## Part II Bonds and Fixed Income

## BOND STRATEGIES

1. 



Money market yield on T-Bill.

$$
\begin{aligned}
\text { Price of T-bill } & =100-\left[5.75 \times \frac{273}{360}\right] \\
& =95.639583 \\
\text { Money Market Yield } & =\left[\frac{100}{95.639583}-1\right] \times \frac{360}{273} \\
& =6.012155 \%
\end{aligned}
$$

Terminal Value of T-Bond $=4\left[1+R \% x \frac{182}{360}\right]+104$
Price $=101.625 \quad$ Accrued $=2.0109$
Dirty Price $=103.6359$
Hence to breakeven with T-Bill

$$
\begin{aligned}
& (103.6359)\left[1+0.06012155 X \frac{273}{360}\right]=4\left[1+R \% \times \frac{182}{360}\right]+104 \\
& \mathrm{R}=\underline{17.85 \%}
\end{aligned}
$$

2. First we calculate the zero yields

6-month zero $100.50=\frac{103}{1+\mathrm{R}_{1}}$
$\mathrm{R}_{1}=2.4876 \%$
12-month zero $105.28125=\frac{5.5}{1.024876}+\frac{105.5}{\left(1+\mathrm{R}_{2}\right)}$
$\mathrm{R}_{2}=\quad=\quad 2.7570 \%$
Coupon yield is calculated as
$100=\frac{\mathrm{C}}{1.024876}+\frac{(100+\mathrm{C})}{(1.02757)^{2}}$
C $=2.7533 \% \times 2$
Annual Coupon $=\underline{5.5067 \%}=5.51 \%$
Answer = (c)
3. Firstly we must calculate the price of the Treasury Bond

Price $=\frac{2.5}{1.03}+\frac{2.5}{(1.03)^{2}}+\frac{2.5}{(1.03)^{3}}+\frac{102.5}{(1.03)^{4}}=98.142$
Secondly, we calculate the duration of the bond:
Duration $=\frac{\frac{2.5}{1.03}+\frac{2.5(2)}{(1.03)^{2}}+\frac{2.5(3)}{(1.03)^{3}}+\frac{102.5(4)}{(1.03)^{4}}}{98.141}=3.854$ Half Years
Thirdly, we use the duration number to calculate the expected price change for a 100 basis points change in yield:

Expected Change $=\frac{98.141 \times 3.854 \times 0.0050}{1.03}=1.836$
Finally we re-calculate the exact price of the bond using a yield of 7.00\%
Price $\quad=\quad \frac{2.5}{1.035}+\frac{2.5}{(1.035)^{2}}+\frac{2.5}{(1.035)^{3}}+\frac{102.5}{(1.035)^{4}}=96.327$

Actual Price Change is: $98.141-96.327=1.814$

Answer = (a)
4. The Hedge Ratio is obtained from:

$$
\operatorname{HR}=\frac{\underline{\mathrm{P}}_{1}}{\mathrm{P}_{2}} \quad \mathrm{x} \quad \underline{\mathrm{D}}_{1} \quad \mathrm{x} \quad \frac{\left(1+\mathrm{Y}_{2}\right)}{\mathrm{D}_{2}} \quad \mathrm{x} \quad \frac{\text { Change in } \mathrm{Y}_{1}}{\left(1+\mathrm{Y}_{1}\right)} \quad \mathrm{x} \quad \mathrm{Ch}^{1}
$$

Assuming equal absolute changes in yield on both bonds we have:

$$
\mathrm{HR}=\frac{\underline{98.00} \mathrm{x}}{105.00} \quad \underline{5.0} \quad \underset{4.0}{\mathrm{x}} \quad \underline{1.065} 1.064=1.1678
$$

You would therefore require $\$ 10,000,000(1.1678)$ or $\$ 11,678,000$ nominal of the second bond as a hedge.

Answer = (d)
5. Using the HP17B Bond program gives a yield to maturity for a 10-year TBond maturing on March 222013 with settlement on March 222003 as 7.5038\%.

The true annual yield is therefore:

$$
\begin{array}{ll}
(1+\mathrm{R}) & =\left(1+\frac{0.075038}{2}\right)^{2} \\
\mathrm{R} & =7.6446 \%
\end{array}
$$

The price of the zero must be:

$$
\begin{array}{ll}
\mathrm{P} & =\frac{100}{(1.07446)^{10}} \\
\mathrm{P} & =47.87
\end{array}
$$

$\underline{\text { Answer }=(\mathrm{d})}$
6. First we work out the price changes per 100 nominal using the usual equation:

Price x Duration $\mathrm{x} \frac{\text { Basis Point }}{1+\text { Yield }}$
Bond $102 \times 6 \times \frac{0.0001}{1.0675}=0.05733$
Hedge $95 \times 7 \times \frac{0.0001}{1.068}=0.06227$
Hence hedge amount will be

$$
\$ 10,000,000 \times \frac{0.05733}{0.06227}=\$ 9,206,681
$$

or $\$ 9,207,000$ rounded

## Answer is (e)

7. First we must calculate the zero coupon interest rates by bootstrapping.

$$
\begin{aligned}
100 & =\frac{108}{1+\mathrm{R}_{1}} \quad \mathrm{R}_{1}=8 \% \\
98.30 & =\frac{7.5}{1.08}+\frac{107.5}{\left(1+\mathrm{R}_{2}\right)^{2}} \quad \mathrm{R}_{2} \quad=\quad 8.48 \% \\
97.00 & =\frac{7.25}{1.08}+\frac{7.25}{(1.0848)^{2}}+\frac{107.25}{\left(1+\mathrm{R}_{3}\right)^{3}} \quad \mathrm{R}_{3}=8.43 \%
\end{aligned}
$$

Now we use these zero rates to calculate the fair value of the $9 \% 3$ year bond

$$
\begin{aligned}
\text { PRICE } & =\frac{9}{(1.08)}+\frac{9}{(1.0848)^{2}}+\frac{109}{(1.0843)^{3}} \\
& =\underline{101.48}
\end{aligned}
$$

## Answer is (c)

8. Step 1 Work out yield to maturity on bond with HP17B

| Type | $30 / 360$ Annual |
| :--- | :--- |
| Settlement | $30 / 9 / 00$ |
| Maturity | $30 / 9 / 04$ |


| Coupon | $6 \%$ |  |
| :--- | :--- | :--- |
| Call | 100 |  |
| Price | 98 |  |
|  |  |  |
| Gives yield | $=$ | $6.585 \%$ |

Step 2 Calculate bond's duration

$$
\begin{aligned}
\mathrm{D}= & \frac{\frac{6}{1.06585}+\frac{6(2)}{(1.06585)^{2}}+\frac{6(3)}{(1.06585)^{3}}+\frac{106(4)}{(1.06585)^{4}}}{98} \\
& =3.6693
\end{aligned}
$$

Step 3 Calculate duration implied price change for yield reduction of 1\%

$$
\begin{aligned}
\Delta \mathrm{P} & =-3.6693 \times 98 \times\left(\frac{-0.01}{1.06585}\right) \\
& =3.3738
\end{aligned}
$$

Step 4 Calculate actual price change for yield reduction of $1 \%$ using HP17B

| Yield $=5.5$ | 5.585\% |  |
| :---: | :---: | :---: |
| Gives |  |  |
| Price $=10$ | 101.4517 |  |
| PRICE CHANGE | = | 101.4517-98 |
|  | = | 3.4517 |


| Hence DURATION CHANGE | $=$ | 3.3738 |
| :---: | :--- | :--- |
| CONVEXITY CHANGE | $=$ | 0.0779 |
| TOTAL CHANGE | $=$ | 3.4517 |

## Answer is (b)

9. First we must calculate the zero coupon interest rates by bootstrapping.
$100=\frac{108}{1+\mathrm{R}_{1}} \quad \mathrm{R}_{1} \quad=\quad 8 \%$

$$
\begin{aligned}
& 98.30=\frac{7.5}{1.08}+\frac{107.5}{\left(1+\mathrm{R}_{2}\right)^{2}} \quad \mathrm{R}_{2} \quad=8.4768 \% \\
& 97.00=\frac{7.25}{1.08}+\frac{7.25}{(1.084768)^{2}}+\frac{107.25}{\left(1+\mathrm{R}_{3}\right)^{3}} \quad \mathrm{R}_{3}=8.4316 \%
\end{aligned}
$$

Now we use these zero rates to calculate the fair value of the $9 \% 3$ year bond

$$
\begin{aligned}
\text { PRICE } & =\frac{9}{(1.08)}+\frac{9}{(1.084768)^{2}}+\frac{109}{(1.084316)^{3}} \\
& =\underline{\underline{101.48}}
\end{aligned}
$$

## Answer is (c)

## 10. Answer is (a)

11. Assuming even yield, etc., as above, the bond with the call disadvantages the investor compared to the bond with no embedded options, while those with the puts give the investor an advantage compared to the bond with no embedded options.

## Answer is (b)

12. The first step is clearly to calculate the zero coupon interest rates by bootstrapping. To keep things simple, we assume equal 6 -month periods.
$100.50=\frac{100+3.5}{1+R_{1 / 2}} \quad R_{1}=\underline{5.97 \%}$
$99.53125=\frac{3}{1+\left(\frac{0.0597}{2}\right)}+\frac{(100+3)}{\left(1+R_{2 / 2}\right)^{2}}$

$$
R_{2}=\underline{6.50 \%}
$$

$104.1875=$

$$
\frac{5}{1+\left(\frac{0.0597}{2}\right)}+\frac{5}{\left[1+\left(\frac{0.065}{2}\right)\right]^{2}}+\frac{(100+5)}{\left(1+\frac{R_{3}}{2}\right)^{3}}
$$

$$
R_{3}=\underline{7.045 \%}
$$

Then we can calculate the price of an eighteen month zero coupon bond by:

$$
\mathrm{P}=\frac{100}{\left[1+\left(\frac{0.07045}{2}\right)\right]^{3}}=90.1354=90-04
$$

## Answer is (b)

13. First we need to calculate the market price of the bond. Using a bond calculator with a 30/360 annual basis, I get a price of 101.7526.

We then calculate the PV of the cash flows

$$
\begin{aligned}
& \frac{6}{1.055}=5.6872 \\
& \frac{6}{(1.055)^{2}}=5.3907 \\
& \frac{6}{(1.055)^{3}}=5.1097 \\
& \frac{106}{(1.055)^{4}}=85.5650
\end{aligned}
$$

Now we calculate the duration from

$$
\begin{aligned}
D & =\frac{5.6872(1)+5.3907(2)+5.1097(3)+85.5650(4)}{101.7526} \\
& =\underline{\underline{3.676}} \text { years }
\end{aligned}
$$

Answer is (d)

