Assessing risk: decision-making under uncertainty

On pages 259-61 of *Smart Thinking* we learned a simple method of calculating uncertainty using Bayes’s Theorem. To help you develop your skills I have listed a number of problems below that you can use to practise.

Exercise 1:

As you have been feeling under the weather recently you make an appointment to see your doctor. After she has examined you and given you the all-clear, she asks whether you would agree to take part in a screening programme to test for a genetic defect that has been linked with an aggressive form of cancer in later life. Although you agree, you ask if you can have more information about it. She tells you that the defect has been found in just 1 per cent of people. Quite reasonably you ask about the reliability of the test and she tells you that 90 per cent of tests for the defect successfully detect it, while only 9.6 per cent of the tests result in false positives.

If you were to get a positive test result, what is the probability that you actually have the defect?

Answer:

As we did in *Smart Thinking* set out clearly the four pieces of information you need to determine the base rate (1 and 2 below) and the reliability of the test (3 and 4 below).

Base rate

1. Chances of having the defect – 1 per cent.

2. Chances of not having the defect – 99 per cent.

Reliability

3. Test is accurate – 90 per cent of those who have the defect test

 positive; 90.4 per cent without it test negative.

4. Test is inaccurate – 10 per cent of those with the defect test negative;

 9.6 per cent without it test positive.

Now calculate A and B as we did on page 260 of *Smart Thinking* and substitute the values in the formula.

A = 1 x 3 = 1 x 0.9 = 0.9

B = 2 x 4 = 0.99 x 9.6 = 9.5

Now enter these values into the formula:

 0.9 = 0.9 = 8.65%

 0.9 + 9.5 10.4

Exercise 2:

In a small coastal town north of Manchester it rains on one third of the days. Each day the local paper gives a prediction of whether or not it will rain the following day. Three quarters of rainy days and three fifths of dry days are correctly predicted. Given that today’s paper predicts rain for tomorrow, what is the probability that it will actually rain tomorrow?

Answer:

Base rate

1. Chances of it raining – ⅓ .

2. Chances of not raining – ⅔ .

Reliability

3. Predictions are accurate – ¾ of rainy days; ⅗ of dry days.

4. Predictions are inaccurate – ¼ of rainy days; ⅖ of dry days.

Now calculate A and B:

A = 1 x 3 = ⅓ x ¾ = ¼

B = 2 x 4 = ⅔ x ⅖ = ⁴⁄₁₅

Now enter the values in the formula:

 ¼ = ¼ = 60 = 15 = 4.839%

 ¼ + ⁴⁄₁₅ 31 124 31

 60

Exercise 3:

In an exam 60 per cent of students know the correct answer to a particular problem. However, there is a 15 per cent chance that a student will choose the wrong answer even if he/she knows the right answer and there is also a 25 per cent chance that a student who doesn’t know it will guess correctly. If a student did get the answer right, what is the chance that this student really knows the answer?

Answer:

Base rate

1. Chances of knowing the right answer – 60 per cent.

2. Chances of not knowing the right answer – 40 per cent.

Reliability

3. Accuracy – 85 per cent of students who know the right answer choose it; 75 per cent of students who don’t know it choose the wrong answer.

4. Inaccuracy – 15 per cent of students who know the right answer choose the wrong one; 25 per cent of students who don’t know the right answer choose it.

Now calculate A and B:

A = 1 x 3 = 0.6 x 0.85 = 0.51

B = 2 x 4 = 0.4 x 0.75 = 0.3

Now enter these values into the formula:

0.51 = 0.51 = 62.96%

 0.51 + 0.3 0.81

Exercise 4:

In recent times recessions appear to have occurred every ten years. A think tank has created a model of the economy to predict recessions. It is 80 per cent reliable in predicting them when indeed they are coming and 10 per cent reliable when they are not. If the model predicts a recession what are the chances that a recession will, in fact, come?

Answer:

Base rate:

 1. Chances of this year having a recession – 10 per cent.

 2. Chances of not having a recession – 90 per cent.

Reliability:

 3. The model is accurate – in 80 per cent of those years in which it

 predicted a recession one occurred; in 10 per cent of those in which it

 predicted there would not be one, a recession did not occur.

 4. The model is inaccurate – in 20 per cent of those years in which it

 predicted a recession, it did not occur; in 90 per cent of those years in

 which it predicted there would not be a recession, one occurred.

A = 1 x 3 = 0.1 x 0.8 = 0.08

B = 2 x 4 = 0.9 x 0.9 = 0.81

 0.08 = 0.08 = 0.0899 = 8.99%

 0.08 + 0.81 0.89

Exercise 5:

You’re watching a Premier League game between Manchester United and Leicester City in a bar in London. You see someone in the bar who is obviously a Manchester United supporter and you wonder, ‘What is the probability that they were actually born within 25 miles of Manchester?’

Calculate the probability of this, given the following:

* The probability that a randomly selected person in a typical local bar in London was born within 25 miles of Manchester is 1 in 20.
* The probability that a person born within 25 miles of Manchester actually supports Manchester United is 7 in 10.
* The probability that a person not born within 25 miles of Manchester supports Manchester United is 1 in 10.

Answer:

As in the questions above, make clear the four points, calculate A and B and then enter the values in the formula. What we are trying to find out is the value of the following:

 P(U/B) x P(B)

 P(U/B) x P(B) + P(U/BO) x P(BO)

ie.

P(U/B) = Probability of being MU supporter and born in Manchester

P(B) = Probability of being born in Manchester

P(U/BO) = Probability of being a MU supporter and being born outside Manchester

P(BO) = Probability of being born outside Manchester

1. The probability that someone is a Manchester United supporter and was born in Manchester – 7/10.

2. The probability that someone is a Manchester United supporter and was not born in Manchester – 1/10.

3. The probability of randomly finding someone in a bar who was born in Manchester – 1/20; the probability of randomly finding someone in a bar who was born outside Manchester and is not a Manchester United supporter – 9/10.

4. The probability of randomly finding someone in a bar who was born in Manchester and is not a Manchester United supporter – 3/10; probability of randomly finding someone in a bar who was not born in Manchester – 19/20.

 0.7 x 0.05

 (0.7 x 0.05) + (0.1 x 0.95)

 = 0.035

 0.035 + 0.095

 = 0.035 = 0.2692 = 26.92%

 0.13

The following are more general probability problems. They will give you the chance to practise your skills and learn different techniques and principles that you will need to solve similar problems both in your professional life and in psychometric problems.

Exercise 6:

There are 100 students applying for summer jobs in a university’s geology/geography department. Ten of the students have never taken a course in geology or geography. Sixty-three of the students have taken at least one geology course. Eighty-one have taken at least one geography course.

What is the probability that of the 100 applicants any student selected at random has taken either geography or geology, but not both?

How many students have taken at least one course in both geology and geography?

Answer:

If 81 students had taken a course in geography, then only 9 students out of the 90 (10 took neither) took only geology. Since 63 students out of 90 had taken geology, that leaves 27 who had taken only geography.

27 + 9 = 36 out of the 100 applicants

The answer is 36 per cent or nine out of 25.

Since 36 students took either geography or geology and 10 took neither, that leaves 54 per cent who took at least one class in both.[[1]](#endnote-1)1

Exercise: 7

From a well-shuffled pack of cards, what is the probability of being dealt in succession the Ace, the King, the Queen and the Jack of hearts?

Answer:

The probability of being dealt the ace is 1 in 52. Now, with one card less in the pack the probability of being dealt the king is 1 in 51 and so on: the queen, 1 in 50 and the jack 1 in 49. Then, to calculate the answer, you need to multiply all of these figures together:

52 x 51 x 50 x 49 = 6,497,400

So the answer is 1 in 6,497,400

Exercise 8:

If you were to toss a coin six times, what is the probability that it would come up heads each time?

Answer:

The probability of heads coming up on a single toss of the coin is 1 out of 2. This is unchanged from one toss to another. So to calculate the probability of six tosses coming up heads we need to multiply the probability of each toss together:

½ x ½ x ½ x ½ x ½ x ½ = ⅟₆₄

So the probability is 1 in 64.

Exercise 9:

You buy two tickets for a prize draw and you are told that you have a two-in-thirty chance of winning something. So what are your odds?

Answer:

You calculate the odds of a favourable event occurring by dividing the probability of the favourable event by the probability of an unfavourable event.

Odds of a favourable event = probability of the favourable event

 probability of an unfavourable event

 ²⁄₃₀ = ²⁄₂₈

 ²⁸⁄₃₀

So, the odds are 2 to 28.

1. 1 Terry Stickels, *The Big Book of Mind-Bending Puzzles* (New York: Sterling, 2002), p. 32. [↑](#endnote-ref-1)