1.

a) If K=10 and N=40 production is $Y = \sqrt{10 \cdot 40} = 20$.

b) If K=20 and N=80 production is $Y = \sqrt{20 \cdot 80} = 40$.

c) With this production function there is constant returns to scale so a doubling of inputs leads to a doubling of production.

2.

a) The capital stock is reduced from K^* to K_0. With a lower capital stock, the return on investment is higher so there will be high saving and investment until the capital stock is back at the steady state.

b)

K, Y

K

Y

time

t₀

I

I

time

t

r

r

time

t
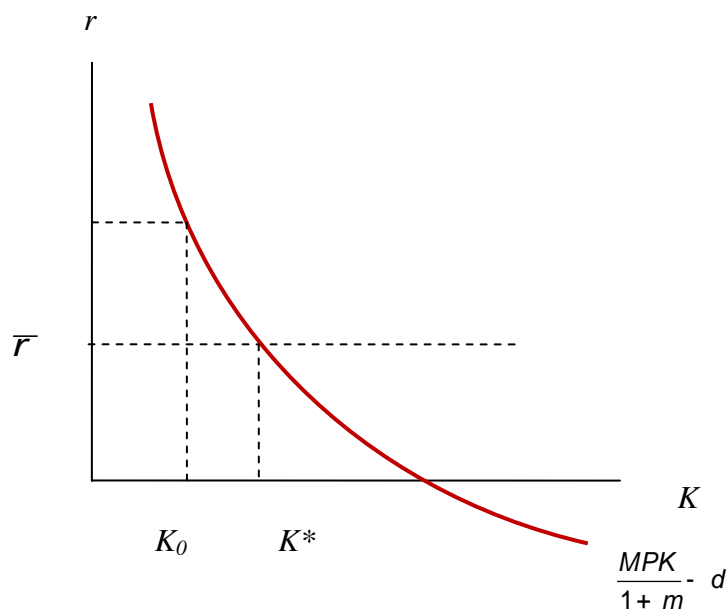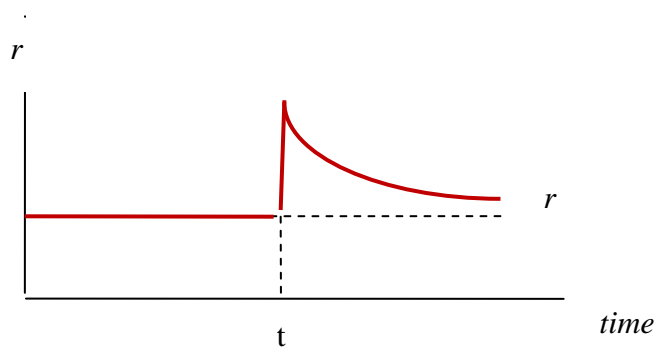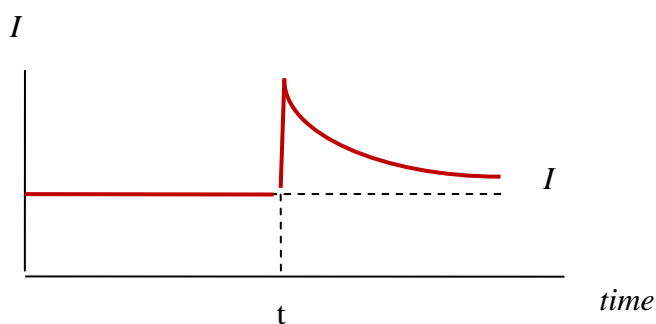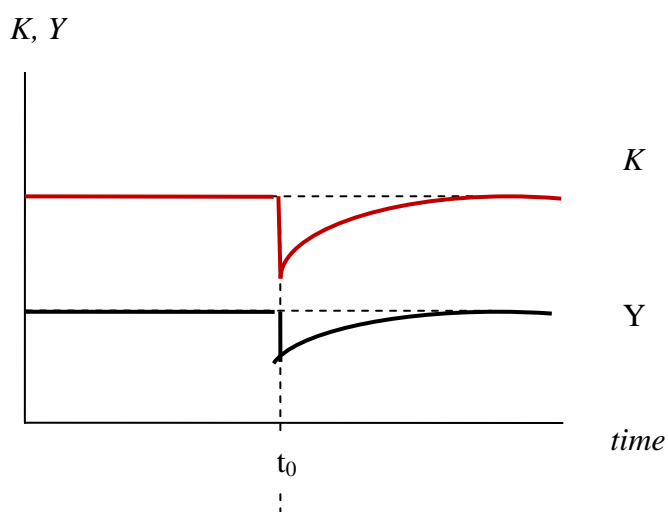
3.

a) As before, the capital stock is reduced. If five percent of the working-age population dies the marginal product of capital will be lower for each level of capital so the schedule showing the real return on capital will shift inwards. The steady state level of capital is reduced and so is the long run level of production.

r

$\overline{r}$

K

$\dfrac{MPK}{1+m} - d$

K_0 $K*$ $K*$

b)

K, Y

K

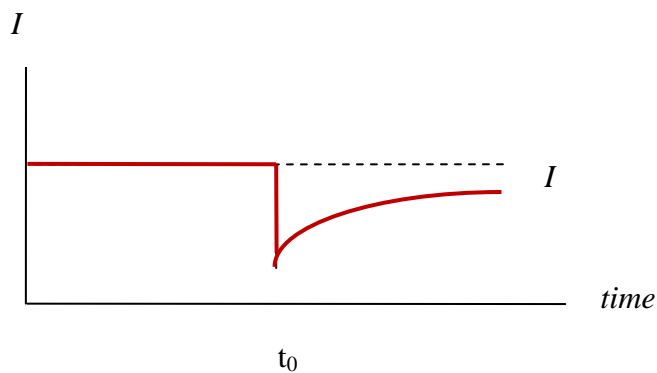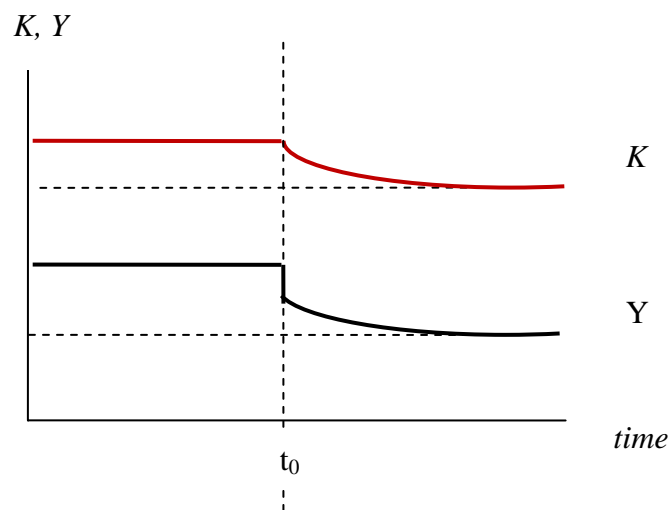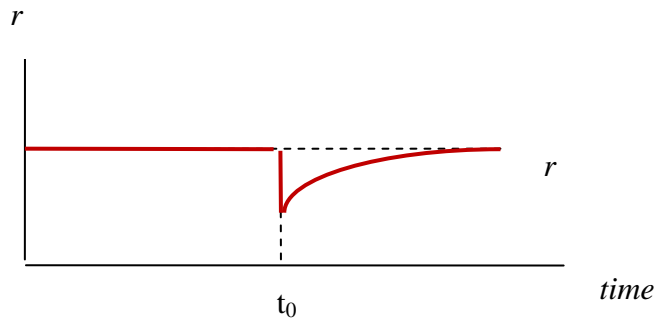Y

t_0

I

I

t_0
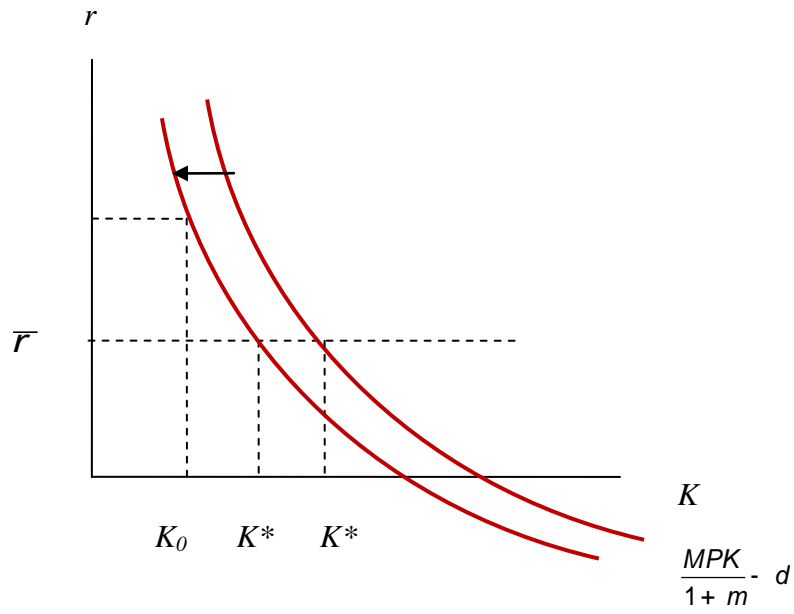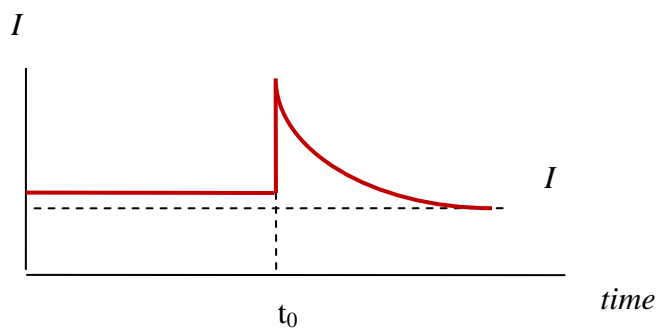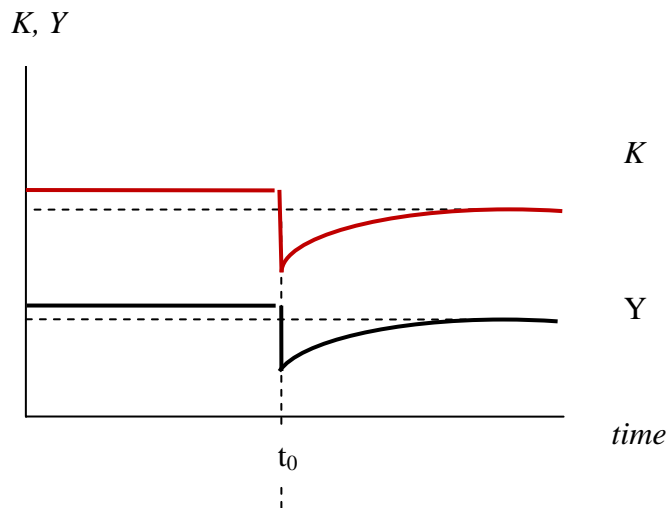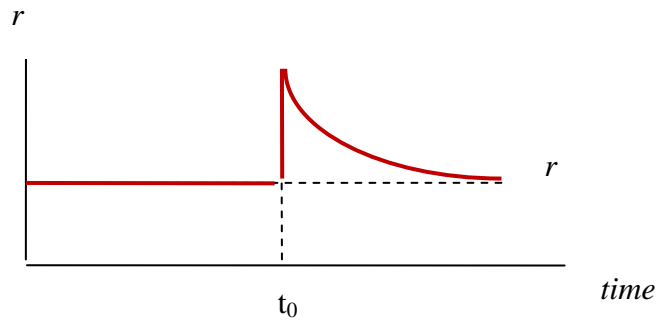
time

time

4. Let us add working time to the model and assume that the production function is $Y = F(K, EHN)$ where H is working time.

a) $\dfrac{f'(k)}{1+\mu} - \delta = r$ where $k = \dfrac{K}{EHN}$.

b)   The steady state capital stock and production will fall by 5 percent.

c)

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5. For convergence to occur countries must have access to the same technology and the same level of education. Then the return to investment will be higher in a country with a small capital stock per worker. This means that there will be more saving and investment in the country with a small capital stock, so it will catch up with the countries which have a big capital stock per worker. (If the poor country can borrow abroad it can speed up investments and convergence; see chapter 13.)

6. This is a matter of judgement, but a number of institutional factors are missing in poor countries: infrastructure such as transportation systems, educational systems, judicial system and law and order. These factors reduce the return to private investments.

7. In a growing economy, consumers expect to have higher income in the future than today. If the interest rate was equal to the subjective rate of discount, this would mean that consumers would want to consume more than their income today so as to smooth consumption over time. But in a closed economy, they can only consume the income that is not used for investment. The real interest rate has to be higher than the subjective rate of discount in order to "convince" consumers not to consume more than is available. (In a completely open economy, the real interest rate is determined by the world real interest rate; see Chapter 13.)

8. Since much research and development (R&D) produces knowledge that can be freely copied by others, those who do it do not get the full return on their investment. Therefore, there is a case for subsidizing some kinds of R&D.

9. There may be externalities from educated workers as their knowledge spreads to other workers. Another reason for subsidising education is that children, or their parents, may not have the financial resources to make profitable investments in education, i.e. there are liquidity constraints.

10. If there was no depreciation of capital
   a) the long run levels of the capital stock and production would be higher.

b) the growth rates of the capital stock and production would be unaffected. They would still grow at the same rate as the effective workers: $g+n$.

11. If capital income is taxed at the rate τ the incentive to invest is reduced and the long run levels of capital and income will fall.
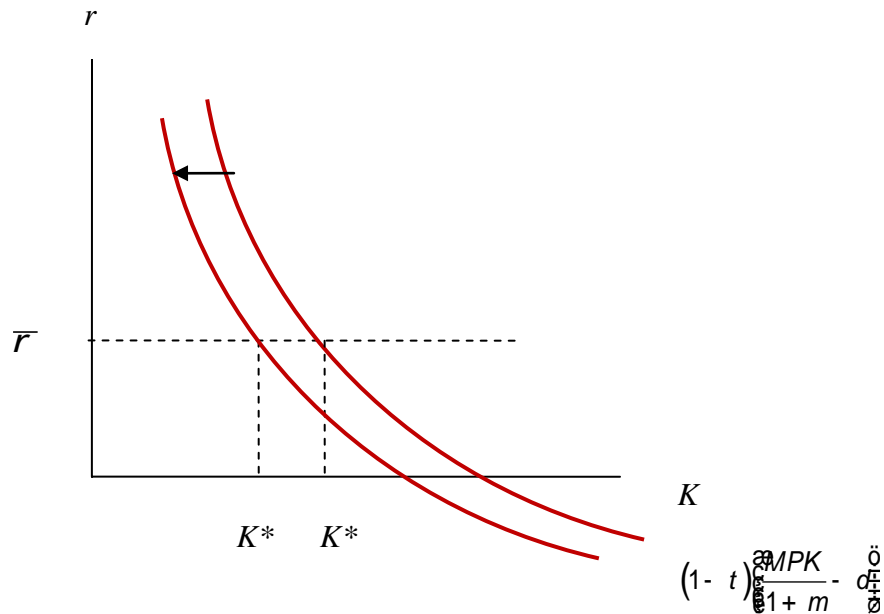
12.

a) The first order condition for consumption says that the marginal utility of consumption today divided by the discounted marginal utility of consumption next year should be equal to the relative price between goods today and goods next year i.e. one plus the real interest rate.

b) In this case, the marginal utility is $1/C_t$ so we have

$$\frac{C_{t+1}}{C_t} = \frac{1+r_{t+1}}{1+\rho}$$

$$1+r = (1+\rho)(1+g) \quad r = (1+\rho)(1+g) - 1 = 1.03\cdot 1.03 - 1 = 0.0609 \approx 0.06.$$

Approximately: $r \approx \rho + g = 0.03 + 0.03 = 0.06$.

c) If the real (net) return is taxed at the rate τ the above first order condition still holds with respect to the after-tax return, so we have

$$\frac{C_{t+1}}{C_t} = \frac{1+(1-\tau)r_{t+1}}{1+\rho}$$

$$r = \frac{(1+\rho)(1+g)-1}{1-0.3} = \frac{1.03\cdot 1.03 - 1}{0.7} = \frac{0.0609}{0.7} \approx 0.09$$

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The required return on capital increases to about 9 percent so the capital stock will be lower.

d) If the mafia extracts a fraction λ of the *gross* real return on investments we have

$$\frac{C_{t+1}}{C_t} = \frac{(1-\lambda)(1+r_{t+1})}{1+\rho}$$

$$1+r = \frac{(1+\rho)(1+g)}{1-\lambda} = \frac{1.03\cdot 1.03}{0.9} \approx 1.18.$$

The required pre-extortion return on investment increases to about 18 percent.

13. $r = 0.06:$ $\dfrac{Y}{L} = \sqrt{\dfrac{1/3}{(1+.10)(0.06+0.06)}}\cdot 1000\cdot(1-0.10) = 1430$

$r = 0.09:$ $\dfrac{Y}{L} = \sqrt{\dfrac{1/3}{(1+.10)(0.09+0.06)}}\cdot 1000\cdot(1-0.10) = 1279$

$r = 0.18:$ $\dfrac{Y}{L} = \sqrt{\dfrac{1/3}{(1+.10)(0.18+0.06)}}\cdot 1000\cdot(1-0.10) = 1011.$

A higher required return reduces the capital stock and production per worker.

14. In practice, ownership of capital is very uneven so a tax on capital income can be used to redistribute from the rich to the poor. The problem is that in the long run it will reduce the capital stock and the level of income in the country.

15. For a given capital stock it gives an increase in production which is approximately 5 percent times 2/3 i.e. 3.3 percent:

$$\frac{Y_2}{Y_1} = \frac{K^{1/3}(1.05EN)^{2/3}N}{K^{1/3}(EN)^{2/3}N} = 1.05^{2/3} \approx 1.033.$$

In the long run, the capital stock will increase and production will increase 5 percent.

16. The immigration has the same effect on production and the capital stock as an increase in E in the previous exercise.