

17

TAKING ACCOUNT OF RANDOMNESS

OBJECTIVES

- To understand the reasons for using simulation in solving stochastic models
- To demonstrate the technique of simulation for simple problems using the tabular method
- To develop simple models of financial systems in Excel

INTRODUCTION

The quantitative techniques that you have met so far have allowed analytical solutions to problems to be found. For example, the inventory control (EOQ) model is a simple formula that will enable you to calculate the order quantity that will minimize inventory costs (see Chapter 16). These techniques or models are called 'deterministic' because it is assumed that the variables are known precisely. However, there is another model class called 'stochastic'. A stochastic model has at least one variable that does not have a single value – it has many possible values defined by some probability distribution. A queuing system is an example of a stochastic model and even relatively simple queuing models are difficult to solve analytically. Queuing models require special software to develop and run them. However one type of stochastic system can be solved using a spreadsheet. These are systems where time is not such an important element. We usually refer to this type of simulation as Monte Carlo simulation. This chapter looks at the use of simulation to solve both types of simulation.

QUANTITATIVE METHODS IN ACTION

Merrill Lynch and the credit crisis of 2008

On 15 September 2008 the Bank of America Corporation announced that it had agreed to acquire Merrill Lynch in a \$50 billion all-stock transaction. At the time many American financial institutions were in trouble and both Fannie Mae and Freddie Mac were effectively nationalized. AIG and Morgan Stanley were also lent money while Lehman Brothers went into liquidation.

Merrill Lynch was a large financial institution and had a multibillion dollar portfolio of loans and investments with over 100 institutions. The case described below shows how Merrill Lynch used liquidity risk models to simulate the effects of different scenarios including those experienced in 2008.

Merrill Lynch in common with other investment companies provided short term loans to companies. This is called *revolving credit lines*. These loans are effectively provided by individual clients with investment accounts at Merrill Lynch. A company seeking a credit line normally only does so as a backup in case other forms of (cheaper) funding aren't available due to adverse market conditions for example. At any one time a company may have a number of credit lines that mature at different times. They can repay any credit line before the end of the term but must repay it at the end of the period unless Merrill Lynch agrees to renew it. It will base its decision on whether to renew on the credit rating of the company, the history of the company's use of

the credit line and other external factors. In general terms greater use of its credit line by a company usually signals that the credit rating of the company is deteriorating.

Merrill Lynch developed a Monte Carlo simulation model that simulated the monthly credit line usage for each company over a 5 year time period. During this time period credit ratings of companies change, credit lines expire, some get renewed and some companies will repay credit lines early. All these events are stochastic which means that they vary in some random way. There is also likely to be some correlation between companies in the same sector and this was included in the model. Another part of the model used Markov processes that modelled the way that monthly credit ratings by companies change. Finally there was an expert system that provided rules for when a company would pay off a loan early or the bank would renew or terminate expiring lines of credit.

The output from over 5000 replications of the model gave liquidity requirements each month over the 5-year time period. By using simulation in this way Merrill Lynch were able to see the effect of different scenarios. As a result of this model they increased their portfolio by 60% to over \$13 billion.

Source: T. Duffy et al., *Interfaces*, Sept/Oct 2005, Vol. 35, No. 5, 2005, pp. 353–69.

Queuing problems

There are many instances of queueing situations. Whenever you go to the supermarket or the bank you will inevitably have to join a queue to be served. Why is this? The reason is that the arrival of customers to a service facility is unpredictable. Although you may know that 30 people an hour will arrive to be served, you cannot predict *when* they will arrive. You may get 10 people in the first 5 minutes, then no one for another 10 minutes. This is just like tossing a coin 10 times; although you may *expect* 5 heads and 5 tails, you wouldn't be that surprised to get 7 heads or even 10 heads. Just like tossing a coin, the average of 30 people an hour will be achieved over a long period of time, but in the short term unpredictable results can happen. This unpredictable behaviour means that it is very difficult to avoid queues. You could increase the capacity of the service facility but you would then find that this expensive resource is lying idle much of the time. The solution to queueing problems is therefore a compromise between having excessive queues and an underutilized resource.

Some simple queueing problems can be solved analytically, but the vast majority have to be solved using a technique called simulation. The following example illustrates a typical queueing problem.

example 17.1

Passengers arriving at a suburban rail ticket office during the morning peak commuter period frequently have to wait for service. There is one clerk who issues tickets and provides an information service for passengers. The manager has received complaints regarding the time passengers spend in the queue waiting to be served, and she wishes to investigate possible methods of reducing the queueing time. Possible ideas include employing a second ticket clerk who could either share the work of the existing clerk or perhaps handle enquiries only. Another idea may be to collect fares on the train. The manager decided to collect data on arrivals and service times over a number of days, and her figures are summarised in Tables 17.1 and 17.2.

Table 17.1 Arrival time distribution

Inter-arrival time (secs)	Frequency (%)
0 to under 30	55
30 to under 60	30
60 to under 90	10
90 to under 120	5

Table 17.2 Service time distribution

Service time (secs)	Frequency (%)
20 to under 30	17
30 to under 40	28
40 to under 50	25
50 to under 60	20
60 to under 90	10

activity 17.1

Describe the components of the queueing system inherent in this example.

You probably realize that passengers must first arrive and then either join a queue or go straight to be served. Once served, passengers leave the 'system'. This can be better described by the means of a diagram.

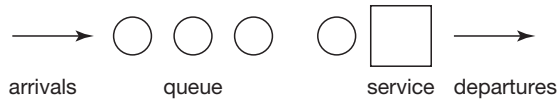


Figure 17.1 Ticket office system

activity
17.2

The manager has a few ideas for reducing the queues. Why shouldn't she just try them out and see which is the best?

This method is probably done in many cases, but it can be costly and may disrupt the entire system if you are not careful. In some cases experimenting with the real system can be dangerous. For example, trying out changes to the safety devices of a nuclear power station is not recommended! Developing a model of a system and experimenting on this is much cheaper, safer and less disruptive. Of course, the model needs to be accurate and much time is spent by analysts *validating* their model. A model can only be an approximation of the real system and the validation checks will tell you how close your model is to the real system. For instance, the model might not have allowed for the fact that passengers 'balk' from a queue; that is, do not wait to be served. If this is important it can be included and the model re-validated.

Random numbers

Although you may be sure that 30 customers turn up to be served every hour, you cannot be sure *when* during that hour they will arrive. You may get 10 people in the first minute and then no one may turn up for the next 15 minutes. In other words, there is a *randomness* in the way customers are likely to arrive. This randomness is apparent in most systems and is the reason why deterministic models are not very good at solving real problems. In order for simulation to take this randomness into account, *random numbers* are used.

True random numbers can only be generated by physical devices such as a roulette wheel which ensures that the distribution is uniform; that is, each number has an equal chance of being picked. In addition the sequence of numbers so produced is non-repeatable. However, most simulations are carried out on a computer and the random numbers in this case are generated by a formula within the computer. Although the random numbers produced are not true random numbers, they behave like true random numbers and are called *pseudo random numbers*. The random numbers included in the table in the Appendix were generated by a computer and are therefore pseudo random numbers.

The purpose of random numbers is to allow you (or the computer) to randomly select an arrival or service time from the appropriate distribution. The frequency tables given in the example represent the distribution of arrival time and service time for the ticket office. Random numbers can be arranged in any order, and for this case two-digit numbers would match the percentage format in the tables. If you look at this table you will see that 55% of inter-arrival times are in the range 0 to under 30 seconds. The random numbers 00 to 54 (or 01 to 55) could therefore be used to represent this time band.

(activity)
17.3

The next time band in Table 17.1 is 30 to under 60 seconds. What random numbers would you use to represent this band? Repeat this procedure for the last two bands.

Since the frequency is 30%, the random numbers should be 55 to 84 (or 56 to 85) inclusive. For the last two bands the random numbers should be 85 to 94 (86 to 95) and 95 to 99 (96 to 00) inclusive.

If a computer package were used to simulate this system, then a routine within the program would generate a random number and then obtain the appropriate inter-arrival time by interpolation. For example, if the random number 15 was generated then the inter-arrival time would be 8.2 seconds $\left(\frac{15}{55} \text{ of } 30\right)$. However, when manually carrying out a simulation it is much easier to represent each time band by its mid point, so any random number between 00 to 54 would correspond to an inter-arrival time of 15 seconds. This can be repeated for all bands, and to do this you may find it easier to write down the cumulative frequencies, as in Table 17.3.

Table 17.3 Allocation of random numbers

Inter-arrival time mid point	Frequency (%)	Cumulative frequency	Random numbers
15	55	55	00–54
45	30	85	55–84
75	10	95	85–94
105	5	100	95–99

(activity)
17.4

Apply this procedure to the service time distribution (Table 17.2).

Again, a particular service time would be represented by a range of random numbers. For example, a service time of 55 seconds would be represented by the random numbers 70 to 89.

Table 17.4 Solution to Activity 17.4

Service time mid point	Frequency (%)	Cumulative frequency	Random numbers
25	17	17	00–16
35	28	45	17–44
45	25	70	45–69
55	20	90	70–89
75	10	100	90–99

Tabular simulation

The easiest method of demonstrating simulation is by manually simulating a simple system. In order to carry out the simulation of the ticket office manually you would need to obtain a stream of random numbers. These can conveniently be obtained from tables (see Appendix). These numbers would then be used to sample from the arrival and service time distributions. For example, suppose that the first few random

numbers are 08, 72, 87 and 46. The random number 08 represents an inter-arrival time of 15 seconds, so the first passenger arrives at a *clock* time 15. There is no queue, so this passenger can be served immediately. The random number 72 corresponds to a service time of 55 seconds, so this service will end at a clock time of 70 (15 + 55). The next customer arrives 75 seconds after the first, so the clock time of 90 is after the end of the last service. The second passenger can also be served immediately. The service time is 45 seconds, so the passenger departs at 135. This information is best displayed in *tabular* format similar to Table 17.5. The last column of this table allows the waiting time of each passenger to be recorded.

Table 17.5 Tabular format

Arrivals			Service				Waiting time
RNo	Inter-arr.	Clock time	RNo	Time	Starts	Ends	
08	15	15	72	55	15	70	0
87	75	90	46	45	90	135	0

activity
17.5

Continue the tabular simulation using the following random numbers:
15, 96, 04, 00, 52, 27, 46, 73, 95, 76, 10, 25, 02, 11

As you can see in Table 17.6, the calculations are not difficult, but they are tedious and time consuming.

Table 17.6 Simulation of the ticket office system

Arrivals			Service				Waiting time
RNo	Inter-arr.	Clock time	RNo	Time	Starts	Ends	
08	15	15	72	55	15	70	0
87	75	90	46	45	90	135	0
15	15	105	96	75	135	210	30
04	15	120	00	25	210	235	90
52	15	135	27	35	235	270	100
46	15	150	73	55	270	325	120
95	105	255	76	55	325	380	70
10	15	270	25	35	380	415	110
02	15	285	11	25	415	440	130

activity
17.6

What is the mean waiting time for the simulation above?

The mean waiting time of the first 9 passenger is 72.2 seconds, but it appears that the waiting time is increasing as the simulation proceeds.

Use of computer simulation software

It is not practical to simulate systems manually so computer software is used. At one time they were purpose-built using specialist programming languages, but these days

most simulation packages consist of a graphical user interface (GUI) which allows you to model parts of the system using graphics, a random number generator which generates random numbers to be used to sample from probability distributions, house-keeping routines that stores information on the life cycle of elements of the system and an output generator that produces statistical and other outputs. A popular package is called Simul8 and this package allows users (even those with little knowledge of simulation) to build quite complex systems easily. The model shown in Figure 17.1 was built using Simul8 (and took only a couple of minutes!). This model was simulated for 8 hours and the output obtained is shown in Figure 17.2. Notice that the average queuing time is 81.7 seconds which is not that different from the 72.7 seconds found manually.

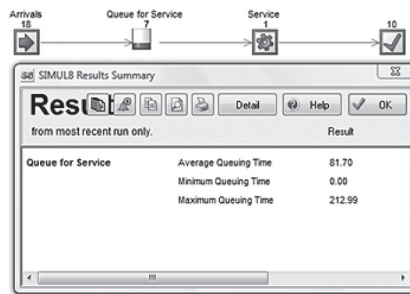


Figure 17.2 The ticket office simulation using Simul8

Monte Carlo simulation

For queuing type systems it is necessary to use specialist simulation software but some systems can be simulated using a spreadsheet. These are systems where time does not have to be modelled in such a precise way and to not involve queuing processes. This type of simulation is often called Monte Carlo simulation to differentiate it from the queuing simulation that we have already met. An example of a system that can be simulated using a spreadsheet is a financial system. Cash flows generated by an investment are likely to be uncertain, particularly when these cash flows occur at some time in the future. The uncertainty in a future cash flow can be represented by a probability distribution and, as in queuing simulation, we can sample from this distribution using random numbers. There is a random number function within Excel, and together with other in built functions it is easy to build a simple simulation model. Example 17.2 illustrates a simplified financial system.

example 17.2

BFC Bank takes in deposits which it uses to make loans to businesses. The pattern of monthly deposits follows a normal distribution with a mean of £20m and a standard deviation of £6.8m. The distribution of monthly loans is again normal with a mean of £8m and a standard deviation of £1.2m. Each month 5% of outstanding loans are repaid and 7% of deposits mature and have to be repaid. It pays 0.5% per month interest on deposits and charges 1% per month on loans. BFC Bank attempts to ensure the total loans do not exceed 50% of total capital from deposits. The problem is that, apart

from the random nature of deposits and loans, there is also a danger that a company will be in default of its loan and in this case the loan is written off. The probability that a company will default on its loan is currently 2% but when this occurs the average amount in default is £20m. This is taken off the available capital at the end of the month.

An Excel model of the process described in Example 17.2 should generate deposits and loans randomly. The Excel function `=Rand()` generates a random number between 0 and 1. However the problem with using this function is that every time Excel does another calculation a new random number will be generated. It is possible to turn off automatic calculation under Calculation Options in the Formulas tab. You then must remember to do the calculation manually when required. However a better method is to generate a large quantity of random numbers in a separate sheet using `Rand()` and then copy and Paste Special/Values only back into this sheet or to another sheet. Figure 17.3 illustrates this method. Each column of random numbers will represent a *stream* of random numbers and you can use values from these streams in the simulation model. The advantage of this method is that you can use the same random number streams when trying out different scenarios (for example, different probability of loan defaults). You are then able to make a direct comparison between scenarios and not have to worry about the effect that different random numbers have had on the results.

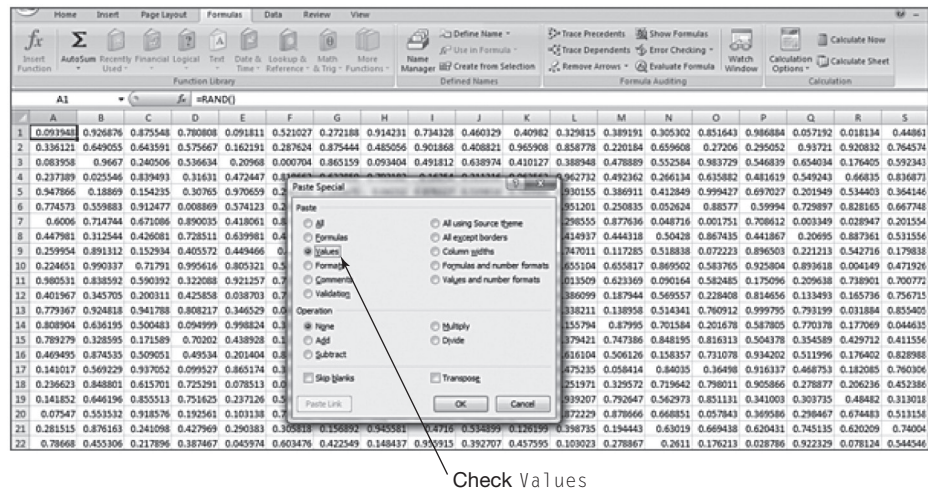


Figure 17.3 Generation of random numbers

From Example 17.2 we know that deposits and loans are governed by the normal distribution. To generate a value from the normal distribution in Excel you use the function `Norminv`. This function is illustrated in Figure 17.4 where a sample deposit is generated. A sample loan can also be generated in the same way.

A random number is also used for the loan default. If this random number is less than or equal to 2% a £20m default is generated. The Excel command for this situation is `=IF('Random numbers'!C1>.02,0,20)`. This uses a random number from stream C1 and if this value is greater than 0.02 then the loan default is zero but if not

the loan default is 20. On average there should be two occasions in 100 months when a company defaults.

When developing any Excel model it is often useful to have an input section where parameter values are set. The Excel functions then use these cells as absolute references instead of actual values. The Excel sheet showing the basic Excel inputs and column headings is given in Figure 17.5.

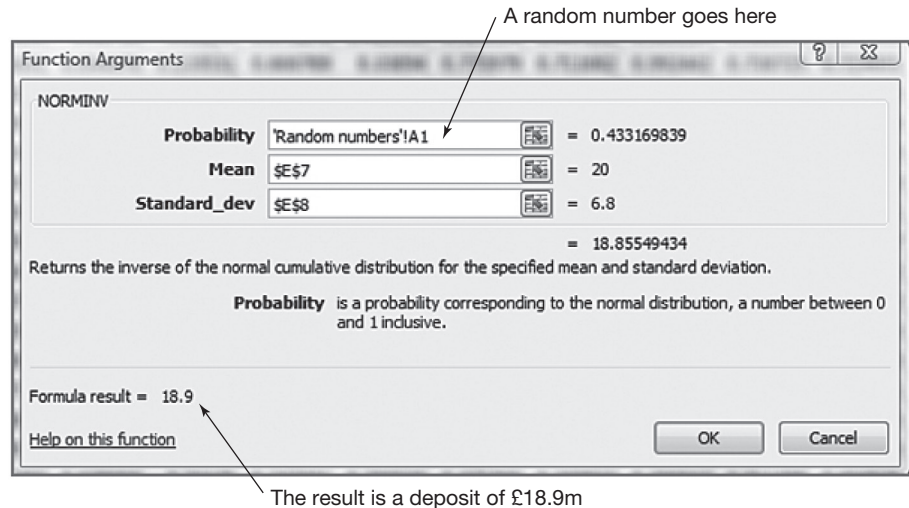


Figure 17.4 Generating a value from the normal distribution

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		SIMULATION MODEL OF BFC BANK											
3													
4		Monthly Interest on deposits			0.5%		% of deposits maturing		7%				
5		Monthly Interest on loans			1%		Prob of loans default		2%				
6		% of loans repaid each month			5%								
7		mean deposit			20								
8		SD of deposit			6.8								
9		Mean loan			8								
10		SD of loan			1.2								
11													
		Deposit made this month	Total deposits held at start of month	Interest paid on deposits (outgoings)	Loans made this month	Total loans outstanding at start of month	Interest on loans (cash inflow)	Deposits maturing at end of month	Loans repaid (cash inflow)	Loans default (cash outflow)	Capital available at end of month	Total on loan at end of month	Ratio of loans to capital
12	Month	month	of month	(outgoings)	this month	at start of month	(cash inflow)	at end of month	(cash inflow)	(cash outflow)	at end of month	at end of month	loans to capital

Figure 17.5 The Excel inputs and column headings

activity
17.7

What formulae are required for:

1. Deposits held at start of month?
2. Interest paid on deposits?
3. Total loans outstanding at start of month?
4. Deposits maturing at end of month?
5. Loans repaid?
6. Capital available at end of month?
7. Total on loan at end of month?
8. Ratio of loans to capital?

The answers to Activity 17.7 are given in Table 17.7.

Table 17.7 Answers to Activity 17.7

Formula	Explanation	Example
1	deposit held at the end of the previous month plus the new deposits this month	=K13+B14
2	0.5% * total deposits held	=\$E\$4*C14
3	loans outstanding from the previous month plus new loans this month	=L13+E14
4	7% * total deposits	=C14*\$I\$4
5	5% * total loans outstanding	=\$E\$6*F14
6	total capital at start of month + interest on loans – interest paid on deposits + loans repaid this month – loans default – loans made this month – deposits maturing this month	=C14+G14 - D14+I14 - J14 - E14 - H14
7	total loans at start of month – loans repaid this month	=F14 - I14
8	Total on loan/total capital	=L14/K14

Month	Deposit made this month	Total deposits held at start of month	Interest paid on deposits (outgoings)	Loans made this month	Total loans outstanding at start of month	Interest on loans (cash inflow)	Deposits maturing at end of month	Loans repaid (cash inflow)	Loans default (cash outflow)	Capital available at end of month	Total on loan at end of month	Ratio of loans to capital
0										200	100	50.0%
1	18.9	218.9	1.1	7.8	107.8	1.1	15.3	5.4	0.0	201.1	102.4	50.9%
2	17.9	219.0	1.1	7.1	109.5	1.1	15.3	5.5	20.0	182.0	104.0	57.2%
3	23.2	205.2	1.0	6.3	110.3	1.1	14.4	5.5	0.0	190.1	104.8	55.1%
4	24.9	215.0	1.1	8.4	113.2	1.1	15.1	5.7	0.0	197.3	107.5	54.5%
5	15.6	212.9	1.1	9.8	117.4	1.2	14.9	5.9	0.0	194.1	111.5	57.4%
6	24.2	218.3	1.1	9.5	121.0	1.2	15.3	6.1	0.0	199.7	115.0	57.6%
7	14.2	213.9	1.1	9.4	124.4	1.2	15.0	6.2	0.0	195.9	118.1	60.3%
8	26.4	222.4	1.1	7.9	126.1	1.3	15.6	6.3	0.0	205.3	119.8	58.3%
9	16.9	222.2	1.1	9.1	128.9	1.3	15.6	6.4	0.0	204.1	122.4	60.0%
10	21.5	225.6	1.1	7.4	129.9	1.3	15.8	6.5	0.0	209.1	123.4	59.0%
11	13.5	222.6	1.1	9.1	132.5	1.3	15.6	6.6	0.0	204.7	125.9	61.5%
12	20.0	224.7	1.1	8.5	134.4	1.3	15.7	6.7	0.0	207.4	127.7	61.5%

Figure 17.6 Twelve months' simulation of BFC Bank

To run this simulation you need to copy the formulae down the number of rows corresponding to the number of months you wish to run the simulation for. So if you want to run the simulation for 12 months copy the formulae down from cells 14 to 25. However before you do this you need to start the simulation with a fixed amount of deposits and loans. In the spreadsheet shown in Figure 17.6 values of £200m and £100m were used. You can download this spreadsheet from the website but if you have developed your own spreadsheet your answers will be different to the ones shown in Figure 17.6 as the random numbers will be different.

(activity 17.8)

Summarize the results from the simulation shown in Figure 17.6

The deposits varied from £13.5m to £26.4m while the loans varied from £6.3m to £9.8m. The capital available from deposits has increased by £7.4m during the month

while loans have increased by £27.7m. Although the ratio of loans to capital started off at 50% the loan default that occurred in month 2 has helped this ratio rise to 57%. This ratio has stayed above 50% for the rest of the 12 months.

(activity)
17.9

Try repeating the simulation using different stream (columns) of random numbers. (You will need to edit the formulae in row 14 and then copy the formulae down to row 25.)

The simulation using this set of numbers is shown in Figure 17.7. From this you will see that the results are slightly different to the first run. In particular there was no default in the first 12 months so the ratio of loans to capital is lower than in the first run. However the ratio is still above 50% for the 12 months.

We normally have to make many different runs of a simulation as the results obtained are affected by the random numbers used. In fact we may need many hundreds if not thousands of runs to get a statistically valid result. This would be difficult to do manually and we can either set up a macro to do this automatically or use a spreadsheet add-on such as @Risk.

12	Month	Deposit made this month	Total deposits held at start of month	Interest paid on deposits (outgoings)	Loans made this month	Total loans outstanding at start of month	Interest on loans (cash inflow)	Deposits maturing at end of month	Loans repaid (cash inflow)	Loans default (cash outflow)	Capital available at end of month	Total on loan at end of month	Ratio of loans to capital
13	0										200	100	50.0%
14	1	19.2	219.2	1.1	6.8	106.8	1.1	15.3	5.3	0.0	202.3	101.5	50.2%
15	2	15.9	218.2	1.1	6.2	107.7	1.1	15.3	5.4	0.0	202.1	102.3	50.6%
16	3	9.6	211.7	1.1	6.3	108.6	1.1	14.8	5.4	0.0	196.1	103.1	52.6%
17	4	15.2	211.3	1.1	8.5	111.6	1.1	14.8	5.6	0.0	193.6	106.1	54.8%
18	5	27.2	220.8	1.1	9.1	115.2	1.2	15.5	5.8	0.0	202.0	109.4	54.2%
19	6	21.1	223.1	1.1	8.0	117.4	1.2	15.6	5.9	0.0	205.4	111.5	54.3%
20	7	12.9	218.3	1.1	7.5	119.0	1.2	15.3	6.0	0.0	201.6	113.1	56.1%
21	8	15.0	216.6	1.1	7.7	120.8	1.2	15.2	6.0	0.0	199.9	114.7	57.4%
22	9	19.7	219.6	1.1	9.2	123.9	1.2	15.4	6.2	0.0	201.3	117.7	58.5%
23	10	29.3	230.6	1.2	7.8	125.5	1.3	16.1	6.3	0.0	213.1	119.2	55.9%
24	11	14.1	227.2	1.1	10.2	129.4	1.3	15.9	6.5	0.0	207.7	123.0	59.2%
25	12	21.7	229.4	1.1	7.7	130.7	1.3	16.1	6.5	0.0	212.3	124.1	58.5%

Figure 17.7 Simulation of BFC using a different stream of numbers

Uses of simulation

Both queuing and Monte Carlo simulation can be used to try out *what-if* experiments. These are ideas you have for changing the real system. In Example 17.1 we discussed possible ideas for changing the way that the ticket office is run in order to reduce queuing time. In Example 17.2 we may want to see the effect on the ratio of loans to capital if the default rate changed. Or we might want to see the effect of a change in the monthly mean deposit or loan rate.

(activity)
17.10

You have decided to halve the mean loan rate from £8m to £4m a month. What effect does this have on the ratio of loans to capital?

If you used the spreadsheet provided on the website you should have found that the ratio has been reduced and is consistently below 50% for the whole 12 months (see

Figure 17.8). If you had used your own spreadsheet you would have got a slightly different result for the reasons given after Activity 17.10.

	A	B	C	D	E	F	G	H	I	J	K	L	M
6		% of loans repaid each month				5%							
7		mean deposit				20							
8		SD of deposit				6.8							
9		Mean loan				4		% of deposits maturing		7%			
10		SD of loan				1.2		Prob of loans default		2%			
11													
12	Month	Deposit made this month	Total deposits held at start of month	Interest paid on deposits (outgoings)	Loans made this month	Total loans outstanding at start of month	Interest on loans received (cash inflow)	Deposits maturing at end of month	Loans repaid (cash inflow)	Loans default (cash outflow)	Capital available at end of month	Total on loan at end of month	Ratio of loans to capital
13	0										200	100	50.0%
14	1	18.9	218.9	1.1	3.8	103.8	1.0	15.3	5.2	0.0	204.9	98.6	48.1%
15	2	17.9	222.8	1.1	3.1	101.7	1.0	15.6	5.1	20.0	189.0	96.6	51.1%
16	3	23.2	212.2	1.1	2.3	98.9	1.0	14.9	4.9	0.0	199.9	94.0	47.0%
17	4	24.9	224.8	1.1	4.4	98.4	1.0	15.7	4.9	0.0	209.5	93.4	44.6%
18	5	15.6	225.0	1.1	5.8	99.3	1.0	15.8	5.0	0.0	208.3	94.3	45.3%
19	6	24.2	232.5	1.2	5.5	99.8	1.0	16.3	5.0	0.0	215.5	94.8	44.0%
20	7	14.2	229.7	1.1	5.4	100.2	1.0	16.1	5.0	0.0	213.1	95.2	44.7%
21	8	26.4	239.6	1.2	3.9	99.1	1.0	16.8	5.0	0.0	223.6	94.2	42.1%
22	9	16.9	240.5	1.2	5.1	99.3	1.0	16.8	5.0	0.0	223.3	94.3	42.2%
23	10	21.5	244.8	1.2	3.4	97.8	1.0	17.1	4.9	0.0	228.9	92.9	40.6%
24	11	13.5	242.4	1.2	5.1	98.0	1.0	17.0	4.9	0.0	225.0	93.1	41.4%
25	12	20.0	245.0	1.2	4.5	97.6	1.0	17.1	4.9	0.0	227.9	92.7	40.7%

Figure 17.8 Halving the mean loan rate

As you may be aware, developing and running simulations is not a straightforward affair. When using random numbers there will be many variations which need to be considered. This is particularly important when comparing what-if experiments. Many runs should be made and statistical analysis (for example, paired t-tests) used to arrive at sound conclusions. You also need to consider *start-up* conditions. We started the Monte Carlo simulation with £200m of capital and £100m of loans but what would have happened if we had used different amounts? In simulation terminology we talk about *warm-up* and *steady state* to remove these start-up conditions. For those of you interested in learning more about simulation see Robinson (2004).

Reflection

After statistics, simulation is the next most common quantitative methods technique used in business and industry.

One of the reasons for this is that modern simulation software is cheap and very easy to use. However, its ease of use also makes simulation one of the techniques that is most easily misused. Part of the problem is that simulation is essentially a sampling device, and knowledge of statistics is required to fully interpret the output from a simulation model. In the wrong hands simulation can provide at best worthless results and at worst errors of modelling or of interpretation which may create misleading conclusions. Simulation can also be a very time consuming technique, not only in the model development stage, but also in the running of the model.

However simulation is a very powerful modelling tool and can save a company millions of pounds by enabling it to test ideas out on a model before trying them for real. There are some situations where testing cannot be done on the real system either because the system doesn't yet exist or because it would be too dangerous to experiment on it. One example of this latter category is in the use of simulation in the transmission and control of MRSA.

In the *Quantitative methods in action* we looked at the use of simulation in the financial sector. Merrill Lynch used simulation to model worse case scenarios although even they probably didn't foresee how bad the situation would get. Fortunately this company survived albeit by becoming part of a larger bank.

Key points

- *Simulation* is used to solve complex systems where it would be difficult to apply analytical techniques.
- Simulation takes into account the *variability* that occurs in everyday life.
- To produce statistical results, it is necessary to make several runs of a *simulation model*.
- *Queuing simulation* requires special software, but *Monte Carlo simulation* can be performed using a spreadsheet.

Further reading

Robinson, S. (2004) *Simulation: The Practice of Model Development and Use*, Wiley.

<http://www.palgrave.com/oakshott-eqm6>

Solutions to all questions are freely available –

Practice questions

- 1 Customers arrive at a single cash dispenser with the inter-arrival time distribution shown in Table 17.8.
The service time is 45 seconds.
Using the random numbers below, manually simulate the system and find the average time spent waiting for service and the utilisation of the cash dispenser.
05, 20, 30, 85, 22, 21, 04, 67, 00, 03

Table 17.8 Inter-arrival time distribution for Question 1

Inter-arrival time (secs)	Frequency (%)
20 to under 50	5
50 to under 100	20
100 to under 150	30
150 to under 200	45

- 2 Customers arrive at a bank, which has only a single cashier, with the inter-arrival time and service time distributions shown in Table 17.9.
Using the random numbers given below, simulate the next 6 arrivals and find the mean time that they spend queueing for the cashier.
04, 10, 59, 07, 38, 98, 01, 75, 48, 91, 04, 12

Table 17.9 Arrival and service time distributions for Question 2

Inter-arrival time (mins)	% of customers	Service time (mins)	% of customers
0 to under 4	30	0 to under 1	0
4 to under 6	40	1 to under 3	50
6 to under 8	20	3 to under 5	40
8 to under 10	10	5 to under 7	10

- 3 Ajax Food Products has its main factory in the centre of Bristol. Lorries arrive at a constant rate from 08.00 to 18.00 five days a week, where they are either loaded or

unloaded using the single loading/unloading bay and on a first-come, first-served basis. The area around the factory is frequently congested with lorries because the loading/unloading depot is not large enough for all arriving lorries to wait. A suggestion has been made that an improvement in numbers queueing might result if priority was given to lorries that required unloading. This is because unloading is generally a faster operation. However, before any decision is made it has been decided to build a simulation model of the current system.

- (a) Briefly discuss the advantages and disadvantages of simulation as a means of experimenting on this system.
- (b) Using the random numbers 42, 17, 38 and 61, demonstrate how 4 unloading times could be generated from the frequency distribution given in Table 17.10.

Table 17.10 Unloading times for Question 3

Time (mins)	Frequency (%)
0 to under 30	20
30 to under 40	35
40 to under 50	22
50 to under 60	15
60 to under 70	8

- 4 Andrew Giles, the Transport Manager at Bristol Tyres, has just returned from a meeting with the Managing Director. Apparently the police have received complaints from local residents about the parking of heavy lorries in the side streets near the factory. This is occurring because there is insufficient room in the depot for lorries to wait to be loaded/unloaded.

To reduce the congestion it has been suggested a second bay be built. However, before this is done a simulation of the current system is to be developed and you have been asked to take on this project.

Your first task was a data collection exercise, and this gave you the following information:

- (i) The depot is open from 0800 to 1800, Mondays to Fridays.
- (ii) Vehicles either require loading or unloading (not both).
- (iii) 70% of lorries require loading and 30% require unloading.
- (iv) The frequency distributions of the loading/unloading operations found by timing a large number of lorries were as shown in Table 17.11.

Table 17.11 Frequency distributions for Question 4

Time (mins)	Loading frequency (%)	Unloading frequency (%)
0 to under 30	20	30
30 to under 40	35	40
40 to under 50	22	25
50 to under 60	15	4
60 to under 70	8	1

The frequency distribution of the inter-arrival time – that is, the time between successive arrivals – was as shown in Table 17.12.

Table 17.12 Inter-arrival time distribution

Time (mins)	Frequency (%)
0 to under 10	15
10 to under 20	40
20 to under 30	30
30 to under 40	5
40 to under 50	5
50 to under 60	3
60 to under 70	2

You have also made the following assumptions:

- (i) The pattern of arrivals is constant throughout the day.
- (ii) A second bay would be used like the first; that is, for loading and unloading.
- (iii) A single queue of lorries would form and a lorry could use either bay on a first-come, first-served basis.
- (iv) Any lorries in the queue at the end of the day would be loaded or unloaded.
 - (a) Explain why simulation is a better method than experimenting on the real system for this problem.
 - (b) How would you go about simulating the system? In your answer you should discuss the run length of the simulation and the number of runs required.
 - (c) Demonstrate the technique of simulation by using the tabular method to simulate 3 hours (180 minutes) of depot operation. What is the average waiting time of the lorries?
Random numbers:
20, 17, 42, 96, 23, 17, 28, 66, 38, 59, 38, 61,
73, 76, 80, 00, 20, 56, 10, 05, 87, 88, 78, 15
 - (d) Repeat the simulation using 2 unloading bays.

Assignment

Either use your Monte Carlo model developed for Example 17.2 or the one on the website to test out the following ideas.

- (a) What effect will an increase in default rate from 2% to 5% have?
- (b) Try changing the 'start-up' capital from £200m to £500m. What effect does this have on the results?
- (c) Try running the simulation for 60 months (5 years).
- (d) Run the simulation for 10 years and compare the first and last 5 years. Are there any differences between the two?
- (e) Use several different streams of random numbers to see how different the results can be.
- (f) Try out other ideas you might have.