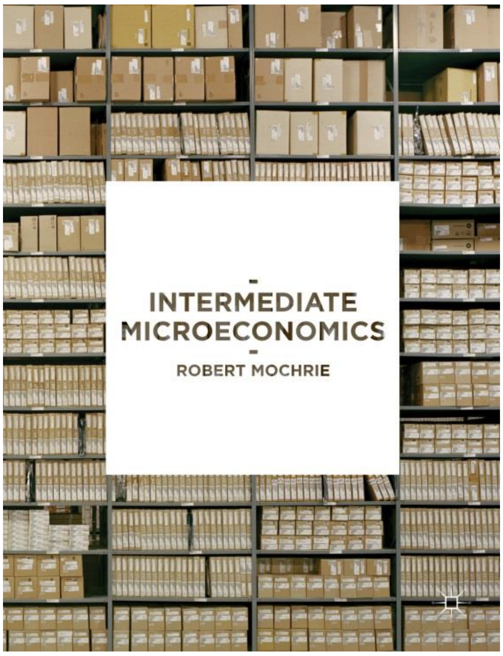
# Solutions Manual: Part III

# Resource Allocation for Firms

## Summary answers to the 'By yourself' questions



## Chapter 10

X10.1 **Confirm that, given input prices $w$ for labour and $r$ for capital, the acquisition cost $A$: $A(L, K) = wL + rK$ is weakly convex.**

*Consider the set of bundles the acquisition cost, $X = \{(L, K): A(L, K) \geq A_0\}$. For all bundles on the boundary of that set, $wL + rK = A$. We recognize this as the equation of straight line, and so any linear combination of two points on the boundary of the set also lies on the boundary of the set. Therefore the acquisition cost is weakly convex.*

X10.2 **A firm has to choose between using technology 1, for which $K/L = \tau_1 = 0.5$, or technology 2, for which $K/L = \tau_2 = \frac{4}{3}$. The firm can produce output $B = 2L$ using the technology 1 and $B = 4L$ using technology 2. Both technologies are perfectly divisible.**

a) **Sketch a diagram showing the input combinations associated with technologies 1 and 2. Indicate on the diagram which is the more capital intensive.**

*Drawing the diagram with capital usage on the horizontal, and labour usage on the vertical, axis, input combinations using technology 1 lie on the line $L = 2K$, an upward sloping straight line passing through the origin, and with gradient 2. Input combinations using technology 2 lie on the line $L = ¾K$. This is upward sloping, passes through the origin, and has gradient 0.75. Technology 2 is more capital intensive.*

b) **Now indicate on the diagram the input combinations for which the firm might be able to produce its target output, $B$ = 1,200. (Remember to calculate the firm's capital usage.)**

*The firm can use any linear combination of technology 1 and technology 2 to produce its output. Using only technology 1, it would need to hire the combination $(K_1, L_1) = (300, 600)$ to produce output $B$ = 1,200, or else, using technology 2, $(K_2, L_2) = (400, 300)$.*

c) **Suppose that the firm decides to hire the input combination $(K, L) = (350, 450)$. It then allocates 150 units of capital and 300 units of labour to production using technology 1, and uses the remaining 200 units of capital and 150 units of labour to produce output using technology 2. Calculate the quantity of output produced using each technology, and the firm's total output.**

*The firm produces 600 units output using technology 1 and 600 units of output using technology 2. By using both technologies, the firm produces an output, $B$ = 1,200.*

d) **Repeat the exercise in part (c) using these total input combinations:**

**i. $(K, L) = (375, 375)$, with 25% of output produced using technology 1; and**

*The firm uses 75 units of capital and 150 units of labour to produce 300 units of output using technology 1; and 300 units of capital and 225 units of labour to produce 900 units of output using technology 2.*

**ii. $(K, L) = (325, 525)$, with 75% of output produced using technology 1.**

*The firm uses 225 units of capital and 450 units of labour to produce 900 units of output using technology 1; and 100 units of capital and 75 units of labour to produce 300 units of output using technology 2.*

e) **On your diagram, indicate the input combinations obtained in parts (c) and (d). Hence sketch the isoquant $X$ = 1,200. [*Note:* An isoquant is a curve on which output stays constant as inputs vary. The isoquant connects the five output combinations that you have found in parts (b), (c) and (d).]**

*The least capital-intensive (most labour-intensive) combination of inputs uses technology 1 only, while the most capital-intensive combination uses technology 2 only. We note that in* *replacing the labour intensive with the capital intensive technology, the firm can substitute one unit of capital for every three units of labour. Starting from the input combination (K, L) (300, 600), any combination of inputs for which $600 - L = -3(300 - K)$, or for which $3K + L = 1{,}500$, and with $600 \geq L \geq 300$ will allow the firm to achieve its production target.*

**f) Recalling our discussion of the properties of utility functions (in Section 5.2.1), what do you conclude from the shape of the isoquant about labour and capital?**
*Labour and capital are perfect substitutes.*

X10.3 **Suppose that the firm hires more labour than it uses at A or more capital than it uses at B. What would happen to production? Complete the isoquant passing through A and B.**
*Hiring more labour than at A (and no more capital), or more capital than at B (and no more labour), there will be no change in production. The firm has reached the highest possible labour-intensity in production at A, and the highest possible capital-intensity at B. The isoquant is therefore horizontal to the right of A and vertical above B.*

X10.4 **We have already confirmed that City Bakers might use the hybrid technology to produce its target output using input bundle C. Using this, explain why City might prefer to use the hybrid technology if the new technology would require the input bundle D to meet the production target. Similarly, explain why City might use the new technology if it could meet the production target using input bundle E.**
*If the new technology requires inputs as at D to produce the same output as the hybrid technology does at C, then the hybrid remains a more efficient use of inputs. If the production target can be achieved using combination E, then it is the new technology that uses resources more efficiently.*

X10.5 **Suppose that a firm is in an industry where it needs no capital. Its production function can be written as $B = 20L^{0.5}$, where *B* is output and *L* is the firm's labour usage.**

**a) Show that if the use of labour doubles, output increases by a factor of $\sqrt{2}$.**
*Define $B_0 = 20L_0^{0.5}$; then $B_1 = 20(2L_0)^{0.5} = 20\sqrt{2}\,L_0^{0.5} = \sqrt{2}\,B_0$.*

**b) Find the first and second derivatives, $\frac{dB}{dL}$ and $\frac{d^2B}{dL^2}$.**

$\frac{dB}{dL} = 10L^{-0.5}$ ; $\frac{d^2B}{dL^2} = -5L^{-1.5}$

**c) Sketch graphs showing *B* and $\frac{dB}{dL}$ as a function of *L* and *K*.**
*Showing labour input, L, on the horizontal axis and output, B, on the vertical axis, the graph is one arm of a parabola, proceeding from the origin, initially vertical, always increasing, but at a decreasing rate.*
*Showing L on the horizontal axis and the derivative, $\frac{dB}{dL}$, on the vertical, this is a downward sloping convex curve that is bounded by the axes.*

X10.6 **How realistic is it to suppose that a firm can undertake production without any capital? [*Note:* You may want to consider the examples of simple businesses that we introduced in Chapter 1.]**
*The very simplest businesses can of course undertake production with minimal capital. A market stall might require little more than the cash to buy a day's produce, and pay rental monthly in advance: it operates essentially by making payments from receipts. A hairdresser will often rent a seat in a salon, operating a separate business within a larger shop. A coffee*

*shop may have more equipment, but even that can be rented, so that the main need is for cash from operations. Even a petrol station might have a limited requirement for capital, depending on whether or not the business operating the service owns the site and machinery. If we define capital as the financial resources required to run the business, and the business simply provides a service using resources that are rented for the purpose, then in general, we would expect the capital requirements to be relatively limited. So the assumption that a firm might not have capital is an abstraction from reality, but for small enough businesses, it may not be a poor assumption.*

X10.7 **Suppose that a firm requires an input bundle of labour and capital ($L$, $K$), with its output given by the production function, $B = 20L^{0.5}K^{0.5}$.**

a) **Show that if the use of labour, $L$, doubles, while the use of capital, $K$, remains constant, output increases by a factor of $\sqrt{2}$.**

*Start with inputs ($K_0$, $L_0$), used to produce output $B_0$. Increasing use of labour to $L = 2L_0$, output increases to $B_1 = 20(2L)^{0.5}K^{0.5} = 20\sqrt{2}L^{0.5}K^{0.5}$.*

b) **Show that if the use of capital, $K$, doubles, while the use of labour, $L$, remains constant, output increases by a factor of $\sqrt{2}$.**

*Start with inputs ($K_0$, $L_0$), used to produce output $B_0$. Increasing use of capital to $K = 2K_0$, output increases to $B_1 = 20L^{0.5}(2K)^{0.5} = 20\sqrt{2}L^{0.5}K^{0.5}$.*

c) **Using partial differentiation, find the marginal products of labour and capital, $MP_L$ and $MP_K$.**

*Marginal product of labour, $MP_L = \frac{\partial B}{\partial L} = 10L^{-0.5}K^{0.5}$; marginal product of capital, $MP_K = \frac{\partial B}{\partial K} = 10L^{0.5}K^{-0.5}$.*

d) **Confirm that the marginal product of labour is a decreasing function of labour usage, but an increasing function of capital usage; and that the marginal product of capital is a decreasing function of capital usage, but an increasing function of labour usage.**

*We have to differentiate the marginal products, effectively taking the second partial derivatives of output.*

*$MP_L = \frac{\partial B}{\partial L} = 10L^{-0.5}K^{0.5}$; $MP_K = \frac{\partial B}{\partial K} = 10L^{0.5}K^{-0.5}$;*

*And now differentiating the marginal products,*

$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 B}{\partial L^2} = -5L^{-1.5}K^{0.5} < 0$; $\frac{\partial MP_L}{\partial K} = \frac{\partial^2 B}{\partial K \partial L} = 5L^{-0.5}K^{-0.5} > 0$;

$\frac{\partial MP_K}{\partial L} = \frac{\partial^2 B}{\partial L \partial K} = 5L^{-0.5}K^{-0.5} > 0$; $\frac{\partial MP_K}{\partial K} = \frac{\partial^2 B}{\partial K^2} = -5L^{0.5}K^{-1.5} < 0$.

*The marginal product of each factor decreases as usage of that factor increases; while the marginal products increase when the usage of the other factors increase.*

e) **Suppose that both capital and labour usage double. What is the effect on the firm's output, and on the marginal products of labour and capital?**

*Define initial factor usage ($K_0$, $L_0$), and output $B_0 = B(K_0, L_0) = 20K_0^{0.5}L_0^{0.5}$; we also define marginal products, $MP_K(K_0, L_0) = 10K_0^{-0.5}L_0^{0.5}$; $MP_L(K_0, L_0) = 10K_0^{0.5}L_0^{-0.5}$.*

*Doubling inputs to ($2K_0$, $2L_0$), output $B_1 = B(2K_0, 2L_0) = 20(2K_0)^{0.5}(2L_0)^{0.5} = [2^{0.5}]^2B_0$;*

*Applying the argument to marginal products,*

*$MP_K(2K_0, 2L_0) = 10(2K_0)^{-0.5}(2L_0)^{0.5} = [2^{0.5-0.5}]MP_K(K_0, L_0) = MP_K(K_0, L_0)$;*

*while $MP_L(2K_0, 2L_0) = 10(2K_0)^{0.5}(2L_0)^{-0.5} = [2^{0.5-0.5}]MP_L(K_0, L_0) = MP_L(K_0, L_0)$.*

*We see that by doubling both inputs, output also doubles, but marginal products remain unaffected.*

X10.8 **Consider the production functions:**

**(1) $X = K^{0.5} + L^{0.5}$; (2) $Z = [K^{0.5} + L^{0.5}]^2$**

a) **For each of these functions, obtain the marginal products of labour and capital.**

*We obtain the marginal products by partially differentiating the production function with respect to a single factor.*

(1) $MP_K = 0.5K^{-0.5}$; $MP_L = 0.5L^{-0.5}$.

(2) $MP_K = K^{-0.5}[K^{0.5} + L^{0.5}] = 1+\left(\frac{L}{K}\right)^{0.5}$; $MP_L = L^{-0.5}[K^{0.5} + L^{0.5}] = 1+\left(\frac{K}{L}\right)^{0.5}$.

b) **Using partial differentiation, confirm that for both functions the marginal product of labour is decreasing in labour usage; and that the marginal product of capital is decreasing in capital.**

(1) *Differentiating $MP_K$ with respect to K, $\frac{\partial MP_K}{\partial K} = -0.25K^{-1.5}$; and differentiating $MP_L$ with respect to L, $\frac{\partial MP_L}{\partial L} = -0.25L^{-1.5}$. The negative signs of the partial confirm that the marginal product of a factor decreases as usage of that factor increases.*

(2) *Differentiating $MP_K$ with respect to K, $\frac{\partial MP_K}{\partial K} = -0.5L^{0.5}K^{-1.5}$; and differentiating $MP_L$ with respect to L, $\frac{\partial MP_L}{\partial L} = -0.5K^{0.5}L^{-1.5}$.*

c) **Confirm that for function (1) the marginal product of labour remains constant as capital usage increases, but that for function (2) the marginal product of labour increases as capital usage increases.**

(1) *We observe that L does not appear in the expression for $MP_K$; and that K does not appear in the expression for $MP_L$. On differentiating, $\frac{\partial MP_K}{\partial L} = \frac{\partial MP_L}{\partial K} = 0$, and the marginal product of one factor does not depend on usage of the other one.*

(2) *Differentiating $MP_K$ with respect to L, $\frac{\partial MP_K}{\partial L} = 0.5L^{-0.5}K^{-0.5}$; and differentiating $MP_L$ with respect to K, $\frac{\partial MP_L}{\partial K} = 0.5K^{-0.5}L^{-0.5}$. Every factor in both of these expressions is certainly positive, and so the marginal product of one factor increases as usage of the other one increases.*

d) **Suppose that usage of capital and labour doubled. For each production function, explain what would happen to output, to the marginal product of labour, and to the marginal product of capital.**

(1) *Doubling inputs, output will increase by a factor $\sqrt{2}$; the marginal product of labour will decrease by a factor $\frac{1}{\sqrt{2}}$; and the marginal product of capital will also decrease by a factor $\frac{1}{\sqrt{2}}$.*

(2) *Doubling inputs, output will also double. The marginal product of each factor remains constant.*

X10.9 **Why might it be reasonable to expect the marginal product of both factors to be decreasing? [*Hint:* Remember that usage of the other factor is held constant, and that 'Too many cooks spoil the broth.']**

*We expect marginal products to be (eventually) decreasing to prevent the situation where there are continuing increasing marginal returns, so that factor productivity increases constantly as intensity of usage increases. Taking the example of a small business, holding its capital usage constant, were the marginal product of labour always increasing, it would continue to increase the efficiency of production whenever it expanded labour use, so that a single, small factory might dominate an industry.*

X10.10 **Suppose that a firm whose output, *X*, can be modelled using the Cobb-Douglas production function, *X*: *X*(*K*, *L*) = $AK^{a}L^{1-a}$, expands its activities while maintaining its technology. The parameter *a* < 1.**

**a) Show that the marginal product of capital, $MP_K = \frac{\partial X}{\partial K} = \frac{aAL^{1-a}}{K^{1-a}} = \frac{aX}{K}$, and that the marginal product of labour, $MP_L = \frac{\partial X}{\partial L} = \frac{(1-a)AK^{a}}{L^{a}} = \frac{(1-a)X}{L}$.**

*The calculations should be familiar from previous exercises with Cobb-Douglas utility functions. We simply apply the power rule of differentiation and rearrange the expression to obtain the partial derivatives in terms of the input factors; and rearrange it again to obtain the expression in terms of output and the variable input factors.*

**b) Confirm that both marginal products are decreasing functions of the variable factor.**

$\frac{\partial MP_K}{\partial K} = \frac{\partial^2 X}{\partial K^2} = -\frac{a(1-a)AL^{1-a}}{K^{1-a+1}} = -\frac{a(1-a)AL^{1-a}}{K^{2-a}} = -\frac{a(1-a)X}{K^2}$. *By assumption, 0 < a < 1, so 0 < 1 – a < 1. This ensures that every expression within the partial derivative is greater than zero, so that the sign of the partial derivative is given by the negative sign. Since* $\frac{\partial MP_K}{\partial K} < 0$*, the marginal product of capital is decreasing in use of capital. A similar argument holds for use of labour.*

X10.11 **Suppose that with the CES production functions in X10.8, a firm hires only labour. How much output can it produce? How reasonable do you consider this to be?**

*The production functions are*

*(1)* $X = K^{0.5} + L^{0.5}$; *(2)* $Z = [K^{0.5} + L^{0.5}]^2$

*So for K = 0, in case (1),* $X = L^{0.5}$*, while for case 2, Z = L. We allow these firms to produce output with use of capital. We have argued (in X10.6) that even the smallest firm will require some capital, although its production process might be very labour intensive. We might, though, want to rule out the possibility of production with no capital at all.*

X10.12 **In terms of the discussion of substitutes and complements in Chapter 9, how would you define capital and labour in X10.8? What conclusions might we draw about desirable values of the elasticity of substitution?**

*We here define capital and labour as net substitutes, meaning that if the price of capital increases, its use will decrease, but the use of labour will increase. Restricting our attention to CES production functions, we might also conclude from the observation that it is sensible to require firms to use minimum levels of capital and labour input that we should consider capital and labour often to be net complements, in the sense that when the price of capital increases, there is limited substitution of labour, so that usage of both will decrease. This would be consistent with the elasticity of substitution being in the interval between zero and one.*

X10.13 **Explain why the straight-line segments are consistent with production for any given technology being homogeneous of degree 1.**

*The straight line segments mean are consistent with the marginal product of factors remaining constant as scale of production changes using any given technology. By definition, production would then be HOD1.*

X10.14 **The convex shape has a vertical segment above point A and a horizontal segment to the right of point B. Explain briefly why this might occur. [*Hint:* Think about the range of technologies available to the firm.]**

*This results from there being 'most capital-intensive' and 'most labour-intensive' technologies. Additional use of the intensively used factor cannot increase output.*

X10.15 **Suppose that bakeries can choose from a continuum of technologies, with each technology defined by a unique capital: labour ratio, $\tau$: $\tau_1 \leq \tau \leq \tau_2$. As in X10.2 and X10.3, bakeries can combine inputs using different technologies, with output, *B*, linear in capital usage, *K*, for all technologies.**

a) **Using diagrams, demonstrate that it is impossible that firms might choose to use every available technology when there is a concave segment of an isoquant. Discuss why we should now expect isoquants to be smooth curves.**

*We have implicitly assumed that there will be constant returns to scale in production, and that technologies are perfectly divisible. So, if there is a concave segment of the production function, it will be possible to draw a line segment connecting points on that segment, which lies below the curve showing the inputs required to produce a fixed level of output. This line segment represents linear combinations of other technologies that can produce the target output with lower inputs than the pure technology.*

*The concave segment is then rather like the point D in X10.4. To produce the target output using technology D, rather than a linear combination of other technologies that is a hybrid version of the technology, is inefficient.*

*Where there is a continuum of divisible technologies, each of which is more efficient than a combination of alternatives, then we never expect to see firms using combinations of technologies, and so we would not expect small changes in technology to lead to sudden changes in the MRTS, ensuring that isoquants are smooth.*

b) **Discuss how the technology index, $\tau$, and the marginal rate of technical substitution change as production becomes more capital intensive.**

*As production becomes more capital intensive, technology $\tau$: $K = \tau L$ decreases in value. Defining MRTS = $-\frac{MP_K}{MP_L}$, and assuming diminishing marginal products, as we substitute capital for labour, (moving down and to the right on an isoquant), $MP_K$ decreases and $MP_L$ increases. This means that MRTS increases – but since MRTS < 0, the absolute value, |MRTS|, decreases, so that the isoquant becomes flatter.*

X10.16 **Show that if a production function is homogeneous of degree *r*, then the firm faces diminishing returns to scale if *r* < 1, constant returns to scale if *r* = 1, and increasing returns to scale if *r* > 1.**

*By definition of homogeneity, we require for any scalar t, and for all input combination (K, L), that output $X(tK, tL) = t^r X(K, L)$. Increasing scale of production by a factor t leads to an increase in output of factor $t^r$. So, if r < 1, the increase in scale of production is less than the increase in scale of inputs, which means that there are diminishing returns to scale. If r = 1, then the increase in scale of production is equal to the increase in scale of inputs, which means that there are constant returns to scale. If r > 1, then the increase in scale of production is greater than the increase in scale of inputs, which means that there are increasing returns to scale.*

X10.17 **Discuss the effect on our arguments about the creation of a hybrid technology if returns to scale were always decreasing.**

*Were returns to scale always decreasing, then the graph showing inputs required for linear combinations of existing technology to produce a target output would be a segment of a convex curve. This strengthens the claim that isoquants will be convex.*

X10.18 **Use the definition of homogeneity to confirm which of these production functions are homogeneous, stating the degree of homogeneity.**

a) $X = \min[cK, d(L - L_0)]$  b) $X = sK + tL$  c) $X = 50.\ln(K + L)$  d) $X = K^a L^{1-a}$

*The requirement for homogeneity of degree r is that $X(tK, tL) = t^r X(K, L)$.*

*a) For $X(K_1, L_1) = \min[cK_1, d(L_1 - L_0)]$, $X(tK_1, tL_1) = \min[ctK_1, d(tL_1 - L_0)]$. It is sufficient here to note that $d(tL_1 - L_0) = t(dL_1) - dL_0$, and that this expression is not homogeneous.*

*b) For $X(K_1, L_1) = sK_1 + tL_1$, $X(\lambda K_1, \lambda L_1) = s(\lambda K_1) + t(\lambda L_1) = \lambda X(K_1, L_1)$, so that X is HOD1.*

*c) For $X(K_1, L_1) = 50.\ln(K_1 + L_1)$, $X(\lambda K_1, \lambda L_1) = 50.\ln(\lambda K_1 + \lambda L_1) = 50.\ln(\lambda(K_1 + L_1)) = 50\ln\lambda + 50.X(K_1 + L_1)$. The function is not homogeneous.*

*d) For $X(K_1, L_1) = K_1^a L_1^{1-a}$, $X(\lambda K_1, \lambda L_1) = (\lambda K_1)^a(\lambda L_1)^{1-a} = \lambda X(K_1, L_1)$, so that X is HOD1.*

X10.19 **Sketch an isoquant map for the production function in X10.18a, and illustrate the output expansion path. Discuss how the technology of production changes as output increases. [*Hint:* Start by sketching the isoquants for the function $X = \min(cK, dL)$.]**

*Showing usage of capital on the horizontal, and usage of labour on the vertical, axis, we find that the isoquants are L-shaped, but that the vertex of each one lies on the line $L = L_0 + \frac{c}{d}K$. This line is upward sloping, but intersects the vertical axis at $(0, L_0)$.*

X10.20 **Define the marginal products of capital and labour for each of the production functions in X10.18, and – except for part (a) – the marginal rate of technical substitution.**

**a) $X = \min[cK, d(L - L_0)]$ b) $X = sK + tL$ c) $X = 50.\ln(K + L)$ d) $X = K^a L^{1-a}$**

*a) $MP_K = c$ when $\min[cK, d(L - L_0)] = cK$; $MP_L = d$, when $= \min[cK, d(L - L_0)] = d(L - L_0)$*

*b) $MP_K = s$; $MP_L = t$; MRTS $= -\frac{MP_K}{MP_L} = -\frac{s}{t}$.*

*c) $MP_K = \frac{\partial X}{\partial K} = \frac{50}{K+L}$ (Note that $\frac{d}{dx}\ln x = \frac{1}{x}$; we also apply the chain rule here). Similarly, $MP_L = \frac{\partial X}{\partial L} = \frac{50}{K+L}$. MRTS $= -\frac{MP_K}{MP_L} = -1$.*

*d) $MP_K = aK^{(1-a)}L^{1-a}$; $MP_L = (1-a)K^a L^{-a}$; MRTS $= -\frac{MP_K}{MP_L} = -\frac{aL}{(1-a)K}$.*

X10.21 **Suppose that production becomes more capital intensive as output increases. Draw a curve on a diagram that could represent the output expansion path, which illustrates the combination of factor inputs that the firm uses at different production levels. Show the marginal rate of technical substitution for at least three points on this path, and sketch possible isoquants passing through them.**

*Showing capital usage on the horizontal axis and labour usage on the vertical axis, the income expansion path will be upward sloping and concave. The marginal rate of technical substitution is constant, moving along the expansion path. Isoquants should be drawn so that they have the same gradient wherever they meet the expansion path.*

## Chapter 11

X11.1 **Using the definition above, why might we argue that workers who supply labour services also provide a business with short-term financing?**
*It is usual for workers to be paid in arrears, perhaps on a monthly basis. During the month, the wages accrue as a debt within the business' accounts, but are not paid until the end of the month. In some countries, financially distressed businesses will delay making wage payments to workers, effectively using their workers as a source of working capital.*

X11.2 **If a firm becomes insolvent, it is unable to repay all of its debts. Given the accounting definition of equity capital as the residue returned to the owners after payment of debts, what is the capital of an insolvent business?**
*The company has no capital: it should appear as a negative number in the accounts – and if the business owners do not have limited liability, then they would be liable to make good the shortfall in capital to allow the full repayment of all debts.*

X11.3 **In a legal definition, we might consider the owners of a business to be the people who have an entitlement to some share of the money left after the payment of all liabilities.**

**(1) If a business is structured as a *partnership*, then the partners are the owners, but have unlimited liability for the debts of the business.**

**(2) If a business is structured as a *limited company*, then the shareholders are the owners, but have limited liability for the debts of the business.**

**Discuss the different ways in which the owners of these businesses will be affected if an insolvency event occurs.**
*Partners are individually liable for all debts of the firm. In an insolvency event, creditors may choose to pursue partners, either collectively or separately, for full repayment of all debts. Shareholders are strictly liable for the debts of the company to the extent of their unpaid share capital; and since most capital issued 'fully paid,' there will be no additional liability in an insolvency event.*

X11.4 **In the example, we have found the bakery's cost of producing a fixed quantity of baked goods. Assume that the bakery's production target, $B$, is variable, but that factor input prices remain the same. Confirm that Expression 11.5 is the condition for cost minimization, and that the cost function is $C: C(B) = 4B$.**

*We can write the bakery's problem as* $\min\limits_{K,L} 20L + 5K : \left[K^{0.5} + L^{0.5}\right]^2 = B$. *We again apply the solution method set out in Chapter 9, but using the condition from Expression 11.5: that the marginal rate of technical substitution equals the ratio of input prices:*

$MRTS = -\frac{MP_K}{MP_L} = -\left(\frac{L}{K}\right)^{0.5} = -\frac{r}{w} = -0.25$, *so that $K = 16L$.*

*Then $25L = B$, so that $L = 0.04B$ and $K = 0.64B$. Then $C = 20L + 5K = 0.8B + 3.2B = 4B$.*

X11.5 **Assume that a chemical company can produce a volume $D$ of detergent, and that the production function takes the form $D = 98[K^{-1} + L^{-1}]^{-1}$, where $K$ and $L$ are the quantities of factor inputs, hired at prices $w_K = 4$ and $w_L = 25$.**

a) **Obtain the marginal products and the marginal rates of technical substitution.**
*$MP_L = \frac{\partial D}{\partial L} = 98L^{-2}\left(K^{-1}+L^{-1}\right)^{-2} = 98\left(\frac{1}{L}\right)^2\left(\frac{1}{K}+\frac{1}{L}\right)^{-2} = 98\left(\frac{1}{L^2}\right)\left(\frac{KL}{K+L}\right)^2 = 98\left(\frac{K}{K+L}\right)^2$. In the same way, we obtain $MP_K = 98\left(\frac{L}{K+L}\right)^2$. Then $MRTS = -\frac{MP_K}{MP_L} = -\left(\frac{L}{K}\right)^2$.*

b) **Confirm that the first-order condition for cost minimization may be written as $K = 2.5L$.**

*The FOC is that (-1 times) the factor price ratio equals the MRTS, so that* $\frac{w_K}{w_L} = \frac{4}{25} = \left(\frac{L}{K}\right)^2$. *Cross-multiplying,* $4K^2 = 25L^2$, *so that 2K = 5L.*

**c) By substituting for *K* in the production function or otherwise, obtain the cost minimizing factor input combination and the total cost of production.**

*Cost minimizing inputs: to achieve output D, firm hires (K, L):* $98[K^{-1} + L^{-1}]^{-1} = D$, *or else so that* $98\left[\frac{1}{2.5L} + \frac{1}{L}\right]^{-1} = 98\left[\frac{3.5}{2.5L}\right]^{-1} = 98\left(\frac{5}{7}\right)L = 70L = D$; *so that* $L = \frac{D}{70}$; *and with K = 2.5L,* $K = \frac{D}{28}$. *Cost* $C(D) = 4K + 25L = \frac{D}{7} + \frac{5D}{14} = 0.5D$.

**d) Confirm that for the input combination (*K*, *L*) = (100, 250), output *D* = 7,000, and that the first-order condition for cost minimization is satisfied.**

*For (K, L) = (100, 250),* $D(K, L) = 98[K^{-1} + L^{-1}]^{-1} = 98\left(\frac{1}{100} + \frac{1}{250}\right)^{-1} = 98*50\left(\frac{7}{10}\right)^{-1} = 14*500$.

**e) Obtain the equation of the isoquant that passes through the input combination given in part d).**

*We may evaluate the production function as* $\left(\frac{1}{K} + \frac{1}{L}\right)^{-1} = \frac{500}{7}$; *or 7KL = 500(K + L).*

**f) Sketch a graph showing this isoquant, the output expansion path, and the isocost line that is tangent to the isoquant.**

*We draw the diagram, showing usage of capital on the horizontal, and usage of labour on the vertical, axis. It will be useful to refer to results derived in Chapter 9, especially around X9.23. Writing the equation of the isoquant explicitly, with L a function of K, the isoquant consists of all input combinations (K, L):* $L = \frac{500K}{7K-500}$. *It follows that the isoquant can only be evaluated for* $K > \frac{500}{7}$, *and that as* $K \to \infty$, $L \to \frac{500}{7}$. *The curve is therefore downward sloping and convex, bounded by asymptotes* $K = \frac{500}{7}$ *and* $L = \frac{500}{7}$; *passes through (K, L) = (250, 100), and there has gradient* $MRTS = -\frac{4}{25}$.

X11.6 **Assume that a creamery's output of cheese is *F*. We may write $F = 8K^{1/3}L^{2/3}$, where *K* and *L* are the quantities of factor inputs, hired at prices $w_K$ and $w_L$.**

**a) Obtain the marginal products and the marginal rates of technical substitution.**

*Marginal product of capital,* $MP_K = \frac{\partial F}{\partial K} = \frac{8}{3}\left(\frac{L}{K}\right)^{\frac{2}{3}}$; *marginal product of labour,* $MP_L = \frac{\partial F}{\partial L} = \frac{16}{3}\left(\frac{K}{L}\right)^{\frac{1}{3}}$; *Marginal rate of technical substitution,* $MRTS = -\frac{MP_K}{MP_L} = -\frac{L}{2K}$.

**b) Confirm that the first-order condition for cost minimization may be written as 2*rK* = *wL*.**

*FOC:* $MRTS = -\frac{r}{w}$, *so* $\frac{L}{2K} = \frac{r}{w}$, *and the result follows from cross-multiplication.*

**c) By substituting for *K* in the production function or otherwise, obtain the cost minimizing factor input combination and the total cost of production when *r* = 4*w*. [*Note:* This will be a function of *w*.]**

*To produce output F, the firm hires (K, L):* $8K^{\frac{1}{3}}L^{\frac{2}{3}} = F$; *and since* $K = \frac{w}{2r}L$, $8\left(\frac{w}{2r}\right)^{\frac{1}{3}}L = F$; *and so* $L = \frac{F}{8}\left(\frac{2r}{w}\right)^{\frac{1}{3}}$ *and* $K = \frac{F}{8}\left(\frac{w}{2r}\right)^{\frac{2}{3}}$. *The cost of production is then*

$C(F, w, r) = rK + wL = w\left[\frac{F}{8}\left(\frac{2r}{w}\right)^{\frac{1}{3}}\right] + r\left[\frac{F}{8}\left(\frac{w}{2r}\right)^{\frac{2}{3}}\right] = \left(2^{\frac{1}{3}} + 2^{-\frac{2}{3}}\right)\left(\frac{F}{8}r^{\frac{1}{3}}w^{\frac{2}{3}}\right) = 3*2^{-\frac{11}{3}}Fr^{\frac{1}{3}}w^{\frac{2}{3}}$.

*When r = 4w,* $C(F, w, r) = 3*2^{-\frac{11}{3}}F(4w)^{\frac{1}{3}}w^{\frac{2}{3}} = \frac{3}{8}Fw$.

**d) On a diagram, sketch the isoquant that passes through the input combination ($K$, $L$) = (5, 40). Confirm that the first-order condition is satisfied at this point and add to the diagram the isocost line that is tangent to the isoquant.**

*Employing input combination (K, L) = (5, 40), F =* $8.5^{\frac{1}{3}}(5*8)^{\frac{2}{3}} = 8.5.4 = 160$. *For this input combination, MRTS =* $-\frac{L}{2K} = -4$. *Given that r = 4w, the first-order condition, MRTS =* $-\frac{r}{w}$ *is satisfied. Sketching a diagram with the usage of capital on the horizontal, and the usage of labour on the vertical, axis, the isoquant through (K, L) = (5, 40) is a downward sloping, convex curve, with asymptotes the axes (so that it approaches but does not touch them). At (5, 40), the gradient of the indifference curve is -4.*

*Given that the cost of production, C =* $\frac{3}{8}Fw$, *in this case C = 60w. We can write the equation of the isocost line as L – 40 = -4(K – 5) or L + 4K = 60.*

X11.7 **[*Hard*] Assume that a firm produces output $B$, and that it faces a CES production function with elasticity of substitution $\sigma = (1 - a)^{-1}$. Then using quantity $K$ of capital and quantity $L$ of labour, the firm is able to produce:**

$$B = [K^a + L^a]^{1/a}$$

**We also assume that the firm is able to hire any quantity of capital at interest rate $r$ and any quantity of labour at wage $w$.**

**a) Obtain the marginal product functions and the marginal rate of technical substitution for this production function.**

*Marginal products of capital and labour,* $MP_K = K^{-(1-a)}\left[K^a + L^a\right]^{\frac{1-a}{a}}$; *and* $MP_L = L^{-(1-a)}\left[K^a + L^a\right]^{\frac{1-a}{a}}$; *MRTS =* $\left(\frac{L}{K}\right)^{-(1-a)}$. *These results have already been obtained for the utility function U:* $U(K, L) = [K^a + L^a]^{1/a}$. *See exercises X9.9 – 11 for further examples.*

**b) Confirm that the first-order condition for cost minimization, which is also the equation of the income expansion path, may be written:** $\frac{L}{K} = \left(\frac{r}{w}\right)^{\frac{1}{1-a}}$.

*This result follows immediately from the requirement that MRTS =* $-\frac{r}{w}$.

**c) Confirm that the indirect demands for capital and labour may be written:**

$$K = B\left[\frac{r^{-\frac{a}{1-a}}}{r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}}\right]^{\frac{1}{a}} \text{ and } L = B\left[\frac{w^{-\frac{a}{1-a}}}{r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}}\right]^{\frac{1}{a}}$$

*This has the same derivation as the compensated demand functions, specified in Expression [9.19], and studied in X9.27.*

**d) Hence or otherwise, confirm that the total cost of producing output $B$ is as given in Expression 11.7.**

*The cost of production, C(B, r, w) is calculated in the same way as the expenditure function for a consumer; the necessary cost of achieving a fixed level of output. The derivation is similar to that of Expression [9.22].*

$$C = rB\left[\frac{r^{-\frac{a}{1-a}}}{r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}}\right]^{\frac{1}{a}} + wB\left[\frac{w^{-\frac{a}{1-a}}}{r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}}\right]^{\frac{1}{a}} = B\left\{\frac{r^{\left(1-\frac{1}{1-a}\right)} + w^{\left(1-\frac{1}{1-a}\right)}}{\left(r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right)^{\frac{1}{a}}}\right\}$$

$$= B\left\{r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right\}^{1-\frac{1}{a}} = B\left\{r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right\}^{-\left(\frac{1-a}{a}\right)}$$

X11.8 **Complete the argument relating to economic efficiency, explaining why the bakery achieves both technical and economic efficiency when using input bundle P.**
*Technical efficiency: impossible for the bakery to reduce use of inputs and maintain output. Satisfied when inputs on isoquant. Economic efficiency: impossible for the bakery to alter composition of inputs and maintain output and costs. Satisfied since MRTS is equal to the input price ratio, with P the least-cost, acceptable bundle.*

X11.9 **For the following total cost functions, calculate the average and marginal cost functions, sketching graphs of each as output, *X*, varies between 1 and 5 units.**
**a) $C = kX$ b) $C = 1 + x$ c) $C = 9 + x$ d) $C = rX^2$ e) $C = 1 + x + x^2$**
*Remember that we define average cost AC: AC =* $\frac{C(X)}{X}$*, and marginal cost MC: MC =* $\frac{dC}{dx}$.

*a)* $AC(X) = MC(X) = k$

*b)* $AC(X) = \frac{1+X}{X} = 1 + \frac{1}{X}$; $MC(X) = \frac{d}{dx}(1+X) = 1$

*c)* $AC(X) = \frac{9+X}{X} = 1 + \frac{9}{X}$; $MC(X) = \frac{d}{dx}(9+X) = 1$

*d)* $AC(X) = \frac{rX^2}{X} = rX$; $MC(X) = \frac{d}{dx}\left(rX^2\right) = 2rX$

*e)* $AC(X) = \frac{1+X+X^2}{X} = X + 1 + \frac{1}{X}$; $MC(X) = 1 + 2x$

X11.10 **Given** $AC(X) = \frac{C(X)}{X}$**, show that the derivative function,** $\frac{dAC}{dX} = \frac{MC(X) - AC(X)}{X}$**. Hence confirm that if marginal cost is greater than average cost, average cost is increasing, and that if average cost is greater than marginal cost, average cost is decreasing.**
*We write* $AC(X) = X^{-1}.C(X)$*, and apply the product rule of differentiation, with* $g(X) = X^{-1}$ *and* $h(X) = C(X)$*. Then* $\frac{dg}{dX} = -X^{-2}$ *and* $\frac{dh}{dX} = MC(X)$*. By the product rule,*
$\frac{dAC}{dX} = g(X)\frac{dh}{dX} + h(X)\frac{dg}{dX} = X^{-1}MC(X) - X^{-2}C(X) = \frac{1}{X}\left(MC(X) - \frac{C(X)}{X}\right) = \frac{1}{X}\left(MC(X) - AC(X)\right)$.
*The sign of the derivative,* $\frac{dAC}{dX}$ *is then the sign of the expression MC(X) – AC(X). So, if MC(X) > AC(X), then the derivative is greater than zero, and average cost, AC, is increasing in output, X. Similarly, if MC(X) < AC(X), then the derivative is less than zero, and average cost, AC, is decreasing in output, X.*

X11.11 **Using the results of Exercise 11.10, differentiate the average cost function twice to obtain the conditions that must be satisfied for there to be a minimum value of average cost. Confirm that for the function used in X11.9e, these conditions are satisfied when output *x* = 1.**
*Given that* $\frac{dAC}{dX} = \frac{1}{X}\left(MC(X) - AC(X)\right)$*, differentiating the derivative,*
$\frac{d^2AC}{dX^2} = X^{-1}\left(\frac{dMC(X)}{dX} - \frac{dAC(X)}{dX}\right) - X^{-2}\left(MC(X) - AC(X)\right) = \frac{1}{X}\left(\frac{dMC(X)}{dX} - 2\frac{dAC(X)}{dX}\right)$*. The first-order condition for a turning point is that* $\frac{dAC}{dX} = 0$*, so when this condition is satisfied, the second-order condition for a minimum,* $\frac{d^2AC}{dX^2} > 0$ *will be satisfied if* $\frac{dMC}{dX} > 0$*. So, for there to be a minimum*

*value of AC, the average cost, AC, is neither increasing, not decreasing in output X, and the marginal cost, MC, is increasing in X.*

*Given* $AC(X) = \frac{1+X+X^2}{X} = X + 1 + \frac{1}{X}$; $MC(X) = 1 + 2X$, $\frac{dAC}{dX} = 1 - X^{-2}$ *and* $\frac{d^2AC}{dX^2} = 2X^{-3}$, *and* $\frac{dMC}{dX} = 2$.

*Evaluating these expressions when X = 1, the answers follow immediately.*

X11.12 **In Figure 11.4, we show the marginal and average costs for output $Q_0$.**

**a) Explain this geometric representation of these cost measures.**

*Marginal cost is the instantaneous rate of change of total cost. So, at point Z, marginal cost is shown as the gradient of the total cost curve. Average cost is the cost per unit, or the total cost divided by the number of outputs. This is the gradient of the chord OZ.*

**b) Demonstrate that the result obtained in X11.10 (when *MC* > *AC*, *AC* is increasing) holds in this case.**

*A sketch should be sufficient. With output measured on the horizontal, and total cost of production on the vertical, axis, the total cost curve begins from the origin, and is upward sloping and convex. Drawing a line from the origin to any point on the curve, this line lies above the curve (by definition of convexity). It follows that where this line intersects the total cost curve, it is flatter than the total cost curve; and since marginal cost is the gradient of the tangent to the total cost curve, average cost is less than marginal cost.*

X11.13 **Adapt Figure 11.4, so that the firm has fixed costs (or so that total cost *C*(0) > 0), with total costs increasing at an increasing rate.**

**a) Confirm that when output is zero, the average cost, *AC*(0), is undefined.**

*As in X11.12, with output measured on the horizontal, and total cost of production on the vertical, axis, the total cost curve is upward sloping and convex. In this case, though, it begins from a point on the vertical (cost) axis, which represents the fixed costs, those incurred when output is zero. With output, X = 0, and total cost C(0) > 0, we cannot define* $AC(0) = \frac{C(0)}{0}$.

**b) On your diagram, indicate clearly the output, $Q_0$, for which average costs are equal to marginal costs.**

*If* $AC(Q_0) = MC(Q_0)$, *then the tangent to* $(Q_0, C(Q_0))$ *also passes through the origin.*

**c) Demonstrate that for output $Q < Q_0$, average cost is higher than marginal cost; while for output $Q > Q_0$, marginal cost is greater than average cost.**

*On the diagram, select two points,* $(Q_1, C(Q_1))$ *and* $(Q_2, C(Q_2))$ *on the total cost curve, chosen so that for one,* $Q_1$: $Q_1 < Q_0$; *and for the other,* $Q_2$: $Q_2 > Q_0$. *For each point, draw the line joining it to the origin. We note that the line joining* $(Q_1, C(Q_1))$ *to the origin is steeper than the total cost curve where they meet, but that where the line joining* $(Q_1, C(Q_1))$ *to the origin meets the total cost curve, it is flatter (than the cost curve). We conclude that for output* $Q = Q_1$, $AC(Q_1) > MC(Q_1)$, *but that for* $Q = Q_2$, $AC(Q_2) > MC(Q_2)$.

**d) Hence or otherwise, confirm that at output $Q = Q_0$, average cost is minimized.**

*In order for a line passing through the origin to intersect the convex total cost curve, it must be at least as steep as the tangent at* $(Q_0, C(Q_0))$. *So the gradient of any line that represents the average cost of some level of output Q is no less than the gradient of the line representing the average cost at* $Q_0$. *Average cost is minimized at output* $Q_0$.

X11.14 **Extend the argument begun in the previous paragraph to show that, given the assumption of constant returns to scale:**

a) **Factor demands *L* and *K* are linear in output *X*.**

*With constant returns to scale, $X(tK, tL) = tX(K, L)$; $MP_K(tK, tL) = MP_K(K, L)$; and $MP_L(tK, tL) = MP_L(K, L)$. It follows that $MRTS(tK, tL) = MRTS(K, L)$; so that the firm chooses the same technology, irrespective of its output.*

*Suppose that the firm initially produces output, $X_0 = X(K_0, L_0)$, and then changes it to $X_1(K_1, L_1)$. Since the technology of production does not change, then writing $X_1 = tX_0$, it follows that $K_1 = tK_0$, so that $\frac{K_1}{X_1} = \frac{K_0}{X_0}$.*

b) **Total costs are linear in output *X*.**

*Given that total costs, C, are linear in factor inputs, and changes in both factor inputs are linear in output, X, it follows that C is linear in X.*

c) **Average cost, *AC*, equals marginal cost, *MC*.**

*Since total cost is linear in output, we can write $C(X) = cX$, where c is (defined in terms of input prices) does not vary with output X. Marginal cost, $MC(X) = \frac{dC}{dX} = c$, while average cost, $AC(X) = \frac{C}{X} = c$.*

X11.15 **Now extend the argument to show that if there are increasing returns to scale:**

a) **Factor demands *L* and *K* are increasing, but concave, in output *X*. (That is, as output increases, labour and capital usage will increase; but the proportionate increase in use is less than the proportionate increase in output.)**

*With increasing return to scale, $X(tK, tL) = t^r X(K, L)$, where $r > 1$; $MP_K(tK, tL) = MP_K(K, L)$; and $MP_L(tK, tL) = MP_L(K, L)$. It follows that $MRTS(tK, tL) = MRTS(K, L)$; so that the firm chooses the same technology, irrespective of its output.*

*Suppose that the firm initially produces output, $X_0 = X(K_0, L_0)$, and then changes it to $X_1(K_1, L_1)$. Since the technology of production does not change, then writing $X_1 = t^r X_0$, it follows that $K_1 = tK_0$, so that $\frac{K_1}{X_1} = \frac{tK_0}{t^r X_0} = t^{1-r}\frac{K_0}{X_0}$; and since $r > 1$, $t^{1-r} < 1$. Demand for capital does not increase as rapidly as output. The same argument applies for changes in the labour input.*

*The demand for each factor is increasing, but at a decreasing rate; and so the second derivatives of these functions with respect to output will be less than zero, confirming that demands are concave.*

b) **Total costs are increasing, but concave, in output *X*.**

*Given that total costs, C, are linear in factor inputs, and changes in both factor inputs are increasing but concave in output, X, it follows directly that C is increasing, but concave, in X.*

c) **Average cost, *AC*, decreases in output.**

*We have shown that $\frac{K_1}{X_1} = t^{1-r}\frac{K_0}{X_0}$. It follows that the associated total costs $\frac{C_1}{X_1} = \frac{tC_0}{t^r X_0} = t^{1-r}\frac{C_0}{X_0}$*

*Then $AC_1 = t^{1-r}AC_0$, and average cost is decreasing in output.*

d) **Average cost, *AC*, is greater than marginal cost, *MC*.**

*This result follows directly from the definition of average cost, and its properties, as set out in X11.10.*

X11.16 **Repeat the argument of X11.14 to show that if there are decreasing returns to scale:**

a) **Factor demands $L$ and $K$ are increasing, but convex, in output $X$. (That is, as output increases, labour and capital usage will increase; but the proportionate increase in use is greater than the proportionate increase in output.)**
*With decreasing return to scale, $X(tK, tL) = t^rX(K, L)$, where $r < 1$; $MP_K(tK, tL) = MP_K(K, L)$; and $MP_L(tK, tL) = MP_L(K, L)$. It follows that $MRTS(tK, tL) = MRTS(K, L)$; so that the firm chooses the same technology, irrespective of its output.*
*Suppose that the firm initially produces output, $X_0 = X(K_0, L_0)$, and then changes it to $X_1(K_1, L_1)$. Since the technology of production does not change, then writing $X_1 = t^rX_0$, it follows that $K_1 = tK_0$, so that $\frac{K_1}{X_1} = \frac{tK_0}{t^rX_0} = t^{1-r}\frac{K_0}{X_0}$; and since $r < 1$, $t^{1-r} > 1$. Demand for capital increases more rapidly than output. The same argument applies for changes in the labour input.*
*The demand for each factor is increasing, but at an increasing rate; and so the second derivatives of these functions with respect to output will be greater than zero, confirming that demands are convex.*

b) **Total costs are increasing, but convex, in output *X*.**
*Given that total costs, C, are linear in factor inputs, and changes in both factor inputs are increasing but convex in output, X, it follows directly that C is increasing, but convex, in X.*

c) **Average cost, *AC*, increases in output.**
*We have shown that $\frac{K_1}{X_1} = t^{1-r}\frac{K_0}{X_0}$. It follows that the associated total costs $\frac{C_1}{X_1} = \frac{tC_0}{t^rX_0} = t^{1-r}\frac{C_0}{X_0}$*
*Then $AC_1 = t^{1-r}AC_0$, and average cost is increasing in output.*

d) **Average cost, *AC*, is less than marginal cost, *MC*.**
*This result follows directly from the definition of average cost, and its properties, as set out in X11.10.*

X11.17 **A production function is homogeneous of degree $r$ if $X(tK, tL) = t^rX(K, L)$ for all values of $t$, $K$ and $L$. Confirm that the results of X11.14–X11.16 hold in the case of functions that are respectively homogeneous of degrees one, greater than one and less than one; and that where a production function is homogeneous of degree $r$, the factor demand and total cost functions are homogeneous of degree $1/r$ in output.**
*We have separately confirmed that production functions that are homogeneous of degree r exhibit returns to scale that are increasing if $r > 1$, constant, if $r = 1$, and diminishing if $r < 1$. The arguments relating to factor demands then follow directly from the assumption of homogeneity in the production function, as required.*

X11.18 **A cost-minimizing firm faces the production function in Expression 11.19, and fixed factor costs $w$ and $r$. Using the arguments developed in Section 9.2.1, where we found the Hicksian demands when a consumer's utility takes the form of a CES function, and also from X11.11, where we found the total cost function a CES production function, confirm that:**

a) **The first-order condition for cost minimization is still given by Expression 11.16.**
*It may be helpful to write $B^{\frac{1}{\rho}} = \left[K^a + L^a\right]^{\frac{1}{a}}$. Then we see that the expression on the right hand side is has the (hopefully familiar) CES form; and so the first-order condition for cost minimization will be unchanged. Differentiating implicitly:*
$\frac{1}{\rho}B^{\frac{1-\rho}{\rho}}\frac{\partial B}{\partial K} = \left(aK^{a-1}\right)\frac{1}{a}\left[K^a + L^a\right]^{\frac{1}{a}-1} = \left(\frac{B^{\frac{1}{\rho}}}{K}\right)^{1-a}$. *Similarly, we find:* $\frac{1}{\rho}B^{\frac{1-\rho}{\rho}}\frac{\partial B}{\partial L} = \left(\frac{B^{\frac{1}{\rho}}}{L}\right)^{1-a}$. *Since the expressions on the left hand side are identical apart from the partial derivatives, in taking their ratio, we are left with the ratio of marginal products, or (minus one times) the marginal*

*rate of technical substitution. The ratio of the right hand side expressions is, as required, $\left(\frac{K}{L}\right)^{1-a}$.*

**b) The total cost function is:**

$$C(B,r,w) = B^{\frac{1}{\rho}}.\left(r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right)^{-\frac{1-a}{a}} \qquad [11.20]$$

*This result follows directly from the argument of Chapter 9, but allowing for the returns to scale in production.*

**c) The average cost function is:**

$$AC(B,r,w) = B^{\frac{1-\rho}{\rho}}.\left(r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right)^{-\frac{1-a}{a}} \qquad [11.21]$$

*This follows directly from the definition of average cost.*

**d) The marginal cost function is:**

$$MC(B,r,w) = \frac{B^{\frac{1-\rho}{\rho}}}{\rho}.\left(r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right)^{-\frac{1-a}{a}} \qquad [11.22]$$

*This follows directly from the definition of marginal cost.*

**e) The total cost function is homogeneous of degree $1/\rho$.**
*Applying the requirement that a function, z: z = z(x) is homogeneous of degree r, if z(tx) = $t^r$z(x), then this result follows by inspection.*

**f) The average and marginal cost functions are homogeneous of degree $(1 - \rho)/\rho$.**
*Applying the requirement that a function, z: z = z(x) is homogeneous of degree r, if z(tx) = $t^r$z(x), then this result follows by inspection.*

**g) If $\rho < 1$, total costs are increasing and convex, average costs are increasing, and *MC* > *AC*; but if $\rho > 1$, then total costs are increasing and concave, average costs are decreasing, and *MC* < *AC*.**
*This is a particular case of the arguments developed in X11.14 – 17. Differentiating total costs twice, we obtain $\frac{dC}{dB} = \frac{1}{\rho}B^{\frac{1-\rho}{\rho}}c$, where c = $\left(r^{-\frac{a}{1-a}} + w^{-\frac{a}{1-a}}\right)^{-\frac{1-a}{a}}$ is the marginal cost; and $\frac{d^2C}{dB^2} = \frac{1-\rho}{\rho^2}B^{\frac{1-2\rho}{\rho}}c$. We immediately see that the function is increasing, and is convex when $\rho >$ 1, but concave when $\rho < 1$ (and therefore that marginal cost is positive, but increasing only if $\rho > 1$).*

*Differentiating average cost, we obtain $\frac{dAC}{dB} = \frac{1-\rho}{\rho}B^{\frac{1-2\rho}{\rho}}c$. Noting that $\frac{dAC}{dB} = \rho\frac{dMC}{dB}$, it follows that marginal cost is positive, but increasing only if $\rho > 1$. Lastly, if $\rho > 1$, AC > MC.*

X11.19 **It is possible to demonstrate the nature of total and marginal returns to scale for the curve in Figure 11.6a without using Figure 11.6b.**

**a) Thinking in terms of total costs, restate the condition that the nature of marginal returns to scale is determined by the sign of the derivative of the marginal cost function.**
*There will be diminishing marginal returns to scale if the marginal cost is increasing, or if $\frac{d^2C}{dQ^2} > 0$.*

**b) Demonstrate that the firm experiences diminishing total returns to scale only when output is greater than $Q_0$.**

*For the firm to experience diminishing returns to scale, average cost must be increasing. In Figure 11.6a, we see that the average cost is initially large (it would be the tangent to the total cost curve at the origin, but then, as output increases, average cost decreases. This continues until output reaches $Q_0$; and then average and marginal cost are equal. Increasing output further, the slope of the lines connecting points on the total cost curve to the origin increase, indicating increasing average cost.*

X11.20 **Consider the total cost function $C = 1 + x + x^2$.**

**a) Write down expressions for average cost and marginal cost.**

*AC(x) =* $\frac{1}{x}+1+x$ *; MC(x) = 1 + 2x*

**b) Confirm that average cost is decreasing if $x < 1$, increasing if $x > 1$, and is neither increasing nor decreasing when $x = 0$. Hence or otherwise confirm that average cost is at a minimum when $x = 1$.**

*For there to be a minimum of average cost, we require* $\frac{dAC}{dX}=0$ *and* $\frac{d^2AC}{dX^2}>0$

$\frac{dAC}{dX}=-x^{-2}+1=0$ *if* $x=1$ *(we ignore the possibility that x = -1). Differentiating for a second time,* $\frac{d^2AC}{dX^2}=2x^{-3}>0$, *when x = 1.*

**c) Confirm that marginal cost is always increasing.**

*Marginal cost is increasing if* $\frac{dMC}{dx}>0$, *and here* $\frac{dMC}{dx}=2>0$

**d) State the nature of returns to scale for this total cost function.**

*There are diminishing marginal returns to scale everywhere; increasing returns to scale for x < 1; and diminishing returns for higher outputs.*

**e) Sketch the graph of the total cost function, showing clearly where average costs are minimized.**

*Sketching a graph with output, x, on the horizontal axis, and total cost, C(x), on the vertical axis, the total cost curve is part of a parabola, upward sloping and convex for all values of $x \geq 0$, and intersecting the vertical axis at C = 1. We see that when x = 1, C(1) = 3, and the marginal cost, MC(1) = AC(1) = 3. The line C = 3x is tangent to the C(x) curve.*

X11.21 **Consider the total cost function, $C = x(54 – 12x + x^2)$.**

**a) Write down expressions for average cost and marginal cost.**

*Average cost: AC(x) =* $\frac{C}{x}$ *= $54 – 12x + x^2$*

*Marginal cost: MC(x) =* $\frac{dC}{dx}$ *= $54 – 24x + 3x^2$*

**b) Confirm that average cost is decreasing if $x < 6$, increasing if $x > 6$, and is neither increasing nor decreasing when $x = 0$. Hence or otherwise confirm that average cost is at a minimum when $x = 6$.**

*We differentiate the average cost function:* $\frac{dAC}{dx}=-12+2x$. *If* $\frac{dAC}{dx}>0$, *or if x > 6, then AC is increasing in output, x; if* $\frac{dAC}{dx}<0$, *or if x < 6, then AC is decreasing in output, x; and if* $\frac{dAC}{dx}=0$, *so that x = 6, then AC is neither increasing nor decreasing in output, x.*

*For AC to be at a minimum when x = 6, we require* $\frac{d^2AC}{dx^2} > 0$. *Differentiating a second time,* $\frac{d^2AC}{dx^2} = 2$.

**c) Confirm that marginal cost is decreasing if *x* < 4, increasing if *x* > 4, and is neither increasing nor decreasing when *x* = 0. Hence or otherwise confirm that average cost is at a minimum when *x* = 4.**

*We differentiate the marginal cost function:* $\frac{dMC}{dx} = -24 + 6x$. *If* $\frac{dMC}{dx} > 0$, *or if x > 4, then MC is increasing in output, x; if* $\frac{dMC}{dx} < 0$, *or if x < 4, then MC is decreasing in output, x; and if* $\frac{dMC}{dx} = 0$, *so that x = 4, then MC is neither increasing nor decreasing in output, x.*

*For MC to be at a minimum when x = 4, we require* $\frac{d^2MC}{dx^2} > 0$. *Differentiating a second time,* $\frac{d^2AC}{dx^2} = 6$.

**d) State the nature of returns to scale in this case.**

*Increasing (marginal) returns to scale when x < 4; diminishing marginal returns, but increasing returns when 4 < x < 6; and diminishing (marginal) returns when x > 6.*

**e) Sketch the graphs of the average and marginal cost functions, showing clearly where average and marginal costs are minimized.**

*Showing output on the horizontal and average and marginal costs on the vertical axis, the graph of average cost function is a parabola, beginning from (0, 54), with a minimum point at (6, 18). The graph of marginal cost also passes through (0, 54) and (6, 18), but also reaching a minimum at (4, 6).*

**Chapter 12**

X12.1 **Compare a bridge and a tunnel as examples of fixed investments. [*Note:* To start your argument, consider the old London Bridge, which was reconstructed in Lake Havasu, Arizona, between 1968 and 1970. Could anything similar be done with a tunnel?] For each, what do you consider to be 'the long run' and 'the short run'?**

*A tunnel is effectively fixed in position. There are a few examples of bridges (and other buildings) that have been dismantled carefully and rebuilt; although they are more usually adapted over time and destroyed when making way for newer structures. We can think of there being two stages in a bridge or tunnel project: construction and maintenance. Defining the capital input as the funding required the project, we might think of the capital as being fixed for the whole lifetime of the project. Construction also requires relatively intensive use of labour, compared with the maintenance phase. In the case of the tunnel, the capital is absolutely fixed, since the funding required in order for maintenance to proceed should be included in the capital.*

*In both cases, the short-run is the period in which capital is fixed. For both a bridge and a tunnel, the short-run will include the whole of the construction phase, which may last several years. Thereafter, through maintenance, the short-run may be a period of months, or, in the case of a substantial adaptation of use, a year or more.*

X12.2 **Hospitals are large complex projects. In recent years, it has become increasingly common for health service providers to commission specialist companies to 'build, own, operate and transfer' new medical facilities. As the name suggests, the management company agrees to provide all services in the facility for a period of 25–30 years, at the end of which it transfers ownership to the health service provider.**

**a) Discuss possible advantages and disadvantages of this approach.**

*The business of the health care provider need not involve substantial expertise in the management of the facilities in which it operates, whereas this is exactly the business of the management company. By separating the two parts of the activity, we allow for a degree of specialization in each. Against this, the relationship between the care provider and the management company will involve negotiation and the management of complex contracts, which will add to the costs of running the facility.*

**b) Why would each party want to tie the other into a long-run contract?**

*For the management company, building and equipping the hospital will form the initial short-run. During this construction period, it is unlikely to receive sufficient payments to meet its costs; we can think of the capital as being largely fixed at this time. As with other capital projects, once the facility is operational, it is possible that there will be a need for further capital investment. For example, over 25 years, we expect there to be unanticipated advances in health care, changing the ways in which the hospital operates. To take advantage of such opportunities, we expect the short-run in particular units of the hospital to be longer than a year.*

*Both parties want to enter into a long-run contract: usage fees paid over the operating period should cover the total costs of building and operation. The long-run contract effectively covers what might be the whole life of the facility. At the end of the contract, it is possible that it will be more efficient to reconstruct the facility, reflecting changes in needs.*

**c) What do you consider to be 'the long run' in this case?**

*The very long run might be considered to be the whole 25 years; although we should recognize that during that period, it will be possible to renegotiate the underlying contract. Nonetheless, capital is committed at the start of that period, with the expectation that the* *return on capital will accrue over the whole of that time. Within that very long run, there is a long run consisting of the construction phase, and then most likely a sequence of long-run periods between refurbishment activities.*

X12.3 **Many developed countries have substantially reduced their coal-mining industries in recent years, in many cases without extracting all recoverable reserves. What are the fixed assets associated with a mine? Why might it be desirable for a mining company to enter into a long-run contract with an electricity-generation company? How should we define 'the long run' in this case?**

*The main fixed asset is the mine workings. These structures are similar to tunnels, and like a tunnel they are truly fixed. In this sense, the long run for a mine is its whole life, and so before committing to development, we might expect the owners to ensure that they have a market for a substantial proportion of the likely output. A contract with an electricity generation company is likely to be the most effective way of doing this, since the economies of scale in power generation mean that coal-fired power stations are also capital-intensive projects that can only be profitable if they are operated for many years.*

X12.4 **In managing labour inputs during a recession, firms sometimes ask workers to take extended holidays. Why would a firm not use its right to terminate contracts and hire new workers? [*Hint*: Think about the transactions costs involved in hiring new workers, and the nature of the employment contract.]**

*In many advanced economies, standard labour contracts give workers substantial rights in the event that an employer wishes to dismiss them, or negotiate contract termination. Identifying, and training, new workers might also be an expensive process, especially where work involves particular skills.*

*In a recession, demand is reduced temporarily. The costs of getting rid of workers, and then some months later, when demand increases, hiring replacements may well exceed the costs of effectively paying workers to do nothing for a short period of time.*

X12.5 **It is common for firms to enter into 'rolling contracts' with senior managers. These have a fixed termination date, but can be renewed for a fixed period before termination. Why might a firm choose to hire most workers using an open-ended contract, but to hire senior managers using a sequence of rolling contracts?**

*An open-ended contract allows both parties to terminate it with a relatively short period of notice at any time. This provides a relatively high degree of certainty to employees that their employment will continue (although there may have to be renegotiation of terms over time). It also suits the needs of the employer, who needs staff to undertake specific roles.*

*With senior managers, we consider that the work undertaken has strategic value, and that having the ability to terminate the contract at relatively short notice in the case of unsatisfactory performance protects the interests of the business.*

## Chapter 13

X13.1 **Explain why the assumption that firms only produce the quantity of output that they can sell ensures market clearing.**

*If all firms produce the output that they expect to be able to sell (and their expectations are correct), then the market supply will be equal to the market demand, so that the market clears.*

X13.2 **Why should we never expect to see a firm making losses in the long run? [*Hint:* Think about what it means when a firm produces zero output.]**

*In the long-run, the firm can sell all of its assets, and so quit the market. Both revenue and costs are then zero. This opportunity to exit is available to all firms in the long-run, so we do not expect to see losses being made.*

X13.3 **Sketch a diagram in which a firm never makes profits, but minimizes losses at some output $Q_0$. Why might such a firm choose to produce output $Q_0$ rather than stopping production? [*Hint:* Suppose that there is a fixed element in total costs, so that $TC(0) > 0$, and that total costs increase at an increasing rate, while total revenues are linear.]**

*Sketch the diagram with output, Q, on the horizontal axis, and total revenue, R, and total costs, C, measured on the vertical axis. Draw in a linear total cost function that intersects the vertical axis at some point $(0, C_0)$, and which has gradient $MC(Q) = c_1$. Sketch a total revenue curve, which starts from the origin, is inverted U-shaped, and which always lies below the total cost curve. Identify output $Q_0$, at which the gradient of (the tangent to) the total revenue curve, $\frac{dR}{dQ} = c$, so that marginal revenue equals marginal cost. At $Q_0$, the gap between total cost and total revenue is minimized, so that in maximizing profit, the firm minimizes its losses. In particular, we note that shutting down production, and incurring costs $C_0$ leads to a larger loss than producing output $Q_0$ and making a loss of $C(Q_0) – R(Q_0)$.*

X13.4 **Confirm that $AR(Q) = p$, so that the average revenue and the inverse demand functions are identical.**

*Define total revenue R: R = p.Q. Then average revenue AR = $\frac{pQ}{Q} = p$. Defining the inverse demand, p: p = p(Q), then we can write AR(Q) = p(Q).*

X13.5 **Define the average revenues of a firm as total revenues per unit sold, so that $AR(Q) = TR(Q)/Q$. Confirm that if the market price, $p_0$, does not change as the firm increases its output, then its total revenue function is linear, while its average revenue, $AR(Q)$, and marginal revenue, $MR(Q)$, are constant and equal to the price, $p_0$.**

*Define total revenue R: R = $p_0$.Q. With p constant, R is linear in Q. Average revenue AR = $\frac{pQ}{Q} = p_0$. Marginal revenue, MR = $\frac{dR}{dQ} = p_0$*

X13.6 **Given the definition of average revenues as revenue per unit sold:**

a) **Differentiate the function with respect to output $Q$, confirming that $AR'(Q) = [AR(Q) – MR(Q)]/Q$.**

*Define AR: AR(Q) = $\frac{R(Q)}{Q}$. Differentiating, we apply the product rule, so that* $AR'(Q) = \frac{dAR}{dQ} = \frac{1}{Q}\cdot\frac{dR}{dQ} - \frac{AR}{Q^2} = \frac{1}{Q}\left(MR(Q) - AR(Q)\right)$.

b) **Show that if average revenues are constant, $AR(Q) = MR(Q)$; but that if average revenues are decreasing, $AR(Q) > MR(Q)$.**

*For constant average revenues, $\frac{dAR}{dQ}=0$, so AR(Q) = MR(Q); and for decreasing average revenues, $\frac{dAR}{dQ}<0$, so AR(Q) > MR(Q).*

X13.7 **Given a linear cost function, $C(Q) = cQ$ and a linear inverse demand function $p(Q) = a - bQ$.**

a) **Write down the firm's total revenue function, and obtain its average and marginal revenue functions.**

*Total revenue, $R = p(Q)Q = (a - bQ)Q$; so average revenue, $AR = \frac{R}{Q} = a - bQ$; and marginal revenue, $MR = \frac{dR}{dQ} = a - 2bQ$.*

b) **Obtain the firm's average and marginal cost functions.**

*Total cost, $C = C(Q)$; so average cost, $AC = \frac{C}{Q} = c$; and marginal cost, $MC = \frac{dC}{dQ} = C$.*

c) **Show that the firm generates zero revenue if it stops production or else if it produces $Q_1 = \frac{a}{b}$.**

*If R = 0, then $(a - bQ)Q = 0$, so either $a - bQ = 0$, so that $Q = \frac{a}{b}$; or else $Q = 0$.*

d) **The question is wrongly stated in the textbook: it should be**
**Show that the firm maximizes revenues by producing output $Q^M = a/2b$.**

*For the firm to maximize revenue, we require $\frac{dR}{dQ}=0$ and $\frac{d^2R}{dQ^2}<0$.*

*For $\frac{dR}{dQ}=0$, $a - 2bQ^M = 0$, and so $Q^M = \frac{a}{2b}$. Differentiating a second time, $\frac{d^2R}{dQ^2} = -2b < 0$, as required for there to be a maximum of R at $Q^M = \frac{a}{2b}$.*

e) **Show that the firm breaks even by stopping production, or else by producing $Q_0 = (a - c)/b$.**

*For the firm to break even, profit $\Pi(Q) = R(Q) - C(Q) = 0$. Then $(a - bQ)Q - cQ = (a - c - bQ)Q = 0$, and either $Q = 0$, or $a - c - bQ_0 = 0$, so that $Q_0 = \frac{a-c}{b}$.*

f) **Show that the firm maximizes profits by producing output $Q^* = (a - c)/2b$.**

*For the firm to maximize profits, we require $\frac{d\Pi}{dQ}=0$ and $\frac{d^2\Pi}{dQ^2}<0$.*

*For $\frac{d\Pi}{dQ}=0$, $a - c - 2bQ^* = 0$, and so $Q^* = \frac{a-c}{2b}$. Differentiating a second time, $\frac{d^2\Pi}{dQ^2} = -2b < 0$, as required for there to be a maximum of $\Pi$ at $Q^M = \frac{a-c}{2b}$.*

g) **Sketch two diagrams, one showing the outputs $Q^*$, $Q^M$, $Q_0$ and $Q_1$ on total revenue and cost curves; the other showing these outputs in relation to average and marginal revenues, and average and marginal costs.**

*In both diagrams, preferably with one placed directly below the other, measuring output, Q, on the horizontal axis of both, and then in the upper one measuring total revenue and total cost on the vertical axis, and marginal and average measures in the lower one:*

*Total cost curve is a straight line starting from the origin, with gradient c. Total revenue curve is a parabola, passing through the points (0, 0), $\left(\frac{a}{2b}, \frac{a^2}{4b}\right)$, and $\left(\frac{a}{b}, 0\right)$. Profit is the difference between revenue and cost; and so takes the value zero at (0, 0) and $\left(\frac{a-c}{b}, 0\right)$, reaching a maximum at $\left(\frac{a-c}{2b}, \frac{(a-c)^2}{4b}\right)$.*

*Marginal cost and average costs curves are coincident: a horizontal line, with equation AC(Q) = MC(Q) = c. Average revenue curve is a downward sloping straight line, passing through the* *horizontal axis at $\left(\frac{a}{b},0\right)$ and the vertical axis at (0, a). Marginal revenue curve is a downward sloping straight line, passing through the horizontal axis at $\left(\frac{a}{2b},0\right)$ and the vertical axis at (0, a).*

X13.8 **Confirm that for the linear cost function, $C = cQ$, and the linear inverse demand function, $p = a - bQ$, the conditions for a profit maximum are satisfied for output $Q^* = \frac{a-c}{2b}$, so that price $p^* = \frac{a+c}{2}$ and the maximum profit $\Pi^* = \frac{(a-c)^2}{4b}$. Sketch a diagram showing the average and marginal revenue curves, the average and marginal cost curves, the profit maximizing output, $Q^*$, the price, $p^*$, and the total profit.**

*We have already established in X13.7 that the profit maximizing conditions are satisfied for output, $Q^* = \frac{a-c}{2b}$. Substituting into the inverse demand function, we find that $p^* = a - bQ^*$ $= p^* = \frac{a+c}{2}$, so that the firm makes profits $(p^* - c)Q^* = \frac{(a-c)^2}{4b}$*

*The diagram should be identical to the one drawn in X13.7h) showing average and marginal measures of revenue and cost.*

X13.9 **Confirm that for the quadratic cost function $C = cQ^2$ and the linear inverse demand function $p = p_0$, so that the market price does not change with the level of output, the conditions for a profit maximum are satisfied for output $Q^* = p_0/(2c)$, so that the maximum profit $\Pi^* = \frac{p_0^2}{4c}$. Sketch a diagram showing the average and marginal revenue curves, the average and marginal cost curves, the profit maximizing output, $Q^*$, the price, $p_0$, and the total profit.**

*The firm seek to maximize profit $\Pi = p.Q - C(Q) = p_0Q - cQ^2$. We obtain the profit maximizing output $Q^*$ as the level of output for which $\frac{d\Pi}{dQ} = 0$, and $\frac{d^2\Pi}{dQ^2} < 0$.*

*Differentiating the expression for profit with respect to Q, $\frac{d\Pi}{dQ} = p_0 - 2cQ = 0$ for $Q^* = \frac{p_0}{2c}$.*

*The firm then makes profits $\Pi^* = \Pi(Q^*) = \frac{p_0^2}{2c} - c\left(\frac{p_0}{2c}\right)^2 = 2\left(\frac{p_0^2}{4c}\right) - \frac{p_0^2}{4c} = \frac{p_0^2}{4c}$.*

*We sketch a diagram showing the level of output on the horizontal axis, and measures of costs and revenues on the vertical axis. The average and marginal revenue curves are coincident, a horizontal line with equation $AR(Q) = MR(Q) = p_0$. The average cost $AC(Q) = \frac{C}{Q} = \frac{cQ^2}{Q} = cQ$; and so its graph will be an upward sloping straight line with gradient c. At the profit maximizing output $Q^* = \frac{p_0}{2c}$, $AC(Q^*) = 0.5p_0$. The marginal cost curve $\frac{dC}{dQ} = 2cQ$, is also represented on the diagram by a straight line passing through the origin, but it has gradient 2c, and at $Q = \frac{p_0}{2c}$, $MC(Q^*) = p_0$, so that the first-order condition, marginal cost equals marginal revenue is satisfied for this output. We represent profit as a rectangular area, of width $Q^* = \frac{p_0}{2c}$, and height $p_0 - MC(Q^*) = 0.5p_0$, so that the area is $\Pi^* = \frac{p_0^2}{4c}$.*

X13.10 **Confirm the following statements:**

**a)** **If a firm is a price taker, selling the output, $Q$, at a fixed price, $p_0$, which does not change as $Q$ changes, then the firm's total revenues are $R = p_0Q$; its average revenue is $AR = p_0$; and its marginal revenue is $MR = p_0$.**

*For a price taker, total revenue $R = p(Q)Q = p_0Q$; so defining average revenue, AR: $AR(Q) = \frac{R}{Q}$, $AR(Q) = p_0$; and marginal revenue MR: $MR(Q) = \frac{dR}{dQ} = p_0$.*

**b) For such a firm, the profit maximizing conditions stated in Proposition 13.1 will hold at an output $Q^*$, defined so that $MC(Q^*) = MR(Q^*)$ and $MC'(Q^*) \geq 0$.**

*Define the firm's profit, $\Pi$: $\Pi(Q) = R(Q) - C(Q) = p_0Q - C(Q)$. The firm maximizes its profits by choosing output $Q^*$, for which $\frac{d\Pi}{dQ} = 0$ and $\frac{d^2\Pi}{dQ^2} < 0$.*

*Differentiating the expression for profit, $\frac{d\Pi}{dQ} = p_0 - \frac{dC}{dQ} = MR(Q) - MC(Q)$. So the first-order condition is satisfied when $\frac{d\Pi}{dQ} = 0$; or $p_0 = \frac{dC}{dQ}$; or $MR(Q) = MC(Q)$.*

*Differentiating a second time, $\frac{d^2\Pi}{dQ^2} = -\frac{d^2C}{dQ^2} = -\frac{dMC}{dQ}$, so that the second-order condition will be satisfied if $\frac{dMC}{dQ} > 0$.*

**c) For a price taker that maximizes at output $Q^*$, the firm cannot face increasing returns to scale in production at $Q^*$.**

*From part b), we see that the cost function is increasing and convex. This means that the firm's underlying factor demands are also increasing and convex, which is inconsistent with the presence of increasing returns to scale.*

X13.11 **Suppose that a firm is a price taker, but that it faces increasing *marginal* returns to scale. Sketch a diagram showing the relationship between average and marginal revenues and average and marginal costs. Indicate on your diagram the output $Q_0$ for which $MC = MR$. Confirm that $MR'(Q_0) > MC'(Q_0)$, and that the firm makes a loss when output is $Q_0$. Confirm that by producing at $Q_0$ the firm maximizes its loss, rather than its profit.**

*If a firm is a price taker, then we can be certain that the market price $p_0 = MR(Q) = AR(Q)$. That is, marginal and average revenue are both equal to the market price irrespective of the quantity produced.*

*If the firm faces increasing marginal returns (but not increasing returns), then we can be quite certain that both marginal cost and average cost will be decreasing functions of output, and that at any output, marginal cost will be greater than average cost. For average cost AC: $AC(Q) = \frac{C(Q)}{Q}$ and marginal cost MC: $MC(Q) = \frac{dC}{dQ}$, $MC(Q) > AC(Q)$, and $\frac{dMC}{dQ} - \frac{dAC}{dQ} < 0$.*

*In a diagram, with the firm's output measured on the horizontal axis and cost and revenue measures on the vertical axis, the marginal and average revenue functions will be represented by a single, horizontal line, $p_0 = MR(Q) = AR(Q)$. The graph of the marginal cost function will be downward sloping (everywhere). The graph of the average cost function will also be downward sloping, but will lie above the graph of marginal cost.*

*Now consider the conditions for profit maximization. The first-order condition is that marginal revenue and marginal cost are equal. In the diagram, we show this at output $Q_0$, where the graph of marginal revenue meets the graph of marginal cost and $p_0 = MR(Q_0) = MC(Q_0)$. Note that since $AC(Q_0) > MC(Q_0) = p_0$, the firm makes losses.*

*The second-order condition is that marginal cost is increasing more rapidly that marginal revenue. When $Q = Q_0$. $\frac{dMC}{dQ} < 0$, and $\frac{dMR}{dQ} = 0$. In the diagram, the marginal cost curve cuts through the horizontal MR curve from above. The second-order condition is not satisfied. Indeed, with $\frac{dMC}{dQ} < \frac{dMR}{dQ}$, at $Q_0$, the rate of increase of costs falls below the rate of increase of revenues, so that the condition for a profit minimum (here a loss maximum) are satisfied.*

X13.12 **By referring to Figure 13.3 or otherwise, confirm the following:**

**a) There are two values of output at which the first-order condition for a profit maximum, $MC(Q) = MR(Q)$, is satisfied.**

*The condition $MC(Q) = MR(Q)$ is satisfied where price equals marginal cost, and we see in the diagram that $MC(Q) = p$ for two different outputs.*

**b) If $Q \geq Q_1$, then the second-order condition, $MC'(Q) \geq MR'(Q)$, is satisfied.**

*For any output $Q \geq Q_1$, $\frac{dMC}{dQ} > 0$; and since $MR(Q) = p_0$, $\frac{dMR}{dQ} = 0$. The second-order condition is then satisfied.*

**c) The unique profit maximizing output is $Q^*$.**

*There are only two outputs for which $p = MC(Q)$, and only for $Q^*$ is $Q > Q_1$, so that the second-order condition is satisfied.*

d) **When output is $Q^*$, total profit $\Pi(Q^*) = [p_0 - AC(Q^*)].Q^*$.**

*Writing $p_0 = AR(Q^*)$, we define the profit per unit $AR(Q^*) - AC(Q^*)$.*
*Then $Q^*[AR(Q^*) - AC(Q^*)] = R(Q^*) - C(Q^*) = \Pi(Q^*)$.*

X13.13 **Adapting Figure 13.3, sketch diagrams to show how this firm would respond as the market price falls from $p_0$:**

**a) to a level above $AC_{min}$;**
**b) to a level between $AC_{min}$ and $MC_{min}$;**
**c) to a level below $MC_{min}$.**

**In each case, identify the firm's profit maximizing level of output, its average cost of production, and its total profit.**

*As is usual, we draw two panels, with the firm's output measured on the horizontal axis (and both to the same scale; with total revenue and cost measured on the vertical axis of one panel, and average and marginal measures of revenue and cost measured on the vertical axis of the second panel.*

*a) After a price fall, so that $p_1 = AC_{min}$, the total revenue curve becomes flatter, and the marginal (and average) revenue line moves downwards. The total revenue curve still intersects the total cost curve, and the marginal revenue curve still intersects the marginal cost curve at some output greater than $Q_0$, so that the firm can make profits, although they are now smaller than before.*

*b) We have constructed the diagram so that there are no fixed costs. This means that in making its decision, the firm has the option of quitting the market entirely. In this case, where the total revenue curve always lies below the total cost curve, we expect that outcome. We note that although there is an output somewhere between $Q_0$ and $Q_1$ for which $MC(Q) = p_1$, so that the first- and second-order conditions for profit maximization are met, the participation constraint – that production yields greater profits than quitting the market – is not satisfied.*

*c) The decision is exactly the same as in b), but the diagram will be slightly different, in that the total revenue curve is now flatter than the total cost curve for all levels of output, meaning that the marginal cost curve lies above the marginal revenue curve. All production is loss making.*

X13.14 **A firm's total cost function, $C = C(Q) = 15Q - 6Q^2 + Q^3$. It can sell any amount of output it wishes at a price $p_0 = 15$. Write down the firm's total revenue, average revenue, and marginal revenue functions; and also the firm's total profit function.**

*Revenue $R{:}R(Q) = p_0Q = 15Q$; so average revenue and marginal revenue are identical, with $AR(Q) = MR(Q) = 15$.*
*Profit $\Pi$: $\Pi = R(Q) - C(Q) = 15Q - 15Q + 6Q^2 - Q^3 = Q^2(6 - Q)$*

**a) Confirm that the firm breaks even, so that its profit $\Pi(Q) = 0$, if output $Q = 0$ or $Q = 6$.**

*For breakeven, profit $\Pi(Q) = 0$, so either $Q^2 = 0$ (and $Q = 0$), or else $6 - Q = 0$ (and $Q = 6$).*

**b) Obtain the firm's average cost and marginal cost functions.**

*Average cost AC:* $AC(Q) = \frac{C(Q)}{Q} = 15 - 6Q + Q^2$;

*marginal cost, MC:* $MC(Q) = \frac{dC}{dQ} = 15 - 12Q + 3Q^2$

**c) Confirm that *MC*(0) = *AC*(0) = 15; and marginal cost, *MC*, is minimized when *Q* = 2, while average cost, *AC*, is minimized when *Q* = 3. Calculate the minimum values, *MC*(2) and *AC*(3).**

*Evaluating the expressions for MC(Q), and AC(Q) for Q = 0, we obtain MC(0) = AC(0) = 15.*
*To find the minimum value of marginal cost, we differentiate the function, and set the derivative to zero; and then differentiate a second time, checking that where the first-order condition is satisfied, the second derivative is greater than zero.*
*That is, we find the output* $Q_0$ *for which* $\frac{dMC}{dQ} = 6Q - 12 = 0$ *(on rearranging, this is* $Q_0 = 2$*); and then find the second derivative* $\frac{d^2MC}{dQ^2} = 6$*, noting that* $\frac{d^2MC}{dQ^2} > 0$ *for all values of Q. With both conditions satisfied, we can be certain of obtaining a minimum value of marginal cost at Q = 2.*
*To find the minimum value of average cost, we again differentiate the function, and set the derivative to zero; and then differentiate a second time, checking that where the first-order condition is satisfied, the second derivative is greater than zero.*
*That is, we find the output* $Q_1$ *for which* $\frac{dAC}{dQ} = 2Q - 6 = 0$ *(on rearranging, this is* $Q_1 = 3$*); and then find the second derivative* $\frac{d^2AC}{dQ^2} = 2$*, noting that* $\frac{d^2AC}{dQ^2} > 0$ *for all values of Q. With both conditions satisfied, we can be certain of obtaining a minimum value of average cost at Q = 3.*

**d) State the range of outputs *Q* for which the firm faces: (1) increasing total returns to scale; and (2) diminishing marginal returns to scale.**

*Increasing returns to scale where average costs are decreasing, that is for Q < 3.*
*Diminishing marginal returns to scale where MC increasing, that is for Q > 2.*

**e) Confirm that the first-order condition for profit maximization, *p* = *MC*, is satisfied if the firm chooses output *Q* = 0 or *Q* = 4. Confirm that the second-order condition is also satisfied if *Q* = 4.**

*Since p = 15 and* $MC(Q) = 15 - 12Q + 3Q^2$*, the first-order condition for profit maximization is satisfied when p = MC(Q), or when* $15 = 15 - 12Q + 3Q^2$*; in which case,* $3Q(Q - 4) = 0$*. This equation is satisfied if either 3Q = 0 (or Q = 0), or else if Q – 4 = 0, so that Q = 4.*
*Given these two possible values for the profit maximizing output, we consider the second-order condition,* $\frac{d^2C}{dQ^2} > \frac{d^2R}{dQ^2}$*. Since* $MR(Q) = \frac{dR}{dQ} = 15$*,* $\frac{d^2R}{dQ^2} = 0$*, and since* $MC(Q) = \frac{dC}{dQ} = 15 - 12Q + 3Q^2$*,* $\frac{d^2C}{dQ^2} = 6Q - 12$*. We require this expression to take a positive sign when the first-order condition is satisfied; and we see that 6Q – 12 > 0 if Q > 2. The second-order condition for profit maximization is therefore satisfied for Q = 4, but not for Q = 0.*

**f) Confirm that the profit function has a minimum at *Q* = 0 and a point of inflexion at *Q* = 2.**

*Recall that profit* $\Pi$*:* $\Pi = R(Q) - C(Q) = 15Q - 15Q + 6Q^2 - Q^3 = Q^2(6 - Q)$*.*
*In considering the behaviour of this function, we can differentiate it twice, with the first-order condition for stationary values being satisfied when* $\frac{d\Pi}{dQ} = 0$*; the second-order condition for a minimum being satisfied for a value of output,* $Q_1$ *for which the first-order condition is*

*satisfied and* $\frac{d^2\Pi}{dQ^2} > 0$*; and a point of inflexion occurring at a level of output* $Q_2$ *where the first-order condition is not satisfied, but the second derivative* $\frac{d^2\Pi}{dQ^2} = 0$.

*We write the derivatives* $\frac{d\Pi}{dQ} = 12Q - 3Q^2$ *and* $\frac{d^2\Pi}{dQ^2} = 12 - 6Q$*. We already know that the first- and second-order conditions for a minimum are satisfied when Q = 0.*

*Evaluating the derivatives when Q = 2, we obtain* $\frac{d\Pi}{dQ} = 12$ *and* $\frac{d^2\Pi}{dQ^2} = 0$*. The condition for a point of inflexion is satisfied. (At this level of output, the function shifts from being convex to being concave, as it approaches the profit maximizing output).*

g) **For values of output, *Q*, between 0 and 6, sketch: (1) the revenue, cost and profit functions (all on one diagram); and (2) the average and marginal revenue and average and marginal cost functions (all on a second diagram). Clearly mark on your diagrams the key characteristics of these functions, obtained in parts (a)–(g).**

*For diagram (1), we measure the firm's output on the horizontal axis, and (total) revenue, cost and profit on the vertical axis. The graph of total revenue is a straight line, with gradient 15, starting from the origin. (At Q* = 4, the profit maximizing output, R(4) = 60.) The graph of total cost is a cubic, with a minimum turning point at (0, 0), and which is increasing, but concave until the inflexion at Q = 2 (when C = 14). From then on, the curve is convex, increasing at an increasing rate, so that at Q* = 4, C(4) = 28, and the firm makes profit* $\Pi$ *= 32.*

*The graph of profit is also a cubic, and also begins from (0, 0), with an inflexion at (2, 16). In the interval 0 < Q < 2, the profit function is convex, but it becomes concave at Q = 2, increasing at a decreasing rate until Q* = 4, where it reaches its maximum,* $\Pi^*$ *= 32. With costs increasing more rapidly than revenues, the graph intersects the Q-axis at Q = 6, just breaking even at this output and making losses for higher levels of output.*

*For diagram (2), average and marginal revenue are coincident, and represented by the line* $p_0$ *= MR = AR = 15. Average cost is a parabola, intersecting the vertical axis at (0, 15), with a minimum at (3, 6) and increasing for Q > 3. Marginal cost is also a parabola, passing through (0, 15) and (3, 6), but with a minimum at (2, 3).*

h) **Suppose that the market price falls to $p_1$. How should the firm react: (1) if $p_1 > 6$; and (2) if $p_1 < 6$?**

*Recall that* $AC_{min}$ *= 6. If* $p_1$ *= 6, the firm can make profits from sales, and there will be some output Q > 3 that satisfies the first- and second-order conditions for a profit maximum. If Q < 3, then the participation constraint, that the firm makes a positive profit, will not be satisfied. The firm will choose to quit the market rather than continue in a situation where it makes a loss. Even if it is possible to identify a level of output Q > 2 for which the first- and second-order conditions are satisfied, the firm would still make a loss at this level of output.*

X13.15 **For each of the following cost functions, *C* = *C*(*Q*), obtain the average and marginal cost functions, and sketch them. In each case, show that the cost functions exhibit (eventually) diminishing returns to scale. Indicate clearly the firm's supply on your sketches.**

| | | |
|---|---|---|
| **a)** $C(Q) = 0.5Q^2$ | **b)** $C(Q) = 4Q^{1.5}$ | **c)** $C(Q) = Q + Q^3/3$ |
| **d)** $C(Q) = 8Q + Q^2$ | **e)** $C(Q) = 30Q - 9Q^2 + Q^3$ | **f)** $C(Q) = 12 + 8Q + Q^2$ |

*a) The average cost,* $AC(Q) = \frac{C(Q)}{Q} = 0.5Q$*; marginal cost* $MC(Q) = \frac{dC}{dQ} = Q$*. In a diagram with output measured on the horizontal, and average and marginal costs on the vertical, axis, the average cost curve starts from the origin, is upward sloping and has gradient 0.5. The marginal cost curve is also an upward-sloping straight line, which starts from the origin, but it has gradient 1, so is twice as steep as the average cost curve.*

*The supply curve is the upward-sloping segment of the marginal cost curve above the average cost curve; in this case, it is the whole of the marginal cost curve.*

b) *The average cost, AC(Q) =* $\frac{C(Q)}{Q}$ *=* $4Q^{0.5}$*; marginal cost MC(Q) =* $\frac{dC}{dQ}$ *=* $6Q^{0.5}$*. In a diagram with output measured on the horizontal, and average and marginal costs on the vertical, axis, the average cost curve starts from the origin, is upward sloping and initially has an undefined gradient (so that it is vertical). It is one arm of a parabola, and so is concave. The marginal cost curve is also one arm of a parabola, which is vertical at the origin, but its gradient is 1.5 times the gradient of the average cost curve, so that for* $Q > 0$*, the marginal cost curve lies above the average cost curve.*
*The supply curve is the upward-sloping segment of the marginal cost curve above the average cost curve; in this case, it is the whole of the marginal cost curve.*

c) *The average cost, AC(Q) =* $\frac{C(Q)}{Q} = 1 + \frac{Q^2}{3}$ *; marginal cost MC(Q) =* $\frac{dC}{dQ}$ *=* $1 + Q^2$*. In a diagram with output measured on the horizontal, and average and marginal costs on the vertical, axis, the average cost curve is part of a parabola. It is upward sloping and convex everywhere and starts from (0, 1). The marginal cost curve is also an upward-sloping, convex segment of a parabola, which starts from (0, 1). We can see that the coefficient on* $Q^2$ *is larger than for the average cost curve, so that we can be certain that the marginal cost curve lies above the average cost curve.*
*The supply curve is the whole of the marginal cost curve.*

d) *The average cost, AC(Q) =* $\frac{C(Q)}{Q}$ *=* $8 + Q$*; marginal cost MC(Q) =* $\frac{dC}{dQ}$ *=* $8 + 2Q$*. In a diagram with output measured on the horizontal, and average and marginal costs on the vertical, axis, the average cost curve starts from (0, 8), is upward sloping and has gradient 1. The marginal cost curve is also an upward-sloping straight line, which starts from (0, 8), but it has gradient 2, so is twice as steep as the average cost curve.*
*Once again, the supply curve is the whole of the marginal cost curve.*

e) *The average cost, AC(Q) =* $\frac{C(Q)}{Q}$ *=* $30 – 9Q + Q^2$*; marginal cost MC(Q) =* $\frac{dC}{dQ}$ *=* $30 – 18Q + 3Q^2$*. In a diagram with output measured on the horizontal, and average and marginal costs on the vertical axis, the average cost curve is a segment of a parabola, which starts from (0, 30) on the vertical axis. Initially, it slopes downwards, reaching a minimum at* $(Q_0, AC(Q_0)) = (4.5, 9.75)$*, and thereafter increases. It is convex throughout. The marginal cost curve is also a parabola, which starts from (0,30) and passes through (4.5, 9.75), but it has a minimum at (3, 3).*
*The supply curve is the upward-sloping segment of the marginal cost curve above the average cost curve; in this case, it is the marginal cost curve for* $Q \geq 4.5$*.*

f) *The average cost, AC(Q) =* $\frac{C(Q)}{Q}$ *=* $8 + Q$*; marginal cost MC(Q) =* $\frac{dC}{dQ}$ *=* $8 + 2Q$*. In a diagram with output measured on the horizontal, and average and marginal costs on the vertical axis, the average cost curve starts from (0, 8), is upward sloping and has gradient 1. The marginal cost curve is also an upward-sloping straight line, which starts from (0, 8), but it has gradient 2, so is twice as steep as the average cost curve.*
*In this case, the supply curve is the whole of the marginal cost curve.*

X13.16 **Using Expression 13.6, or otherwise:**

a) **Confirm that when output is zero, the firm makes a loss,** $\Pi(0) = -C_0$**.**
*When output,* $Q = 0$*,* $R(0) = C_V(0)$*,* $\Pi_V(0) = R(0) – C_V(0) = 0 = \Pi(0) + C_0$*, so the firm make a loss,* $\Pi(0) = -C_0$*.*

b) **Differentiating twice, confirm that the conditions for profit maximization in Proposition 13.2 will be satisfied, so long we define the firm's objective as being to maximize the profit net of fixed costs,** $\Pi_V$**.**

*Differentiating expression [X13.6], we obtain* $\frac{\partial \Pi_V}{\partial Q} = \frac{\partial \Pi}{\partial Q} + \frac{\partial C_0}{\partial Q} = \frac{\partial R}{\partial Q} - \frac{\partial C_V}{\partial Q}$. *Since* $C_0$ *is constant,* $\frac{\partial C_0}{\partial Q}$ *= 0. Then* $\frac{\partial \Pi_V}{\partial Q} = \frac{\partial \Pi}{\partial Q} = \frac{\partial R}{\partial Q} - \frac{\partial C_V}{\partial Q}$. *Setting this expression to zero, we obtain the usual first-order condition,* $\frac{\partial \Pi_V}{\partial Q} = \frac{\partial \Pi}{\partial Q}$ *=* $MR(Q^*) - MC(Q^*) = 0$. *(Note that* $\frac{\partial C_V}{\partial Q} = MC$ *; since* $\frac{\partial C_0}{\partial Q} = 0$ *.* *Differentiating a second time,* $\frac{\partial^2 \Pi_V}{\partial Q^2} = \frac{\partial^2 \Pi}{\partial Q^2} = \frac{\partial MR}{\partial Q} - \frac{\partial MC}{\partial Q}$ *. For profit maximization, we require this second derivative to be less than zero when output,* $Q = Q^*$*. This is the familiar condition that the rate of change of marginal cost has to be greater than the rate of change of marginal revenue when* $Q = Q^*$*.*

**c) Show that a firm facing short-run cost function,** $C(Q) = 12 + 8Q + Q^2$**, and market price** $p_0 = 14$**, will maximize its short-run profits by choosing an output,** $Q^* = 3$**, at which it makes a loss,** $\Pi(3) = -3$**.**

*In this case, writing* $\Pi_V = 14Q - 8Q - Q^2$, $\frac{d\Pi_V}{dQ} = 6 - 2Q = 0$*, for* $Q^* = 3$*. Since* $\frac{d^2 \Pi_V}{\partial Q} = -2$ *, we see that for* $Q^* = 3$*, the first-order condition,* $\frac{d\Pi_V}{dQ} = 0$ *and the second-order condition,* $\frac{d^2 \Pi_V}{\partial Q} < 0$*, required for a maximum of the profit function, are both satisfied.*

## Chapter 14

X14.1 **Suppose that the market demand curve is decreasing in price, and that the market supply curve is increasing in price. Define the excess market supply as the difference between market supply and market demand at any price. Show that the excess market supply is itself increasing in price, and that there can only be a single value of price and output at which the two curves intersect.**

*Excess market supply, $Q^E$: $Q^E(p) = Q^S(p) – Q^D(p)$, where $Q^S(p)$ is the market supply at price p, and $Q^D(p)$ is the market demand, also at price p. We assume that when p = 0, $Q^S(0) < Q^D(0)$, so that $Q^E(0) < 0$. Differentiating, $\frac{dQ^E}{dp} = \frac{dQ^S}{dp} - \frac{dQ^D}{dp}$. By assumption, $\frac{dQ^S}{dp} > 0$, and $\frac{dQ^D}{dp} < 0$, so $-\frac{dQ^D}{dp} > 0$, and $\frac{dQ^E}{dp} > 0$. As the market price increases, the excess supply increases (towards zero), so that the gap between excess demand and excess supply becomes smaller. We cannot prove here that the excess supply ever reaches zero, but we can be certain that as prices rise, it will not decrease. We can also be certain that if we find a price at which excess supply is greater than zero, it will not decrease back to zero as prices rise. So there cannot be more than one price at which $Q^E(p) = 0$. The market clearing solution $(Q^M(p^M), p^M)$ is unique.*

X14.2 **We assume that market price cannot fall below zero. Suppose that there is excess market supply at any price greater than zero. Sketch a diagram showing this situation. Characterize your solution in terms of the concept of scarcity.**

*In a diagram with quantities measured on the horizontal, and prices on the vertical, axis, the market supply curve will be upward sloping and the market demand curve will be downward sloping. For excess market supply, $Q^E$, when price, p = 0, $Q^S(0) > Q^D(0)$, so that the market supply curve meets the quantity axis to the right of the market demand axis. It follows that as the market price increases, excess market supply continues to increase, and there is no price ($p \geq 0$), for which the market clears.*

*With excess market supply when p = 0, we consider the good to be abundant. Sellers giving the good away for free are unable to get rid of all of their stock of the good.*

X14.3 **Suppose that the graph of market supply is a flat line and that the graph of market demand is a downward-sloping line. Sketch the graphs of market supply, market demand and excess supply.**

*In a diagram with quantities measured on the horizontal, and prices on the vertical, axis, the flat market supply curve indicates that any quantity can be sold at the fixed price, $p = p^M$, while the downward sloping market demand curve indicates that the quantity demanded decreases as the market price increases. The market clears when $p = p^M$.*

*At any price, $p < p^M$, the market supply, $Q^S(p) = 0$, the excess market supply $Q^E(p) = -Q^D(p)$, and so the graph of excess market supply is the reflection of the market demand in the price axis. For $p > p^M$, though, $Q^S(p) \to \infty$, so that the excess supply cannot be measured.*

X14.4 **Adapt Figure 14.1 to show the firm and market supply curves in the very short run.**

*In a diagram with two panels set side by side, and measuring price on the vertical axis for both, with the quantity that the firm produces measured on the horizontal axis of one panel, and the quantity that the industry produces measured on the horizontal panel of the other, in the very short run, the firm supply curve is vertical, and so the market supply curve is also vertical; for both the firm and the market, supply is fixed.*

X14.5 **Adapt Figure 14.1 to show the firm and market supply curves in the short run, with the firm making a loss in the short run but continuing production. Show that this is consistent with profit maximization.**

*In a diagram with two panels set side by side, and measuring monetary values on the vertical axis for both, with the quantity that the firm produces measured on the horizontal axis of one panel, and the quantity that the industry produces measured on the horizontal panel of the other, then in the short run, the firm supply curve is the upward-sloping segment of the firm's marginal cost curve that lies above the firm's average variable cost curve. The market supply will therefore run along the price axis for price, $p^M < AVC_{min}$, the minimum of average variable cost. It then extends horizontally at $p^M = AVC_{min}$, and is then upward sloping when $p^M > AVC_{min}$.*

X14.6 **Consider a firm that has a production function $Q(K, L) = 9K^{1/3}L^{2/3}$. The firm has already acquired the capital stock $K_0 = 1{,}000$, which it will not be able to vary in the short run. It plans to hire $L_0 = 8$ staff.**

a) **Confirm that the firm will be able to produce $Q_0 = 360$ units of output when its plan has been met.**

*Given the production function, $Q(K, L) = 9K^{\frac{1}{3}}L^{\frac{2}{3}}$, setting $K_0 = 1{,}000$ (so that $K^{\frac{1}{3}} = 10$), and $L_0 = 8$ (so that $L_0^{\frac{2}{3}} = 4$), the firm produces output $Q(1{,}000, 8) = 9*10*4 = 360$.*

b) **Applying the usual cost minimizing condition, calculate the factor input price ratio, $w_K/w_L$.**

*The firm plans to be economically efficient, so that we expect the first-order condition, MRTS $= -\frac{MP_K}{MP_L} = -\frac{w_K}{w_L}$ to be satisfied. Partially differentiating the production function, we obtain $MP_K = \frac{\partial Q}{\partial K} = 3\left(\frac{L}{K}\right)^{\frac{2}{3}}$; and $MP_L = \frac{\partial Q}{\partial L} = 6\left(\frac{K}{L}\right)^{\frac{1}{3}}$; so that the ratio $\frac{MP_K}{MP_L} = \frac{L}{2K} = \frac{w_K}{w_L}$. But here, applying the values, $L_0 = 8$ and $K_0 = 1{,}000$, we obtain $\frac{w_K}{w_L} = \frac{8}{2{,}000} = 0.004$.*

c) **Calculate the cost of producing output $Q_0$ given the planned factor usage, when $w_L = 1{,}000$. How much profit does the firm make if the market price $p_0 = 50$?**

*With $w_L = 1{,}000$, then $w_K = 4$ ($= 0.004w_L$). So cost of production $C(360) = w_KK + w_LL = 4*1{,}000 + 1{,}000*8 = 12{,}000$; while revenue $R(360) = 50*360 = 18{,}000$. Profit $\Pi(360) = 18{,}000 - 12{,}000 = 6{,}000$.*

d) **Write down the firm's short-run production function, and hence the firm's short-run total cost function.**

*In short run, we fix capital at $K_0 = 1{,}000$, so production, $Q_S$, is a function of the (variable) labour input, L: $Q_S(L) = 9.(1{,}000)^{\frac{1}{3}}.L^{\frac{2}{3}} = 90L^{\frac{2}{3}}$. Inverting this function, we obtain the demand for labour $L(Q_S) = \left(\frac{Q_S}{90}\right)^{\frac{3}{2}}$. The short run production cost, $C_S$, is of course a function of the target output, $Q_S$; with $C_S(Q_S)$ the sum of the expenditure on (fixed) capital and (variable) labour, so that $C(Q_S) = w_KK_0 + w_LL(Q_S) = 4*1{,}000 + 1{,}000\left(\frac{Q_S}{90}\right)^{\frac{3}{2}} = 1{,}000\left[4 + \left(\frac{Q_S}{90}\right)^{\frac{3}{2}}\right]$.*

e) **Obtain the short-run average total cost, the short-run average variable cost, and the short-run marginal cost. Sketch these functions, and indicate the firm's short-run supply function.**

*Short-run average total cost, $AC(Q_S) = \frac{C(Q_S)}{Q_S} = 1{,}000\left[\frac{4}{Q_S} + \left(\frac{Q_S}{90^3}\right)^{\frac{1}{2}}\right]$.*

*Short run average variable cost, $AVC(Q_S) = \frac{C_V(Q_S)}{Q_S} = 1{,}000\left(\frac{Q_S}{90^3}\right)^{\frac{1}{2}}$.*

*Short-run marginal cost, $MC(Q_S) = \frac{dC}{dQ_S} = 1{,}000\left[\frac{3}{2}\left(\frac{Q_S}{90}\right)^{\frac{1}{2}}\right] = 500\left(\frac{Q_S}{10}\right)^{\frac{1}{2}}$.*

*Since marginal cost is greater than average variable cost, and marginal cost is always increasing in output, the short-run supply has the same form as the marginal cost.*

X14.7 **Show that, in the short run, we can write the firm's profit maximization condition as $MC = w/MP_L$.**

*It may be easier to write the expression as $MC.MP_L = w$. Define the marginal cost as the rate of change of cost with respect to output. Then the wage is the rate of change of cost with respect to the labour input, and the marginal product is the rate of change of output with respect to the labour input. The result then follows by application of the chain rule of differentiation.*

X14.8 **Suppose that it is possible to write a firm's short-run production function, $Q = AL^b$, where parameters $A > 0$, and $b < 1$ are fixed. Obtain the firm's (total) cost function, given that it has already committed itself to paying fixed costs, $F$. Obtain the supply function, showing output, $Q$, as a function of price, $p$, for a given wage, $w$.**

*First, we rewrite the production function so that it defines the demand for labour, L: $L(Q) = \left(\frac{Q}{A}\right)^{\frac{1}{b}}$. We write the cost of production, C: $C(Q) = F + wL(Q) = F + w\left(\frac{Q}{A}\right)^{\frac{1}{b}}$. Then we write the average cost AC: $AC = \frac{C(Q)}{Q} = \frac{F}{Q} + \frac{w}{A^{\frac{1}{b}}}Q^{\frac{1-b}{b}}$; the average variable cost as $AVC = \frac{C_V(Q)}{Q} = \frac{w}{A^{\frac{1}{b}}}Q^{\frac{1-b}{b}}$ and the marginal cost MC: $MC = \frac{dC(Q)}{dQ} = \frac{w}{bA^{\frac{1}{b}}}Q^{\frac{1-b}{b}}$. We see that $AVC < MC$ for all values of Q, and that MC is increasing in Q (since $Q < 1$); so that the short run supply has the same functional form as the marginal cost.*

X14.9 **For the following (long-run) production functions, $Q = Q(K, L)$, write down expressions for:**

i. **the short-run production function, $Q^S = Q^S(K_0, L)$;**
ii. **the marginal product of labour, $MP_L = Q^{S\prime}(K_0, L)$;**
iii. **the short-run demand for labour, $L^S = L(Q^S, K_0)$;**
iv. **the short-run total cost function, $C^S = C(Q^S, K_0)$;**
v. **the short-run marginal cost function, $MC^S = C^{S\prime}(Q^S, K_0)$;**
vi. **the short-run supply function, $Q^S = Q^S(p, w, K_0)$.**

**You may assume that the firm has a capital stock, $K_0 = 10{,}000$, that the cost of a unit of capital $r = 5$, and that the cost of a unit of labour $w = 4$.**

**a) $Q(K, L) = K^{½}L^{½}$; b) $Q(K, L) = 20[K^{½} + L^{½}]$; c) $100(L + K)^{½}$**

*a) For production function, $Q(K, L) = K^{½}L^{½}$*

i. *We rewrite the production function with fixed capital stock, $K_0 = 10{,}000$, so that $Q^S(L) = 100L^{½}$.*
ii. *To obtain the marginal product of labour, we differentiate the short run production function, $Q^S$ with respect to labour usage, L: $\frac{dQ^s}{dL} = 50L^{-0.5}$.*
iii. *To obtain the short run demand for labour, we invert the production function, writing $L^S$ as a function of Q: $L^S(Q) = 0.0001Q^2$*
iv. *The short-run (total) cost, $C^S$ is the sum of expenditures on hiring capital and labour; $C(Q^S, K_0) = w_K K_0 + w_L L^S = 5*10{,}000 + 4*0.0001Q^2 = 50{,}000 + 0.0004Q^2$*
v. *Short-run marginal cost $MC^S$ is the derivative of the short run cost function, so $MC(Q) = \frac{dC}{dQ} = 0.0008Q$.*
vi. *Short run supply: short run average cost, $AC^S$: $AC(Q) = \frac{C^S}{Q^S} = \frac{50{,}000}{Q} + 0.0004Q$. We see that the short-run average variable cost, $AVC^S$ can then be written $AVC(Q^S) = 0.0004Q$, which is increasing. The firm's supply is chosen so that it sells the output for which $p = MC(Q^S)$, or for which $p = 0.0008Q^S$; so that $Q^S = 1{,}250p$.*

b) *For production function, $Q(K, L) = 20[K^{½} + L^{½}]$*

i. *We rewrite the production function with fixed capital stock, $K_0$ = 10,000, so that $Q^S(L) = 20[100 + L^{½}]$.*

ii. *To obtain the marginal product of labour, we differentiate the short run production function, $Q^S$ with respect to labour usage, L: $\frac{dQ^S}{dL} = 10L^{-0.5}$.*

iii. *To obtain the short run demand for labour, we invert the production function, writing $L^S$ as a function of Q: $L^S(Q) = L^{0.5} = 100 - 0.05Q$; so that $L = (100 - 0.05Q)^2 = 10{,}000 - 40Q + 0.0025Q^2$*

iv. *The short-run (total) cost, $C^S$ is the sum of expenditures on hiring capital and labour; $C(Q^S, K_0) = w_K K_0 + w_L L^S = 5*10{,}000 + 4*(10{,}000 - 40Q + 0.0025Q^2) = 90{,}000 - 40Q + 0.01Q^2$*

v. *Short-run marginal cost $MC^S$ is the derivative of the short run cost function, so $MC(Q) = \frac{dC}{dQ} = 0.02Q - 40$.*

vi. *Short run supply: short run average cost, $AC^S$: $AC(Q) = \frac{C^S}{Q^S} = \frac{90{,}000}{Q} - 40 + 0.01Q$. We see that the short-run average variable cost, $AVC^S$ can then be written $AVC(Q^S) = 0.01Q^S - 40$, which is increasing. The firm's supply is chosen so that it sells the output for which $p = MC(Q^S)$, or for which $p = 0.02Q^S - 4{,}000$; so that $Q^S = 50(p + 40)$.*

c) *For production function, $Q(K, L) = 100[K + L]^{½}$*

i. *We rewrite the production function with fixed capital stock, $K_0$ = 10,000, so that $Q^S(L) = 100[10{,}000 + L]^{½}$.*

ii. *To obtain the marginal product of labour, we differentiate the short run production function, $Q^S$ with respect to labour usage, L: $\frac{dQ^S}{dL} = 50(10{,}000 + L)^{-0.5}$.*

iii. *To obtain the short run demand for labour, we invert the production function, writing $L^S$ as a function of Q. Since $0.01Q = (10{,}000 + L)^{0.5}$, it follows that $0.0001Q^2 = 10{,}000 + L$; and that $L^S(Q) = \max(0, 0.0001Q^2 - 10{,}000)$ [and since $0.0001Q^2 = 10{,}000$ if Q = 10,000, we shall assume that Q > 10,000]*

iv. *The short-run (total) cost, $C^S$ is the sum of expenditures on hiring capital and labour; $C(Q^S, K_0) = w_K K_0 + w_L L^S = 5*10{,}000 + 4*[0.0001Q^2 - 10{,}000] = 10{,}000 + 0.0004Q^2$*

v. *Short-run marginal cost $MC^S$ is the derivative of the short run cost function, so $MC(Q) = \frac{dC}{dQ} = 0.0008Q$.*

vi. *Short run supply: short run average cost, $AC^S$: $AC(Q) = \frac{C^S}{Q^S} = \frac{10{,}000}{Q} + 0.0004Q$. We see that the short-run average variable cost, $AVC^S$ can then be written $AVC(Q^S) = 0.0004Q$, which is increasing. The firm's supply is chosen so that it sells the output for which $p = MC(Q^S)$, or for which $p = 0.0008Q^S$; so that $Q^S = 1{,}250p$.*

X14.10 **Sketch the isoquant map for the production function in X14.9c, and discuss whether or not you believe that the firm would behave as suggested in the question.**

*Drawing the isoquant map as a diagram with usage of capital on the horizontal and usage of labour on the vertical axis, the isoquants are parallel lines, reflecting the treatment of capital and labour as perfect substitutes. In this case, capital and labour both have diminishing marginal product, but if there is any difference in price, we would expect the firm to hire one and not the other. In the short run, since the firm has hired capital, it should be with the expectation that it will not be necessary to hire any labour, so that the labour demand that we have identified represents the effect of a positive shock, increasing production above its planned level, $Q^S$ = 10,000.*

X14.11 **Repeat X14.9a for any wage, *w*, and any interest rate, *r*, obtaining the short-run supply as a function of the market price, *p*, the wage, *w*, and the capital stock, $K_0$.**

*For production function, $Q(K, L) = K^{½}L^{½}$*

i. *We rewrite the production function with fixed capital stock, $K_0$, so that $Q^S(L) = K_0^{½}L^{½}$.*

ii. *To obtain the marginal product of labour, we differentiate the short run production function, $Q^S$ with respect to labour usage, L: $\frac{dQ^S}{dL} = 0.5K_0^{0.5}L^{-0.5}$.*

iii. *To obtain the short run demand for labour, we invert the production function, writing $L^S$ as a function of Q: $L^S(Q) = \frac{Q^2}{K_0}$*

iv. *The short-run (total) cost, $C^S$ is the sum of expenditures on hiring capital and labour; $C(Q^S, K_0) = w_K K_0 + w_L L^S = w_K K_0 + w_L \frac{Q^2}{K_0}$*

v. *Short-run marginal cost $MC^S$ is the derivative of the short run cost function, so $MC(Q) = \frac{dC}{dQ} = \frac{2w_L}{K_0}Q$.*

vi. *Short run supply: short run average cost, $AC^S$: $AC(Q) = \frac{C^S}{Q^S} = \frac{w_K K_0}{Q} + \frac{w_L}{K_0}Q$. We see that the short-run average variable cost, $AVC^S$ can then be written $AVC(Q^S) = \frac{w_L}{K_0}Q$, which is increasing. The firm's supply is chosen so that it sells the output for which $p = MC(Q^S)$, or for which $p = \frac{2w_L}{K_0}Q$; so that $Q^S = \frac{p}{2w_L}K_0$.*

*[You may wish to check that the answers here correspond to those in X14.9a.]*

X14.12 **Consider the case where there are 200 firms in the market, each with production function $Q_f = 20K_0^{½}L^{½}$. The price of a unit of capital $r = 3$, and the cost of a unit of labour $w = 12$.**

a) **Write down the quantity of labour that the firm hires as a function of its short-run output, $L = L(Q_f, K_0)$.**

*To obtain the short run demand for labour, we invert the production function, obtaining $L = \frac{Q_f^2}{400K_0}$.*

b) **Write down the firm's short-run cost function, $C = C(Q_f)$, and hence obtain its marginal and average cost functions.**

*The firm has already committed itself to paying the hiring cost of capital, $3K_0$, and now must pay the hiring cost of labour, $12L = \frac{3Q_f^2}{100K_0}$. The cost $C = 3K_0 + \frac{3Q_f^2}{100K_0}$.*

*Average cost is the cost per unit of output, so $AC(Q_f) = \frac{C}{Q_f} = \frac{3K_0}{Q_f} + \frac{3Q_f}{100K_0}$. Marginal cost is the partial derivative of cost with respect to output, so $MC(Q_f) = \frac{\partial C}{\partial Q_f} = \frac{6Q_f}{100K_0}$.*

c) **Show that firms minimize their average costs by producing output $Q_f = 10K_0$.**

*We simply confirm here that the average cost function has a stationary value at $Q_f = 10K_0$, ignoring the second-order condition. Evaluating the partial derivative, $\frac{\partial AC}{\partial Q_f} = -\frac{3K_0}{Q_f^2} + \frac{3}{100K_0}$, for $Q_f = 10K_0$, we require $\frac{\partial AC}{\partial Q_f} = 0$.*

*Since $-\frac{3K_0}{Q_f^2} + \frac{3}{100K_0} = 3\frac{Q_f^2 - 100K_0^2}{100Q_f^2K_0}$, the only way in which $\frac{\partial AC}{\partial Q_f} = 0$ is if $Q_f^2 - 100K_0^2 = (Q_f - 10K_0)(Q_f + 10K_0) = 0$, or if $Q_f - 10K_0 = 0$ (since $Q_f + 10K_0 > 0$); and so we find that $Q_f = 10K_0$*

d) **Confirm that for output $Q_f = 10K_0$, firms' marginal and average costs are equal, with $MC(10K_0) = AC(10K_0) = 0.6$.**

*When $Q_f = 10K_0$, $AC(10K_0) = \frac{3K_0}{10K_0} + \frac{30K_0}{100K_0} = 0.3 + 0.3 = 0.6$; and $MC(10K_0) = \frac{60K_0}{100K_0} = 0.6$.*

**e) Confirm that the market supply function, $Q^S = Q^S(p^M) = 10{,}000K_0p/3$.**

*Firm supply is obtained from the profit maximizing condition, $p = MC(Q_f)$, or $p = \frac{3Q_f}{50K_0}$, so that $Q_f = \frac{50K_0}{3}p$. With 200 firms, each facing the same market conditions, the market supply $Q^S = 200Q_f = \frac{10{,}000K_0}{3}p$*

X14.13 **Now suppose that all 200 firms have already hired capital, $K_0 = 10{,}000$, so that the market supply, $Q^S = 100{,}000{,}000p/3$. Sketch a diagram showing the relationship between firm supply and market supply.**

*This can be done quite straightforwardly on a single diagram, showing the market price on the vertical axis, but measuring both the firm output and the market supply on the horizontal axis. We require two scales; with the firm supply scale 200 times that of the market supply scale (so that one unit for the firm equals 200 units of market supply), the firm supply and the market supply curves, will be identical, an upward sloping line through the origin.*

X14.14 **Sketch graphs of the short-run marginal cost, the short-run average (total) cost and the short-run average variable cost functions for the firm in X14.9b, and explain the relationship between these curves and firm's short-run supply function.**

*Short –run marginal cost: $MC(Q) = \frac{dC}{dQ} = 0.02Q - 40$*

*Short-run average (total) cost: $AC(Q) = \frac{C^S}{Q^S} = 90{,}000 - 40Q + 0.01Q^2$; and short-run average variable cost, $AVC(Q) = 0.01Q - 40$.*

*In a diagram with output, Q, measured on the horizontal axis, and the measures of cost on the vertical axis, the marginal cost curve is an upward-sloping line with gradient 0.02, which we show as starting from its intersection with the horizontal axis at Q = 2,000. (Given the nature of the production function, for Q < 2,000, the firm will hire no labour, undertaking production simply by using capital.) The average variable cost curve is a line with gradient 0.01. We show it as starting from its intersection with the horizontal axis where Q = 4,000. For any value of Q: Q > 2,000, MC(Q) > $AVC_S(Q)$, so that the marginal cost curve forms the short-run supply curve.*

*Setting price equal to marginal cost, we write p = 0.02Q – 40, so that Q = 2,000 + 50p and this is the firm's short run supply.*

*Note that since average fixed cost is not defined for Q = 0, average cost will increase without limit as Q = 0. Average cost has a minimum where Q = 3,000 (and AC =20). The marginal cost curve intersects the average curve at this output. For higher levels of output average cost increases, approaching the average variable cost curve from above, but never meeting it.*

X14.15 **Continuing to work with the production function in X14.9b, suppose that there are *F* firms in the market, all of them identical. Write down an expression for the market supply, $Q^S = Q^S(p)$.**

*Since all firms are identical, they produce the same output, $Q_f(p)$ =2,000 + 50p, so that the market supply, which is the sum of output is simply the number of firms multiplied by the output of each firm: $Q^S(p)$ = (2,000 + 50p)F.*

X14.16 **Now suppose that there are *C* consumers in the market, each of whom has a sum of money $m_i$ to finance consumption of goods. We shall assume that there are two goods available for consumption, the good of interest, and a composite that represents all other goods.**

**We assume that all consumers have the same utility function, $U_i = U(q_i, x_i)$, where $q_i$ is the quantity of the good of interest, and $x_i$ is the quantity of the composite good. For Cobb-Douglas preferences, so that $U(q_i, x_i) = q_i^{0.5} x_i^{0.5}$:**

a) **Calculate each consumer's demand, $q_i$, and hence the market demand, $Q^D = Q^D(p)$.**
*We can rely here on our knowledge of the demand functions associated with a Cobb-Douglas utility function. With money, $m_i$, the consumer spends $p_i q_i = 0.5m_i$ on the good of interest, so that demand $q_i = \frac{m_i}{2p}$. Since all consumers face the same decision, except that they may have different amounts to spend on the goods, the market demand is the sum of individual demands, $Q^D$: $Q^D(p) = \sum_{i=1}^{N} q_i(p) = \sum_{i=1}^{N} \frac{m_i}{2p} = \frac{M}{2p}$*

b) **Sketch the market supply and market demand functions for this good.**
*In a diagram with the market supply and market demand measures on the horizontal axis, and the price measured on the vertical axis, the market supply is an upward-sloping line,starting from its intersection with the horizontal axis, where p = 0 and Q = 2,000F. The market demand curve is a rectangular hyperbola.*

**Show that if there is market clearing, then $p = -20 + \left[400 + \frac{M}{100F}\right]^{0.5}$.**
*Writing the market supply, $Q^S = (2,000 + 50p)F$, and the market demand, $Q^D = \frac{M}{2p}$, then for market clearing,*
*$Q^S = Q^D$, and $2Fp(2,000 + 50p) = M$, so that $100Fp^2 + 4,000Fp - M = 0$. We can solve this equation by apply the standard quadratic formula, and considering only the positive root of this particular expression:*

$$p = \frac{-4,000F + \left[(4,000F)^2 + 400MF\right]^{0.5}}{200F} = -20 + \left[400 + \frac{M}{100F}\right]^{0.5}.$$

X14.17 **Explain why the long-run average cost curve always lies below the short-run average (total) cost curve.**
*Long-run average cost curve reflects the economically efficient use of resources; it is the minimum possible cost of production.*

X14.18 **Explain why the long-run marginal cost curve can lie above the short-run marginal cost curve.**
*When output is less than the planned output, a firm has acquired a greater quantity of fixed inputs than it needs. It therefore responds to shocks in demand by reducing usage of the variable factor by more than it would in the long run. Considering an increase in output but remaining at less than the planned level, the additional costs incurred will be less in the short run than in the long run.*

X14.19 **Suppose that Figure 14.3 is best interpreted as showing how output in the production period deviates from the plans laid earlier.**

a) **Explain how changes in demand conditions might lead to the market price increasing to $p_0$ in the short run.**
*The number of firms in the market is fixed; there is therefore no possibility of short-run entry into, or exit from, the market; and all firms have hired fixed factors of production. We shall simply assume that for some reason, market demand is higher at any price than firms anticipated when hiring the fixed factors. They respond to higher demand by increasing their use of variable factors, individually increasing supply until the market supply at the new market price is equal to the actual market demand.*

**b) Discuss how firms within the industry might respond to such a change in market conditions in the (new) short run.**
*An increase in market price leads to supernormal profits. These supernormal profits will lead existing firms to consider changing the scale of production, and firms that have not previously entered the market to review their decision.*

X14.20 **Characterize the long-run equilibrium for this market, based on the discussion above, and emphasizing the role of entrant firms.**
*We assume that it is possible for firms to enter the market, and that each firm will continue to experience the same production conditions, including factor input prices. This means that irrespective of the size of the market, each firm's supply function will take the same form, so that firms produce output so that price equals marginal cost. So, irrespective of market size, firms will be the same size, and as the market expands, then in the long run, it is simply the number of firms that increase, and not their output.*

X14.21 **Using graphical analysis, explain how the market will respond to a negative demand shock, so that the *actual market demand* is less than the *expected market demand* at any price.**
*In a diagram, with two panels set side-by-side, so that there is a common measure of price on the vertical axis, and in one, the firm supply curve, and in the other, the market supply curve. We allow for eventually diminishing returns to scale, so that there is a price (in the short-run) for which, if production falls below that level, firms will stop production; the firm supply curve therefore begins on the vertical (price) axis, and then jumps to the minimum scale of output, where the market price equals the firm's minimum average variable cost. The firm supply curve, for higher prices, is coincident with its marginal cost curve. We assume that after the fall in demand, the market demand meets the market supply curve at a price that lies above the minimum average variable cost. Then no firm leaves the market but all of the firms in the market reduce production, and make losses equal to the difference between price and average cost times their new output.*

X14.22 **In our discussion in this section, we have assumed eventually decreasing returns to scale, so that the minimum average cost occurs at output $q_0 > 0$. Suppose instead that the average cost minimizing firm output $q_0 = 0$. Discuss how this would affect the predictions of our model.**
*We have seen examples of this, such as where the total cost function is quadratic. Marginal cost is greater than average cost for all levels of output, and the so the whole of the marginal cost curve forms the supply curve.*

X14.23 **We argued in Section 13.2 that where the production function is homogeneous of degree less than 1, AC(0) > 0, and $MC(q_f) > 0$ for all values of output. Discuss the predictions of the model in this case.**
*In this case, the marginal cost curve is also the supply curve. If the market price falls below AC(0), then the market will collapse.*

X14.24 **In Section 13.2, we also argued that increasing returns to scale are inconsistent with profit maximization. Suppose that the firms in a perfectly competitive market face constant returns to scale in production. Adapt Figure 14.4 so that in the short run the market price is (1) above and (2) below the long-run average cost. Explain why in this case it is not possible to make any prediction about the size of firms in the long run.**
*With the assumption of constant returns to scale, in a diagram with output on the horizontal axis, and prices and cost measures shown on the vertical axis, both the firm supply curve and the market supply curve will be horizontal lines, and these will be coincident with the average*

*cost and marginal cost curves. If the market price were to be greater than the average cost, then irrespective of scale, firms would make profits. If the market price were to be less than average cost, then all firms would make losses. We expect that in equilibrium, production will be set so that market supply equals market demand.*

*We cannot, though, say anything about the size of firms in this market. This description would hold equally well if there were to be only one firm in the market, or if the market was to be divided among many small firms, given that all firms face the same marginal cost of production.*

*This suggests that it is not consistent with our model of perfect competition to assume constant returns to scale in production.*