2 PRODUCTION, PRICES, AND THE DISTRIBUTION OF INCOME

1.

a) We can write $Y_i = P_i^{-\sigma} P^{\sigma} Y$ and take the derivative:

$$\frac{dY_i}{dP_i} = -\sigma P_i^{-\sigma-1} P^{\sigma} Y \,,$$

$$\frac{dY_i}{dP_i}\frac{P_i}{Y_i} = -\sigma P_i^{-\sigma-1} P^{\sigma} Y \frac{P_i}{Y_i} = -\sigma \frac{P_i^{-\sigma} P^{\sigma} Y}{Y_i} = -\sigma \,.$$

b) The inverse demand function shows what price you have to set to sell a certain quantity. Solving the demand function for the price we get
$P_i = P Y^{1/\sigma} Y_i^{-1/\sigma}$.

c) $$\frac{dP_i}{dY_i}\frac{Y_i}{P_i} = P_i = P Y^{1/\sigma}\left(-\frac{1}{\sigma}\right) Y_i^{-1/\sigma-1}\frac{Y_i}{P_i} = -\frac{1}{\sigma}\frac{P Y^{1/\sigma} Y_i^{-1/\sigma}}{P_i} = -\frac{1}{\sigma} \,.$$

d) Revenue: $R_i = Y_i P_i = Y_i P Y^{1/\sigma} Y_i^{-1/\sigma} = P Y^{1/\sigma} Y_i^{1-1/\sigma}$.

Marginal revenue: $\dfrac{dR_i}{dY_i} = P Y^{1/\sigma}\left(1-\dfrac{1}{\sigma}\right) Y_i^{-1/\sigma} = \left(1-\dfrac{1}{\sigma}\right) P_i$.

The marginal revenue is smaller than the prices because the firm has to reduce the price in order to sell more.

2.

a) Solving the production function for N_i we get $N_i = Y_i^{\frac{1}{1-\alpha}} K_i^{\frac{-\alpha}{1-\alpha}}$, so the labour cost is

$W_i N_i = W_i K_i^{\frac{-\alpha}{1-\alpha}} Y_i^{\frac{1}{1-\alpha}}$.

The labour cost is higher if we produce more and lower if we have more capital.

b) Taking the derivative with respect to production we get the marginal cost

$$MC = W_i \frac{1}{1-\alpha} Y_i^{\frac{1}{1-\alpha}-1} K_i^{\frac{-\alpha}{1-\alpha}} = \frac{1}{1-\alpha} W_i Y_i^{\frac{\alpha}{1-\alpha}} K_i^{\frac{-\alpha}{1-\alpha}} = \frac{1}{1-\alpha} W_i \left(\frac{Y_i}{K_i}\right)^{\frac{\alpha}{1-\alpha}} .$$

c) The marginal cost is higher if the wage is higher and also if production is higher relative to the capital stock. With higher production for a given capital stock, the firm has to employ more workers. With more workers on a given capital stock, the marginal product of labour is lower, so the marginal cost is higher.

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3.

a) The price is set with a mark-up on marginal cost. The marginal cost is the wage divided by the marginal product of labour. If, for example, one worker earns 100 euros per day and the additional worker increases production (for given capital stock) by 5 units, the cost per unit of the additional units is 20 euros.

b) $MPL = K^{\alpha}(1-\alpha)N^{-\alpha} = (1-\alpha)\dfrac{K^{\alpha}N^{1-\alpha}}{N} = (1-\alpha)\dfrac{Y}{N} = (1-\alpha)APL$.

c) $P = (1+\mu)\dfrac{W}{MPL} = (1+\mu)\dfrac{W}{(1-\alpha)Y/N}$ and thus $\dfrac{WN}{PY} = \dfrac{1-\alpha}{1+\mu}$.

d) $\dfrac{WN}{PY} = \dfrac{1-0.40}{1+0.20} = \dfrac{0.60}{1.20} = \dfrac{1}{2}$.

e) $\dfrac{WN}{PY} = \dfrac{1-0.28}{1+0.08} = \dfrac{0.72}{1.08} = \dfrac{2}{3}$. In this case, capital is a less important production factor and the mark-up is lower, and both factors increase the labour share of income.

4. $P_iY_i - cY_i = P_i\kappa P_i^{-\sigma} - c\kappa P_i^{-\sigma} = \kappa P_i^{1-\sigma} - c\kappa P_i^{-\sigma}$. Taking the derivative with respect to the price we get the first order condition for profit maximization:

$$\frac{dY_i}{dP_i} = (1-\sigma)\kappa P_i^{-\sigma} - c\kappa(-\sigma)P_i^{-\sigma-1} = 0.$$

Cancelling (dividing by) $\kappa P_i^{-\sigma}$ on both sides and solving for the price we get

$P_i = \dfrac{\sigma}{1-\sigma}c = \dfrac{1}{1/\sigma - 1}c$. Ändrade från 1/(1-(1/sigma))

The mark-up on marginal cost (c) depends on the price elasticity σ.

5.

a) Limits on the number of firms that enter individual markets will reduce competition, increase mark-ups and pure profits, and reduce the labour share of income. (As we will see in chapter 5, investment and the long run level of capital will also decrease.)

b) The labour share of income depends on the technology and on the degree of competition in the goods market so laws that increase the bargaining power of unions will not affect the labour share. (As we will see in Chapter 6, the level of income will fall, however, because the natural rate of unemployment will increase.)

c) Increased taxes on non-labour incomes and reduced taxes on wage income will leave the labour share before tax unchanged, so the labour share of after tax income will increase. (As we will see in Chapter 5, the long run level of income may fall, however, because there will be a negative effect on investments and the long run level of the capital stock.)

d) A law stipulating reduced working hours is equivalent to a reduction in the technology factor E. It will reduce the level of income without any effect on the labour share. (In an exercise in chapter 6, we will analyse how employment and long run income is affected.)

6.

a) Production is $Y_1 = K^{\alpha}\left(E\left(1-u\right)L\right)^{1-\alpha} = K^{0.3}\left(E\left(1-0.10\right)L\right)^{0.7}$.
With five percent unemployment we instead have
$Y_2 = K^{\alpha}\left(E\left(1-u\right)L\right)^{1-\alpha} = K^{0.3}\left(E\left(1-0.05\right)L\right)^{0.7}$.

The relative increase in production is found from

$$\frac{Y_2}{Y_1} = \frac{K^{0.3}\left(E\left(1-0.05\right)L\right)^{0.7}}{K^{0.3}\left(E\left(1-0.10\right)L\right)^{0.7}} = \frac{0.95^{0.7}}{0.90^{0.7}} = \left(\frac{0.95}{0.90}\right)^{0.7} \approx 1.039 .$$

Production increases by 3.9 percent which is less than the percentage increase in labour input. With a fixed capital stock, the marginal product of labour is lower than the average product of labour.

b) With higher employment, the marginal product of capital increases, so the incentives to invest increase. In the long run, the capital stock should increase. (This adjustment is analysed in Chapter 5.)

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