δs from (i) 0 to 1.0 s, (ii) 1.0 to 2.0 s, (iii) 2.0 to 3.0 s.

(c) Plot a graph to show how the distance, *s*, changed with time *t*.

(d) Show that the speed is constant and find its value.

Q2 An object rolls down a slope without change of direction . its distance , *s* (in metres) from the bottom of the slope changes with time *t* (in seconds) in accordance with the equation

$$s = 2.5 - 0.10t^2$$

(a) Complete the following table ;-

***t* / seconds**	0.00	1.0	2.0	3.0	4.0	5.0
***s* / metres**	2.5	2.4	2.1	1.60		

(b) Calculate the distance moved Δs from (i) 0 to 1.0 s, (ii) 2.0 to 3.0 s, (iii) 4.0 to 5.0 s.

(c) Plot a graph to show how the distance, *s*, changed with time *t*.

(d) Use your calculations in (b) to explain how the speed of the object changed from 0 to 5 s.

Q3 The speed , v (in m s^{-1}) of an object rolling down a slope changed with time t (in seconds) in accordance with the equation

$$v = 12 + 4t$$

(a) Complete the following table ;-

***t* / seconds**	0.00	1.0	2.0	3.0	4.0	5.0
***v* / m s^{-1}**	12					

(b) Calculate the increase of speed Δv from (i) 0 to 1.0 s, (ii) 2.0 to 3.0 s , (iii) 4.0 to 5.0 s.

(c) Plot a graph to show how the speed , v, changed with time t.

(d) Hence show that the acceleration of the object is 4 m s^{-2}.

Click here for answers to B1

B2. Differentiation of x^n and e^x (p222)

Q1 Use the differenentiation rule $\frac{d}{dx}(x^n) = n x^{n-1}$ to differentiate each of the following;-

(i) $2x^3$, (ii) $4x^5$, (iii) $x^3 - 2x$, (iv) $4x - 3x^2$, (v) $\frac{1}{3}x^4$

Q2 Use the differentiation rule $\frac{d}{dx}(e^{kx}) = k e^x$ to differentiate each of the following ;-

(i) $2e^x$ (ii) e^{3x} (iii) e^{-x} (iv) $3e^{-x}$ (v) e^{-2x}

Q3 Differentiate each of the following functions for s with respect to t to give an expression for $\frac{ds}{dt}$ in terms of t ;-

(i) $s = 5t + 3t^2$, (ii) $s = 4 + 2t - 6t^2$, (iii) $s = 3t - 5t^3$, (iv) $s = 4t^3 - 2t^2$

Q4 For constant acceleration, the displacement of an object $s = ut + \frac{1}{2}at^2$ where u is the initial velocity of the object. Show that

(a) by differentiating s with respect to t , the velocity at time t , $v = u + at$,

(b) by differentiating v with respect to t, the acceleration of the object $= a$.

Click here for answers to B2

B3. Rates of change in exponential decay ;

Capacitor discharge (p222)

Q1 The charge Q (in μC) on a capacitor decays with time t (in seconds) in accordance with the equation $Q = 100\, e^{-0.4t}$

(a) Complete the following table ;-

***t* / seconds**	0.00	1.0	2.0	3.0	4.0	5.0
***Q* / μC**	100					

(b) Calculate the loss of charge ΔQ from (i) 0 to 1.0 s, (ii) 2.0 to 3.0 s, (iii) 4.0 to 5.0 s.

(c) Hence calculate the average current in the interval from (i) 0 to 1.0 s, (iii) 4.0 to 5.0 s.

Q2 The potential difference V (in V) across a capacitor decays with time t (in seconds) in accordance with the equation when the capacitor is discharged through a 2.2 kΩ resistor .

$$V = 12\, e^{-0.3t}$$

(a) Complete the following table ;-

***t* / seconds**	0.00	1.0	2.0	3.0	4.0	5.0
***V* / volts**	12					

(b) Plot a graph to show how the potential difference decreases with time .

(c) Use your graph to determine the time taken for the p.d. to fall to 0.37 of its initial value and hence determine the value of the capacitance.

Click here for answers to B3

Radioactive decay (p363)

Q1 The number of nuclei N of a sample of a radioactive isotope decreases with t (in seconds) in accordance with the equation

$$N = N_o e^{-0.4t} \qquad \text{where } N_o = 5.0 \times 10^{20}.$$

(a) Complete the following table ;-

***t* / seconds**	0.0	1.0	2.0	3.0	4.0	5.0
N	5.0×10^{20}					

(b) Plot a graph to show how N decreases with time .

(c) Use your graph to determine the half life of the sample.

Click here for answers to Radioactive decay

B4. Differentiation of a sine function (p310)

Q1 Use the differentiation rule $\frac{d}{dt}(\sin bt) = b\cos bt$ to differentiate each of the following expressions ;-

(i) $\sin 2t$, (ii) $3\sin t$, (iii) $\sin 0.5t$, (iv) $2\sin 0.5t$, (v) $3\sin 5t$

Q2 Use the differentiation rule $\frac{d}{dt}(\cos bt) = -b\sin bt$ to differentiate each of the following expressions ;-

(i) $\cos 3t$, (ii) $2\cos t$, (iii) $\cos 0.1t$, (iv) $3\cos 4t$, (v) $0.5\cos 4t$

Q3 For a moving object with a displacement s from a fixed point, its velocity $v = \frac{ds}{dt}$ and its acceleration $a = \frac{dv}{dt}$.

Use the above differentiation rules to show that

(i) the velocity of an object $v = \omega r \cos \omega t$, if its displacement s varies sinusoidally with time

in accordance with the equation $s = r \sin \omega t$.

(ii) the acceleration , a , of the object in (i) is given by the equation $a = - \omega^2 s$

Click here for answers to B4

C. GRAPHS

C1. Straight line graphs (p35, p95, p99)

Q1 Write down (a) the gradient , (b) the y-intercept for each of the following straight line equations ,

(i) $y = 2x$, (ii) $y = 3x + 5$, (iii) $y = 6 - 2x$, (iv) $y = 3 + 2x$, (v) $y = {}^3/_2 x - 4$

Q2 (a) Plot each of the following equations on a graph covering the range from $x = -2$ to $x = +2$;-

(i) $y = 4x + 1$, (ii) $y = 2x + 3$

(b) Show that the two lines intercept each other at the point $x = 1$, $y = 5$

Q3 The speed v (in m s^{-1}) of an object varies with time t (in s) in accordance with the equation

$$v = 20 - 4t$$

(a) Sketch a graph to show how the speed varies with time,

(b) Show that (i) the speed of the object at $t = 0$ is 20 m s^{-1} , (ii) the acceleration of the object = 4.0 m s^{-2}.

Click here for answers to C1

C2. Inverse square law (p249, p340, p441)

Q1 The force F (in newtons) between two ions varies with their distance apart r (in metres) in accordance with the equation

$$F = \frac{k}{} \quad \text{, where } k = 2.0 \times 10^{-27} \text{ N m}^2$$

r^2

(a) Complete the following Table ,

r / 10^{-9} m	0.5	1.0	1.5	2.0	2.5	3.0
F / N		2.0×10^{-9}				

(b) Plot a graph to show the variation of force with distance from 0.5 to 3.0 nanometres.

Q2 A geiger counter was used to measure the count rate due to a source of gamma radiation. The count rate C (in counts per second) varied with distance r (in metres) from the source in accordance with the equation

$$C = \frac{2.0}{r^2} .$$

(a) Calculate the count rate at a distance of (i) 0.010 m , (ii) 0.020 metres from the source ,

(b) The background count rate was 0.4 counts per second. Calculate the distance from the source at which the count rate due to the source would equal the background count rate.

Q3 The force , in newtons, between a comet and the Sun varies with distance (in metres) from the centre of the Sun in accordance with the equation

$F = \frac{k}{r^2}$, where $k = 4.0 \times 10^{32}$ N m^2 .

(a) Complete the following Table ,

r / 10^9m	0.50	1.0	1.5	2.0	2.5	3.0
F / N		4.0×10^{14}				

(b) Plot a graph to show the variation of force with distance from 0.5 to 3.0 $\times 10^9$ m.

Click here for answers to C2

C3. Rates of change

dynamics - see Calculus ; B1 Rates of change in dynamics

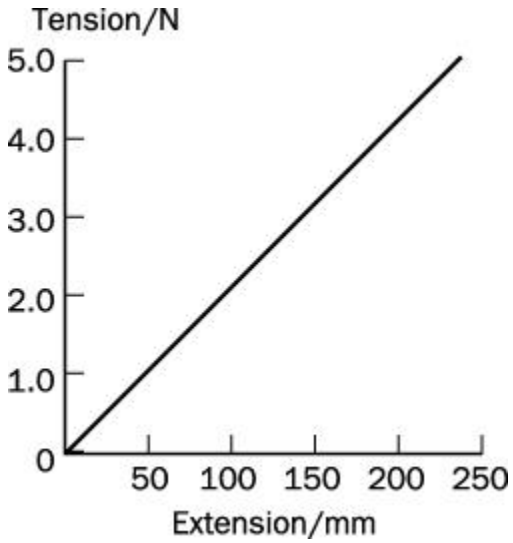
exponential decay - see Calculus ; B3 capacitor discharge

Calculus ; B3 radioactive decay

C4. Area under a curve (p103, p218, p446)

Q1 The graph below shows how the tension T in a spring varies with its extension, e.

Fig 4B Tension v extension for a spring



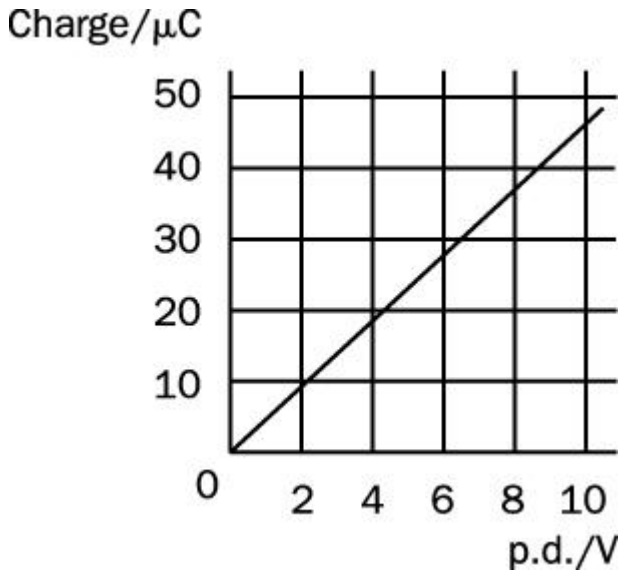
(a) Determine the spring constant k .

(b) (i) Use the graph to determine the energy stored in the spring when its extension is 0.10 m

(ii) Use the equation *Energy stored* = ½ ke^2 to calculate the energy stored at an extension of 0.10 m.

Q2 The graph below shows how the p.d. *V* across a capacitor varies with the charge *Q* stored by the capacitor.

Fig 4C Charge v p.d. for a capacitor



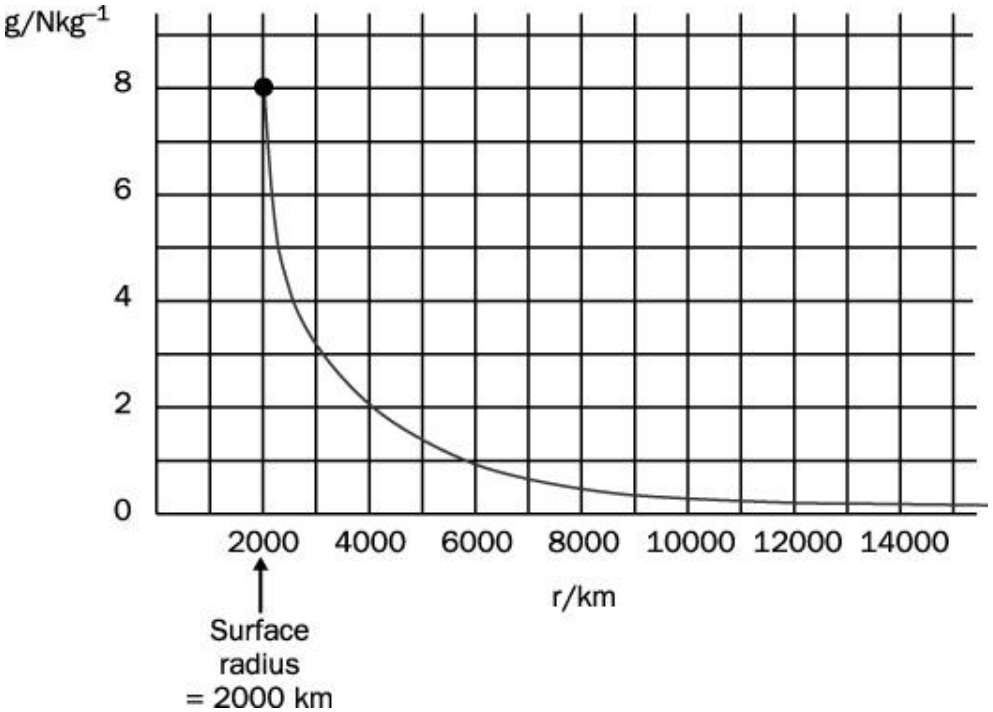
(a) Determine the capacitance *C* of the capacitor.

(b) (i) Use the graph to determine the energy stored in the capacitor when the p.d. is 10.0 V.

(ii) Use the equation $Energy\ stored = \frac{1}{2} C V^2$ to calculate the energy stored at a p.d. of 10.0 V.

Q3 The graph below shows how the gravitational field strength *g* varies with distance *r* from the centre of a planet.

Fig 4D *g* v *r* a spherical planet



(a) Show that the change of gravitational potential energy of a 1 kg mass when it is moved from the surface of the planet to infinity is 16 MJ.

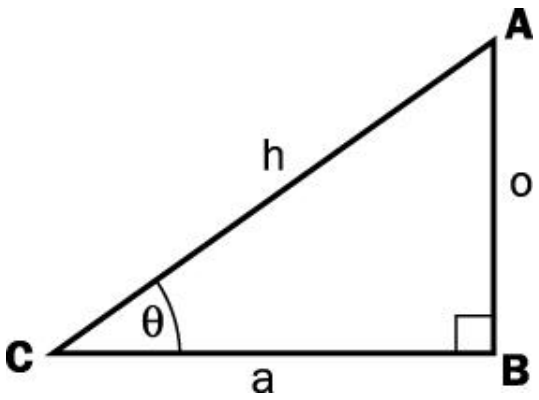
(b) Hence show that the escape speed from the planet is 5600 m s^{-1}.

Click here for answers to C3

D. TRIGONOMETRY

D1. Sin , cos and tan (p13, p54, p148)

Fig 4E



Q1 In Fig 4E, calculate

(a) AB if (i) AC = 5.0 m and $\theta = 30^{o}$, (ii) BC = 8.0 m and $\theta = 60^{o}$

(b) BC if (i) AC = 4.0 m and $\theta = 50^{o}$, (ii) AB = 2.0 m and $\theta = 25^{o}$

(c) AC if (i) AB = 3.0 m and $\theta = 70^{o}$, (ii) BC = 2.5 m and $\theta = 45^{o}$

Q2 Use Fig 4E to explain why

(a) (i) $\sin\theta = \cos(90 - \theta)$, (ii) $\cos\theta = \sin(90 - \theta)$,

(b) (i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$, (ii) $\tan 45 = 1$

Q3 Complete the Table below which refers to the right-angled triangle shown in Fig 4F.

Fig 4F

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
θ / °	30.00			50			60
AB / m		8.0	4.0		12.0	3.0	
BC / m	6.0	4.0		8.0	8.0		

AC / m			5.0			9.0	4.0

Click here for answers to D1

D2. Sine wave formulae (p288, p301, p 456)

Q1 For a frequency , f , of 100 Hz, calculate (i) $2\pi f t$ in radians , (ii) $\sin(2\pi f t)$ when

(a) t = 1.0 ms, (b) 2.0 ms , (c) 2.5 ms

Q2 An alternating voltage varies with time (in seconds) in accordance with the equation

$$V = V_o \sin(2\pi f t)$$

(a) For f = 50 Hz and V_o = 12.0 volts, calculate the voltage when time t is (i) 2.0 ms,

(ii) 4.0 ms , (iii) 7.5 ms , (iv) 16 ms (v) 25 ms ,

(b) For f = 1000 Hz and V_o = 5.0 volts , calculate the least value of time t when V is equal to

(i) 2.5 V, (ii) 3.0 V , (iii) 4.0 V , (iv) 5.0 V

Click here for answers to D2

D3. Circular measures (p431)

Q1 Convert each of the following angles to radians ;-

(a) (i) 30^o , (ii) 45^o , (iii) 90^o , (iv) 135^o , (v) 180^o

(b) Calculate the length s of an arc of a circle of radius 0.20 m that subtends and angle to the centre

of the circle of (a) 30^o , (b) 45^o , (c) 90^o , (d) 135^o , (e) 180^o .

Q2 Calculate the angle in (i) radians, (ii) degrees subtended to the centre of a circle of radius

0.50 m by an arc along the circle of length (a) 0.10 m , (b) 0.20 m, (c) 0.50 m , (d) 2.0 m

Q3 Calculate the radius of a circle which contains a segment with an arc of length 0.15 m that

subtends an angle of (a) 10^o , (b) 30^o to the centre of the circle.

Click here for answers to D3

E. USING A CALCULATOR

E1. Standard form and powers of ten (p2/3)

Q1(a)Write down the symbol for each of the following prefixes and state its value as a power of ten ;-

(i) nano- (ii) micro- (iii) milli- (iv) kilo- (v) mega- (vi) giga-

(b)Write each of the following measurements in standard form in the stated unit;-

(i) 300,000 km /s in m s^{-1} , (ii) 31, 000, 000 s in s , (iii) 590 nm in metres

Q2 For each of the following pairs of numbers , calculate (i) their sum , (ii) their difference and express the answer in standard form ;-

(a) 3615 , 2132 (b) 1.65×10^{4} , 12328 (c) 6.75×10^{19} , 5.41×10^{19}

(d) 3.49×10^{6} , 4.5×10^{5} (e) 3.46×10^{-6} , 2.15×10^{-6}

Q3 For each of the following pairs of numbers , calculate (i) their product (ii) the first number divided by the second number and express the answer in standard form to 3 significant figures ;-

(a) 3615 , 2132 (b) 1.65×10^{4} , 12328 (c) 6.75×10^{19} , 5.41×10^{19}

(d) 3.49×10^{6} , 4.5×10^{5} (e) 3.46×10^{-6} , 2.15×10^{-6}

Click here for answers to E1

E2. Raising a number to a given power (p74, p423)

Q1 (a) Raise each of the following numbers to the fourth power and express the answer in standard form to 2 significant figures ;-

(i) 2.5 , (ii) 3.6×10^{3} , (iii) 4.5×10^{10} , (iv) 7.4×10^{-3} , (v) 0.35

(b) Calculate the cube root of each of the numbers above and express the answer in standard form to 3 significant figures.

Q2 Raise (a) 4.6×10^{8} (b) 6.7×10^{-10} to each of the following powers and express the answer in standard form to 3 significant figures.

(i) 0.2 (ii) 0.5 (iii) 2.5 (iv) $^{1}/_{4}$ (v) $^{2}/_{5}$

Q3 In the equation $p\,q^{n} = 100$, calculate and express the answer in standard form to 3 significant figures

(a) p when q = 120 and n = (i) 4 (ii) 0.5 (iii) 1.4 (iv) -1.67 (v) - 0.5

(b) q when p = 40 and n = (i) 2 (ii) - 0.5 (iii) 1.5 (iv) - 3 (v) 10

Click here for answers to E2

E3. Using the 'ln' key and the 'e' key of a calculator (p221, p 341)

Q1 Calculate (a) e^{x}, (b) e^{-x} and express the answer in standard form to 3 significant figures for x = (i) 0.5, (ii) 1, (iii) 1.5, (iv) 2.0, (v) 2.5

Q2 Calculate ln x and express the answer in standard form to 3 significant figures for

(a) x = (i) 0.01, (ii) 0.1, (iii) 1.0, (iv) 10, (v) 100

(b) x = (i) 0.03, (ii) 0.3, (iii) 3.0, (iv) 30, (v) 300

Q3 (a) Calculate e^{px} and express the answer in standard form to 3 significant figures for x = 1.5 and p = (i) -1.5, (ii) 0, (iii) 1.5

(b) Calculate ln qx and express the answer in standard form to 3 significant figures for x = 10 and q = (i) 0.04, (ii) 0.40, (iii) 4.0

Click here for answers to E3

F. VECTORS

F1 Representing vectors (1.1 , p11)
Parallelogram rule (1.1 , p12)
Resolving a vector into perpendicular components (1.1 , p12)

Q1 (a) On a coordinate grid , draw to scale and measure the x and y components of each of the following vectors ;-

(i) a displacement vector of magnitude 30 m at an angle of 60^{o} North of due East ,

(ii) a force vector of magnitude 100 N at an angle of 70^{o} North of due West,

(b) Check your answers to (a) (i) and (ii) by calculating the x and y components for each of the above vectors.

Q2 (a) Fig 4G shows a force vector of magnitude F at angle θ to the x-axis .

Fig 4G

Calculate the x and y components of the vector if (i) F = 10 N and $\theta = 30^{o}$ (ii) F = 8.0 N and $\theta = 70^{o}$

(b) In the diagram above , calculate the magnitude F and the y-component if (i) $\theta = 60^{o}$ and the x-component = 8.0 N , (ii) $\theta = 45^{o}$ and the x-component = 6.0 N

Q3 An airport is located 10.0 km due North of the centre of a city. An aircraft takes off from the air port and passes over a prominent landmark which is 30.5 km in a direction 35^{o} East of due North from the airport .

(a) (i) On a coordinate grid , using an appropriate scale, mark the locations of the airport , the city centre and the landmark.

(ii) Determine how far East and how far North the landmark is from the city centre.

(b) When the aircraft passes over the landmark, it is travelling on a level flight path at a constant velocity of 120 m s^{-1} in a direction 70^{o} East of due North.

(i) Resolve the aircraft's velocity into a component due East and a component due North.

(ii) Determine how far East and how far North the aircraft is from the city centre 100 s after passing over the landmark.

Click here for answers to F1

F2. Vector addition (pp12-14)

Q1 By means of a scale drawing , determine the resultant of

(a) a force F_1 of 6.0 N acting due East and a force F_2 of 10. 0 N acting due North on a point object ,

(b) a force F_1 of 6.0 N acting due East and a force F_2 of 10. 0 N acting 30^0 east of due North acting on a point object.

Q2 Check your answers to Q1 by means of calculations.

Q3 (a) By means of a scale drawing , determine the resultant of a force of 6.0 N and a force of 9.0 N acting at an angle of (i) 60^o , (ii) 135^o to one another,

(b) Check your answers to (a) by means of calculations.

Click here for answers to F2

F3 i and j vectors (p13)

Q1 (a) Represent each of the following vectors on the same coordinate grid ;- (i) 2.0 i + 3.0 j

(ii) 4.0 i + 2.5 j

(b) Write down in i,j form (i) the sum, (ii) the difference a(i) -a(ii) of the two vectors above

Q2 (a) Express each of the two forces in Q1(a) in i, j form and hence express the resultant of the two forms in i , j form. Hence determine the magnitude of the resultant .

(b) Express each of the two forces in Q1(b) in i, j form and hence express the resultant of the two forms in i , j form. Hence determine the magnitude of the resultant .

Q3 (a) Calculate the magnitude of each of the force vectors , given its i and j components in newtons ;- (i) 3.0 i + 4.5 j (ii) 2.4 i - 5.5 j

(b) Calculate the sum of the above two force vectors, expressing your answer (i) in i, j form , (ii) as a magnitude and an angle to the x-axis .

(c) Calculate the difference ,(i) - (ii) , of the above two force vectors, expressing your answer (i) in i, j form , (ii) as a magnitude and an angle to the x-axis .

Click here for answers to F3