Understanding Structures, 5th edition

Derek Seward

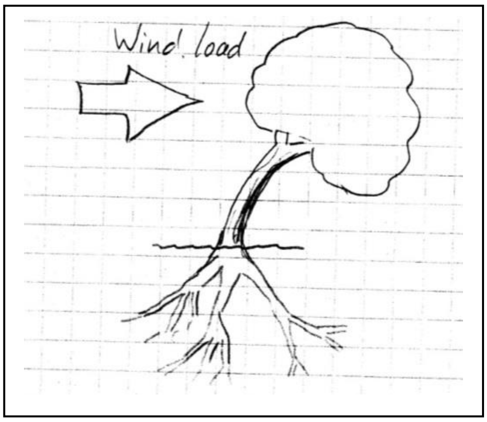
Solutions to end-of-chapter exercises

Contents

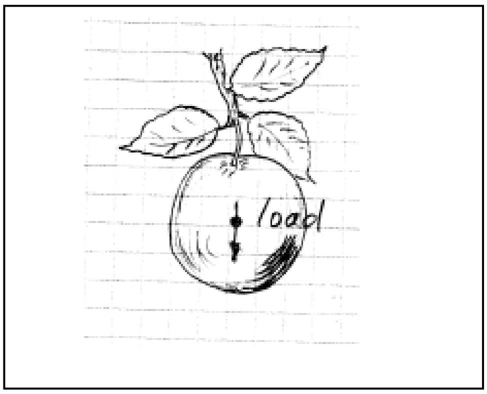
Chapter 1 – Design

E1.1

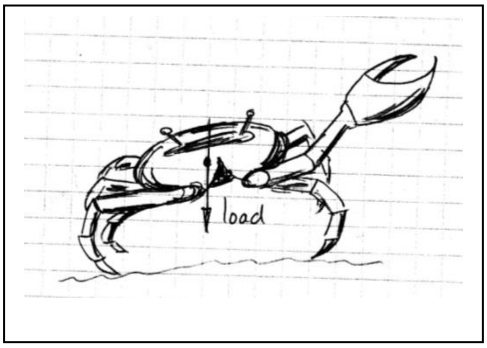
a. A tree trunk acts as a vertical cantilever beam to resist wind force. Roots transfer the load into the ground



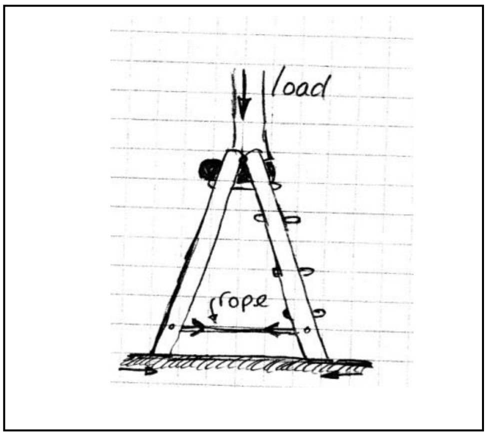
b. An apple stalk is a tension member transferring the weight of the apple back to the tree. Vines and trailing plants are also tension structures



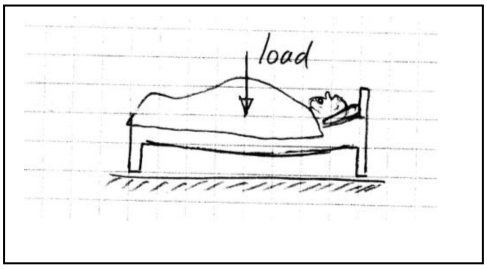
c. Creatures such as crabs have exo-skeletons. This is a "shell" structure which provides the strength and shape on the outside of the flesh.



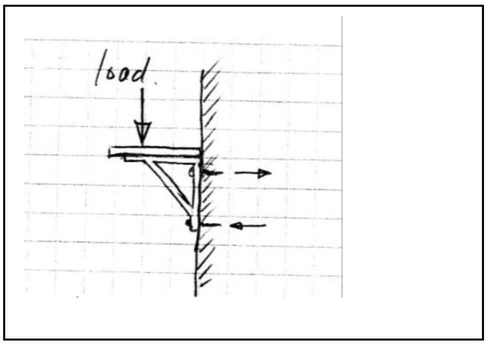
d. Step ladders are an example of a structure to transfer the weight of a person down to the ground. The feet are prevented from moving apart partly by the tensile rope and partly by friction on the ground.



e. A bed is a similar structure to a bridge and consists of a beam spanning between compressive columns.

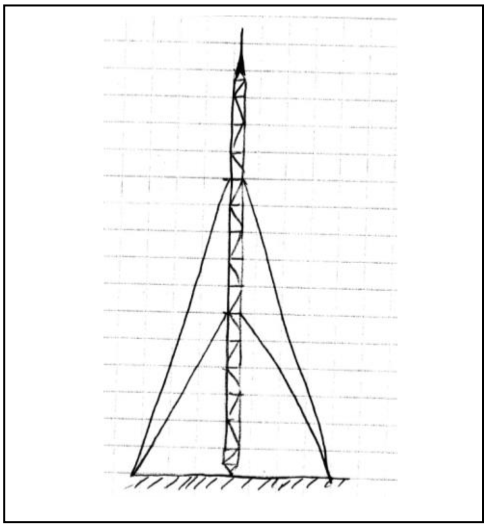


f. The shelf bracket transfers the load back to the wall with the help of screws. The diagonal member is a compressive 'strut' and reduces bending in the bracket.

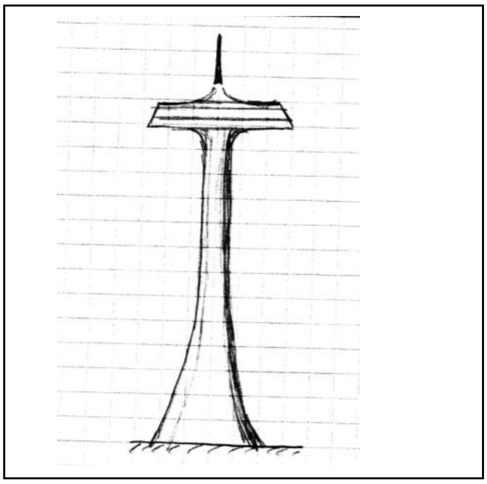


E1.2

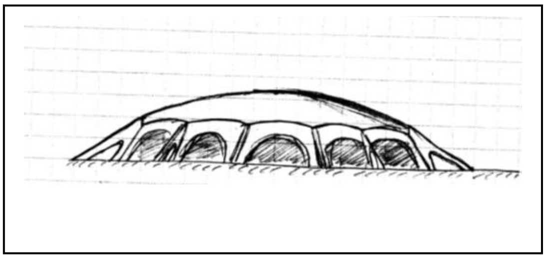
a1. Steel lattice tower with tension guys.



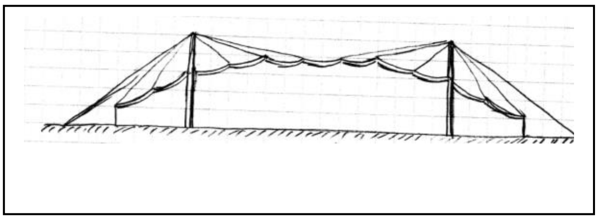
a2. Concrete shell with restaurant.



b1. Shell dome with flying buttresses.



b2. Cable-supported ‘tent’ roof (cross-section)



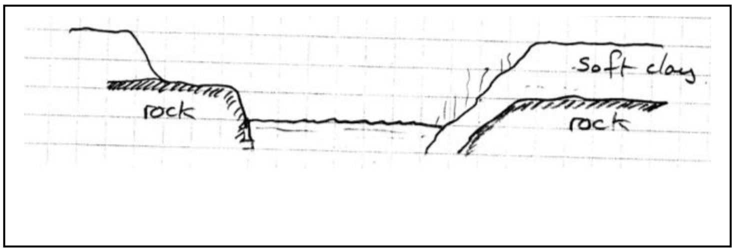
E1.3

<u>Stage 1 – site survey</u>

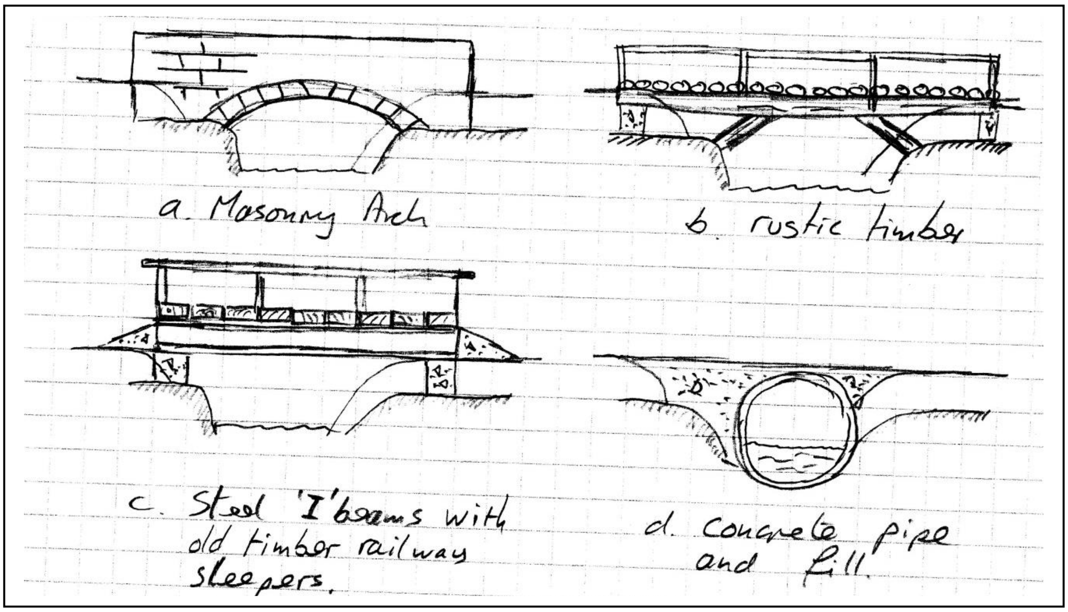
Determine the most suitable point to cross the stream taking into account:

- Stream width
- Height of banks
- Foundation material (any mid-stream rocks for extra support?)
- Location of access roads.

Carry out a detailed survey and produce a cross-section of the chosen point – drawn to scale:



<u>Stage 2 – alternative concepts</u>

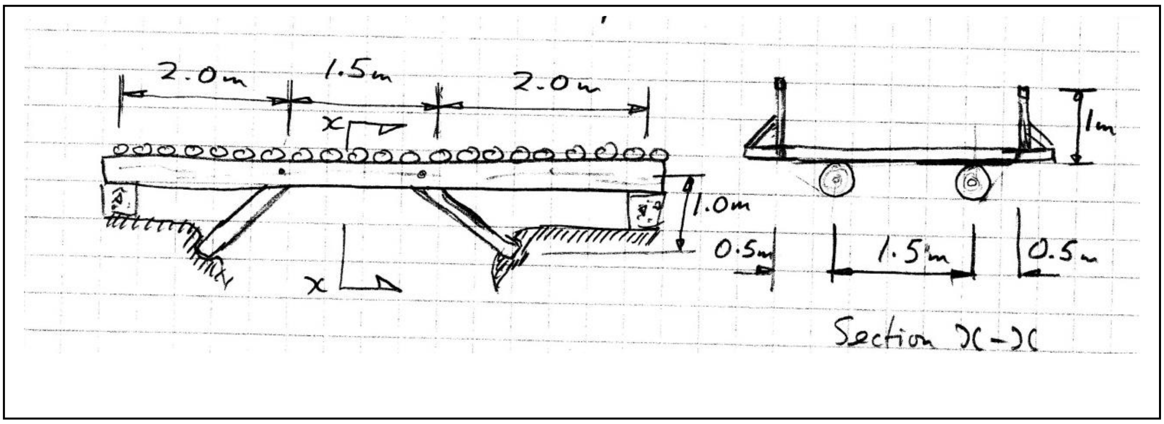


Comments:

a. Best appearance and long life, but expensive and difficult,
b. Reasonably good appearance, fairly cheap and easy but possibly short life,
c. Robust and easy but requires maintenance for long life,
d. Least attractive but cheap and easy.

Choose say b.

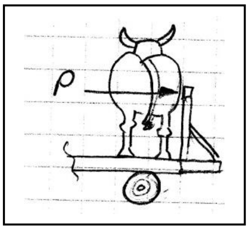
Stage 3 – detailed development



Stage 4 - Assessment of loads

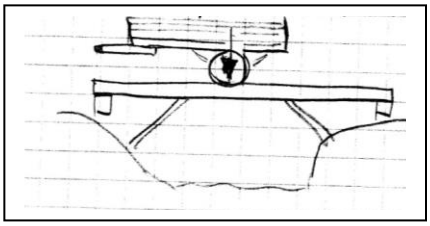
This can be tricky and very much depends upon who has access to the bridge. Is it accessible by the public and emergency vehicles? If this is the case the bridge will need to be designed for loads specified in Eurocode 1-2. If the bridge merely provides a link between two fields and will only be used by the farmer for movement of animals and a tractor and trailer the load requirements may be more relaxed. The local building regulation authority should be contacted for advice.

Assuming use by only the farmer:



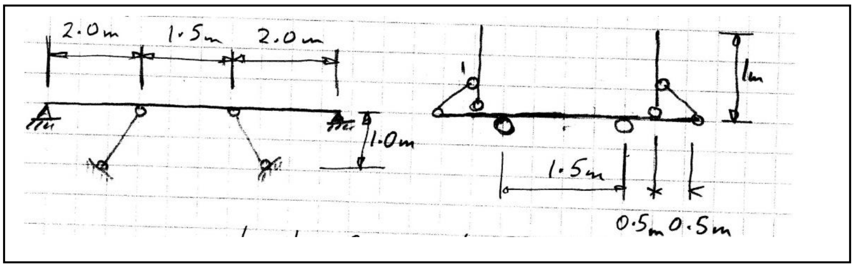
Animals: Design for say the number of cows that can fit on the bridge x weight of one cow. (about 1000 kg each?). Side load, *P*, on the handrail might be 20% of cow weight?

Vehicles: The highest axle loads should be placed at various points in the span.



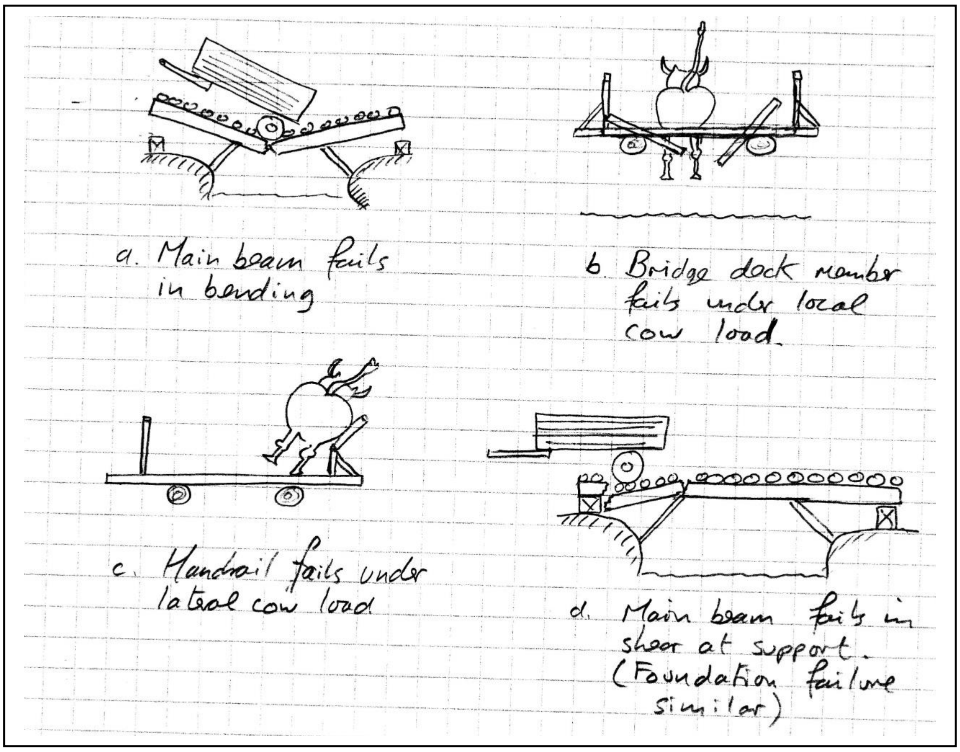
Stage 5 Analysis

Structural model:



Stage 6 – detail design

possible failure modes:



E1.4

a. Permissible stress design

Total load in rod	=	12 + 16	=	28 kN
Area required	=	force/stress	=	28 x 10^3/150
	=	**187 mm²**		

b. Limit state design

Design permanent load	=	12 x 1.35	=	16.2 kN		
Design variable load	=	16 x 1.5	=	<u>24.0 kN</u>		
Total design load			=	40.2 kN		
Design strength	=	f_y/γ_M	=	275/1.0	=	275 N/mm²
Area required	=	40.2 x 10^3/275	=	**146 mm²**		

Chapter 2 - Basics

E2.1

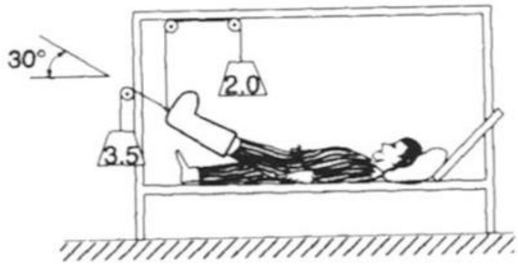
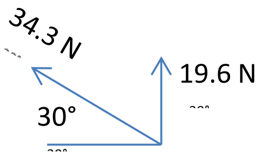


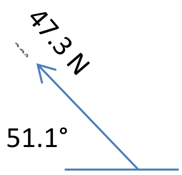
Figure 2.15
Traction weights in kg

a.	Force from 2.0 kg weight	=	2.0 x 9.81	=	**19.6 N**
	Force from 3.5 kg weight	=	3.5 x 9.81	=	**34.3 N**

Structural model:



b.	Horizontal component of force	=	34.3 cos 30°	=	**29.7 N←**
	Vertical component of force	=	19.6 + 34.3 sin 30°	=	**36.8 N↑**
c.	Magnitude of resultant force	=	$\sqrt{(29.7^2 + 36.8^2)}$	=	**47.3 N**
	Direction of resultant force	=	$\tan^{-1}(36.8/29.7)$	=	**51.1°**



d.	Stress in cable	=	Force/Area	=	$34.3/ (\pi \times 1.5^2/4)$
		=	**19.4 N/mm²**		

E2.2

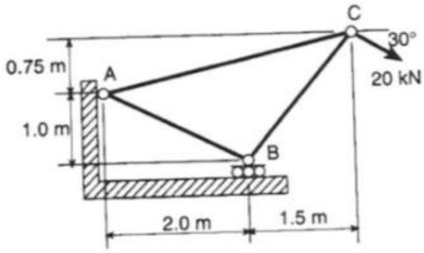
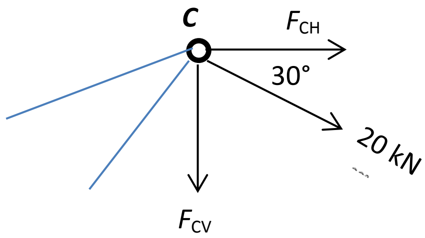


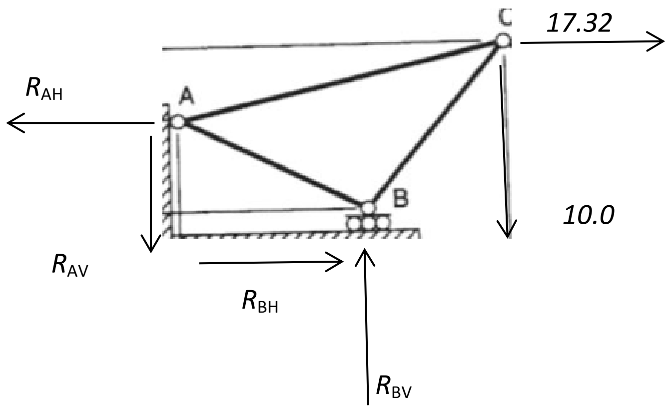
Figure 2.16
Pin-jointed structure



a. F_{CH} = 20 Cos 30° = 17.32 kN

F_{CH} = 20 Sin 30° = 10.0 kN

b.



Support B is a roller ∴ $R_{BH} = 0$

ΣM about A

$$(17.32 \times 0.75) + (10 \times 3.5) = R_{BV} \times 2.0$$

R_{BV} = **<u>24.0 kN↑</u>**

ΣH = 0 R_{AH} = **<u>17.32 kN←</u>**

ΣV = 0 *10.0 +* R_{AV} = 24.0 kN

R_{AV} = **<u>14.0 kN↓</u>**

E2.3

No horizontal forces or reactions

a. ΣM about A

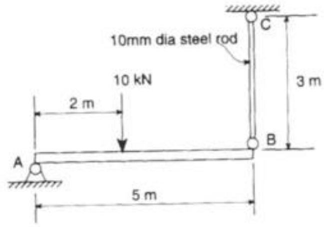


Figure 2.17
Beam and tie structure

10×3 = $5 \times R_A$

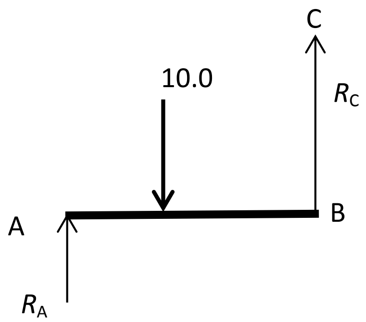
R_A = **<u>6 kN↑</u>**

ΣV = 0

10 = $6 + R_C$

R_C = **<u>4 kN↑</u>** i.e. tension in rod

structural model:



b. Area of rod = $\pi \times 10^2/4$ = 78.5 mm^2

Stress = force/area = $4 \times 10^3/78.5$ = **<u>51 N/mm²</u>**

c. Extension, δ = FL/EA where E = *205* kN/mm^2

δ = $4 \times 10^3 \times 3000/205\,000 \times 78.5$ = **<u>0.75 mm</u>**

Chapter 3 - Materials

E3.1

a. Underground water storage reservoir

Choice: **Reinforced concrete**

Reasons:

- Will resist bending from water pressure
- Can be made waterproof with additives
- Durable and requires no maintenance if cover to reinforcement adequate
- Relatively cheap
- Can be formed on-site from locally available materials

b. Footbridge in chemical plant

Choice: **Fibre composite**

Reasons:

- Resistant to chemical attack
- No maintenance required
- Lightweight and easily transported

c. Portable grandstand

Choice: **Aluminium alloy**

Reasons:

- Lightweight and easily transported
- Good range of extrusions available
- Can be welded
- Can be painted or anodised for good corrosion protection
- Stiffer than fibre composite

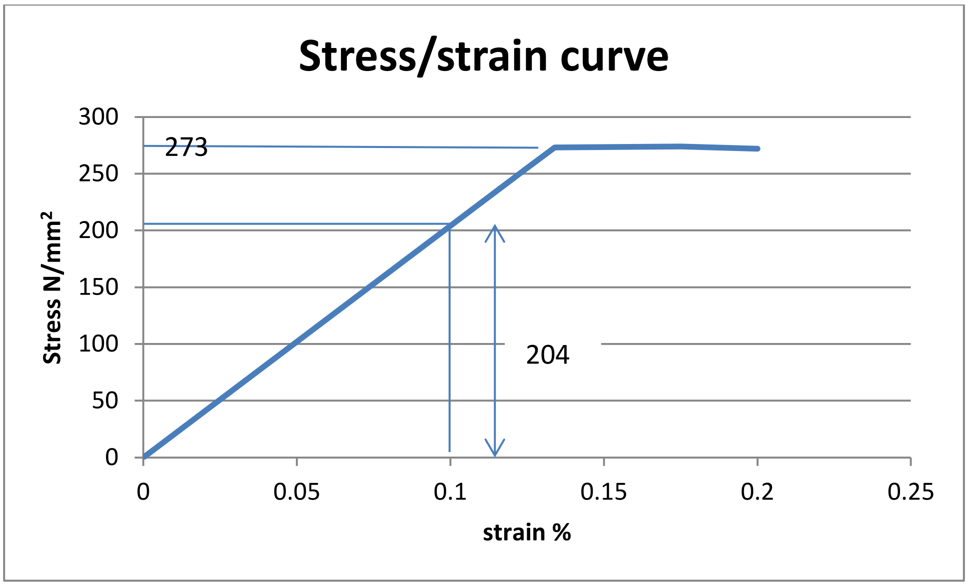
E 3.2

a. Yield stress, f_y, and Modulus of elasticity, *E*
b. Gauge length is the initial length of the specimen for the purposes of measuring extension and hence strain and modulus of elasticity.
c. Plastic behaviour means that materials are ductile. (i.e. they can be deformed by large amounts without losing their strength. The opposite of brittle.
d. Because by the time mild steel has reached its ultimate strength it will have deformed excessively. Keeping below the yield strength means that permanent deformations do not take place.
e. Yield point (i.e. start of plastic behaviour), limit of proportionality (i.e. end of linear behaviour where strain is proportional to stress) and elastic limit (i.e . end of reversible deformation).

E3.3

Area of specimen	=	$\pi/4 \times 5^2$	=	19.63 mm^2
Stress	=	load/area		
% Strain	=	100 x extension/gauge length		

Load (kN)	Extension (mm)	Stress (N/mm^2)	% Strain
0.00	0.00	0	0.000
1.00	0.01	51	0.025
2.00	0.02	102	0.050
3.00	0.03	153	0.075
4.00	0.04	204	0.100
5.00	0.05	255	0.125
5.32	0.06	271	0.150
5.38	0.07	274	0.175
5.34	0.08	272	0.200



From graph:

a. Yield stress, f_y = **274 N/mm^2**

b. Modulus of elasticity, E = 204/0.001 = **204 000 N/mm^2**

Solutions to the end of chapter exercises for *Understanding Structures fifth edition* by Derek Seward

E3.4

Result (x)	**(x-x_m)**	**(x-x_m)^2**
455	-4	16
467	8	64
449	-10	100
452	-7	49
471	12	144
462	3	9
2756		382

Mean x_m = 2756/6 = 459

Standard deviation = $\sqrt{[(x-x_m)^2/(n-1)]}$ = 382/5

= 8.74

Characteristic strength = 459 – 1.59 x 8.74

= 445 N/mm^2

Design strength = Characteristic strength/γ_M

= 445/1.15 = **387 N/mm^2**

Chapter 4 - Loads

E4.1

From *table 4.1* unit weight of steel = 70 kN/m^3

$\therefore$ Weight of of 1 square meter of plate 6 mm thick = 70 x 0.006 = 0.42 kN/m^2

a. Characteristic permanent load = **0.42 kN/m^2**
b. From *table 4.3* variable floor load for factory = 5.0 kN/m^2

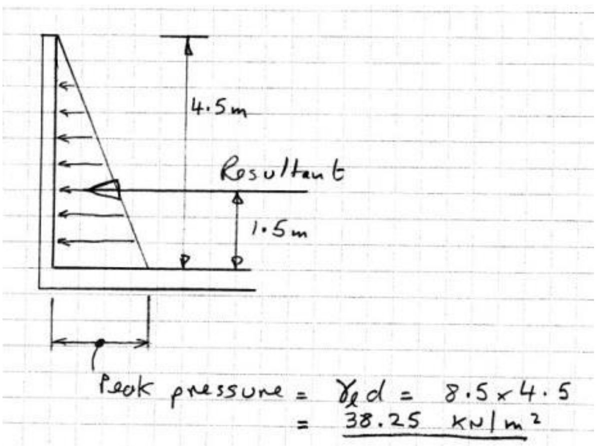
$\therefore$ Total design load = (1.35 x 0.42) + (1.5 x 5.0)

= **8.07 kN/m^2**

E4.2

a.

Resultant occurs at centroid of triangle i.e. 1/3[rd] of height.



a. Force = 38.25 x 4.5/2 = **86.1 kNm**
b. Bending moment = 86.1 x 1.5 = **129.2 kNm**

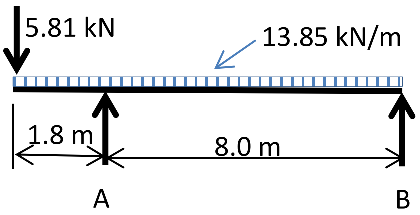
E4.3

	Area of each floor	=	60 x 60	=	3600 m^2
From *table 4.1*	Wt. of floor/m^2	=	25 x 0.2	=	5 kN/m^2
	Wt. of one entire floor	=	3600 x 5	=	18 000 kN
	Wall area per storey	=	4 x 4 x 60	=	960 m^2
	Weight of wall per storey	=	960 x 1.0	=	960 kN
	Total permanent load/storey	=	18 000 + 960	=	18 960 kN
From *table 4.3*	Variable load per floor	=	3600 x 2.5	=	9000 kN
	Total design load	=	110 x [(1.35 x 18 960) + (1.5 x 9000)]		
		=	**4.30×10^6 kN**		

E4.4

The following is for a 1 m wide strip of the floor (into the page):

From *table 4.1*	Permanent load of wall	=	22 x 0.215 x 1.0	=	4.73 kN/m
	Design permanent wall load	=	4.73 x 1.35	=	5.81 kN/m
	Permanent load of slab	=	25 x 0.3	=	7.5 kN/m
	Design permanent slab load	=	1.35 x 7.5	=	10.1 kN/m
From *table 4.3*	Variable load on slab	=	2.5 kN/m		
	Design variable load	=	1.5 x 2.5	=	3.75 kN/m
	Total slab design load	=	10.1 + 3.75	=	13.85 kN/m



Structural model

Load on beam is reaction at A - take moments about B:

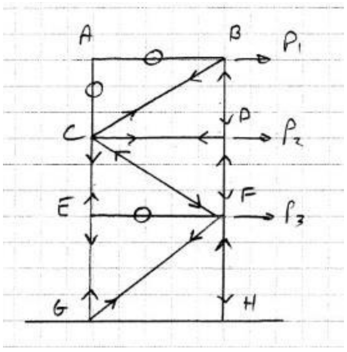
$R_A \times 8.0$ = (5.81 x 9.8) + (13.85 x 9.8 x 9.8/2)

R_A = 90.25 kN/m

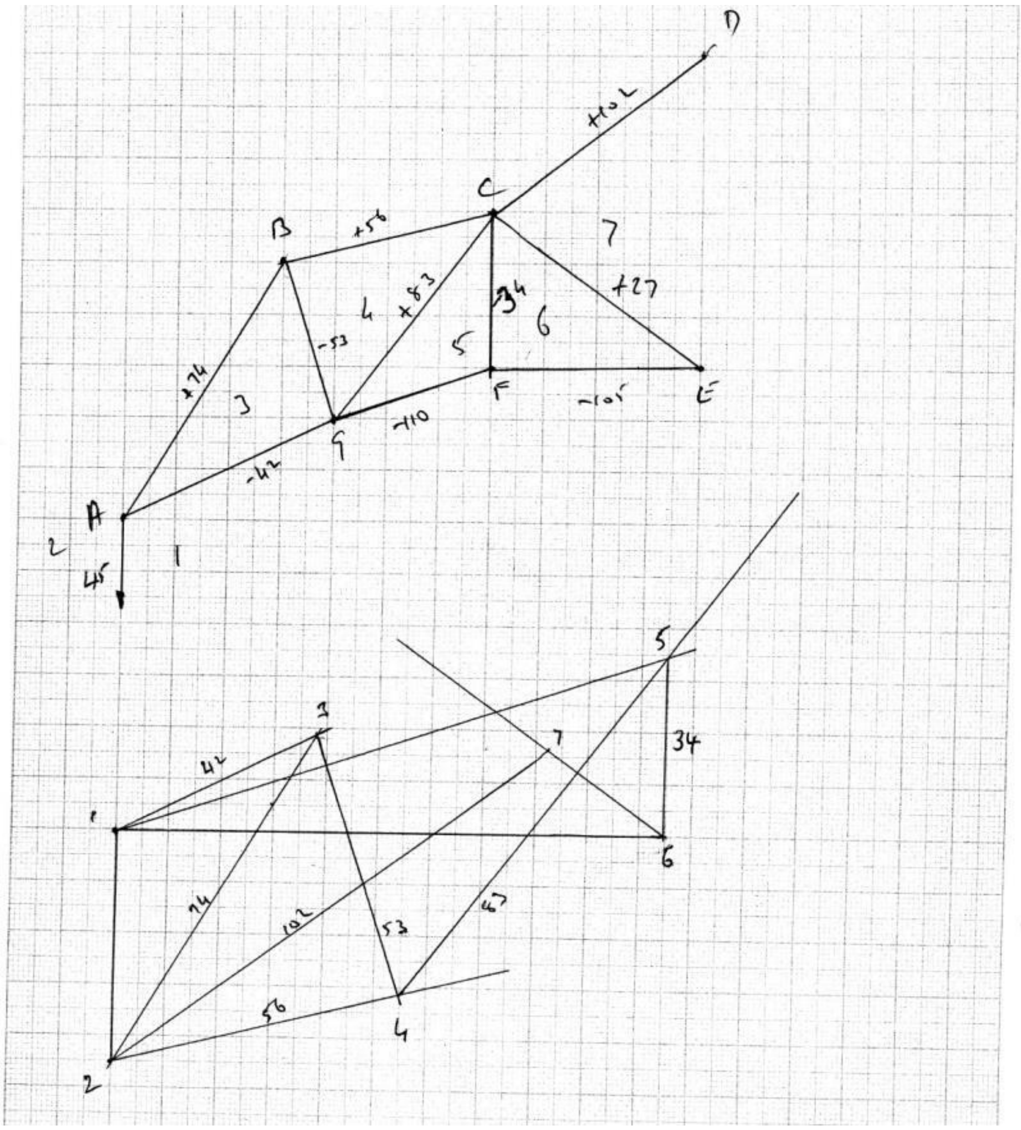
Design self-weight of beam	=	140 x 9.81/10^3 x 1.35	=	1.9 kN/m
Total design load on beam	=	90.25 + 1.9	=	92.15 kN/m

Chapter 5 – Pin-jointed trusses

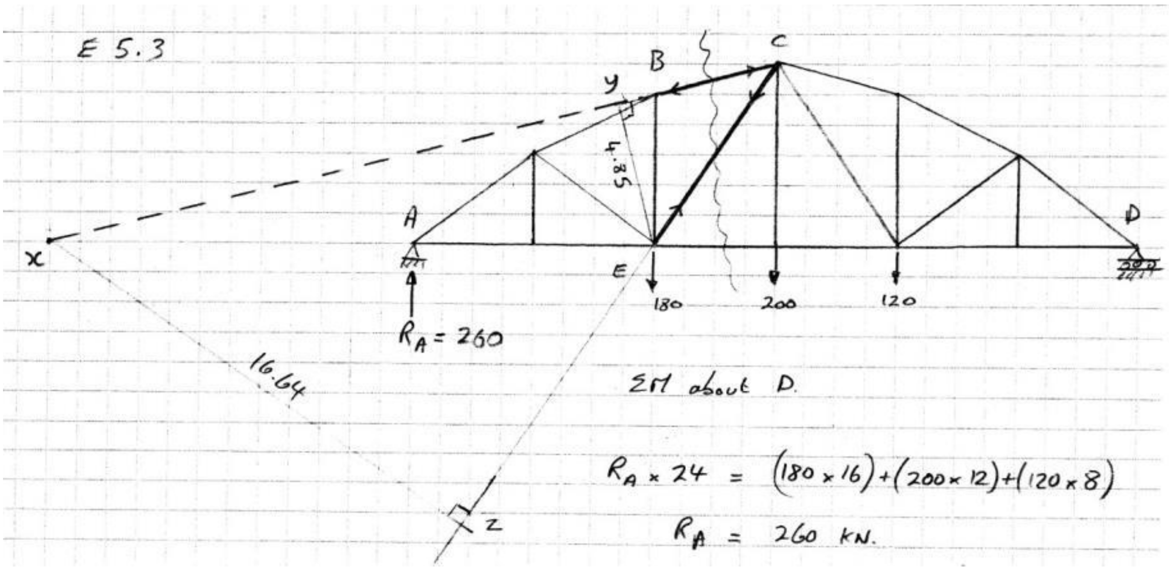
E5.1



E5.2



E5.3



Consider only the left-hand-side of the cut:

ΣM about E for F_{BC}:

$$F_{BC} \times y_E = R_A \times 8$$

$$F_{BC} \times \mathit{4.85} = \mathit{260} \times 8$$

$$F_{BC} = \underline{\textbf{429 kN compression}}$$

ΣM about x for F_{CE}:

$$(180 \times 20) = (260 \times 12) + (F_{CE} \times 16.64)$$

$$F_{CE} = \underline{\textbf{28.8 kN tension}}$$

E5.4

a. $m = 13, j = 6 \;\therefore\; i(1)$
b. $m = 11, j = 5$ + reduced roller support, but formula doesn't work – actually $m + i(1)$
c. $m = 16, j = 8$ extra roller $\therefore\; i(1)$
d. $m = 8, j = 4 \;\therefore\; s$
e. $m = 11, j = 5 \;\therefore\; i(1)$

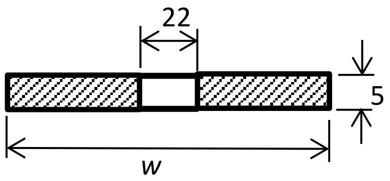
Chapter 6 - Tension

E6.1

From text: $f_y = 275$ N/mm^2, $f_u = 410$ N/mm^2

$\gamma_{M0} = 1.0$, $\gamma_{M2} = 1.1$

Option a)



$$\text{Min. } A_{gross} = \frac{150\times10^3}{1.0} = 545 \text{ mm}^2$$

$$A_{net} = \frac{150\times10^3}{\frac{0.9\times410}{1.1}} = 447 \text{ mm}^2$$

$$W = 447/5 + 22 = 111.4 \text{ say } 115 \text{ mm}$$

A_{gross} = 115 x 5 = **575 mm^2** > 545 mm^2

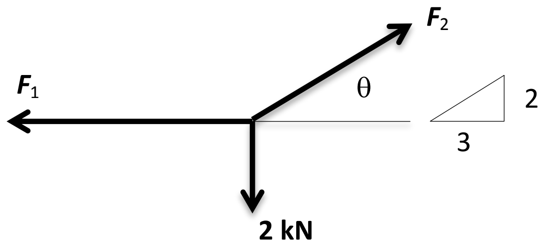
Option b)

A_{gross} = **545 mm^2** as above

From *appendix* **use 60 x 60 x 5 Angle** (A = 582 mm^2)

E6.2

From symmetry consider only half of the structure:



a.

⍰ θ = $\tan^{-1}(2/3)$ = 33.7°

From vertical equilibrium:

vertical component of force F_2 = 2 kN

$\therefore F_2 \text{Sin} 33.7°$ = 2.0

Force in cable at support, F_2 = **3.60 kN**

From horizontal equilibrium:

Force in cable at midspan, F_1 = 3.60Cos 33.7° = **3.00 kN**

b.

Area of cable required = 3600/(1200/5) = 15 mm^2

Diameter = $\sqrt{(15 \times 4/\pi)}$ = 4.4 mm

Use 5mm diameter cable

E6.3

Total design load = (1.35 x 6) + (1.5 x 5 x 2) [where 2 is the width)

= 23.1 kN/m

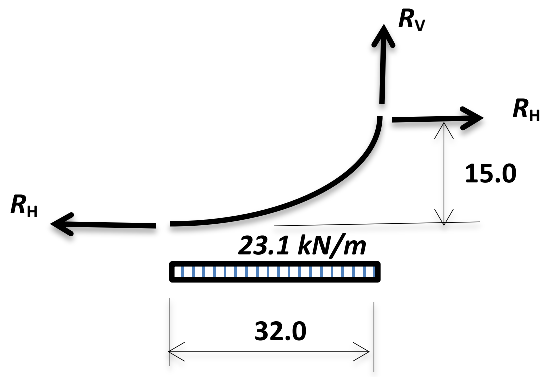


Diagram identical for both main span and side-spans

[6.4] R_V = $wL/2$ = 23.1 x 64/2 = 739 kN

[6.5] R_H = $wL^2/8D$ = $(23.1 \times 64^2)/(8 \times 15)$ = 788 kN

a. Max cable force at support = $0.5 \times \sqrt{(739^2 + 788^2)}$ = **<u>540 kN</u>**

[0.5 above is because of two cables]

b. Compressive force in tower = vert. component in cables at each side

= 2 x 739 = **<u>1478 kN</u>**

c. Area of cable required = Force/(strength/γ_M) = $540 \times 10^3/(1590/3)$
= 1019 mm^2

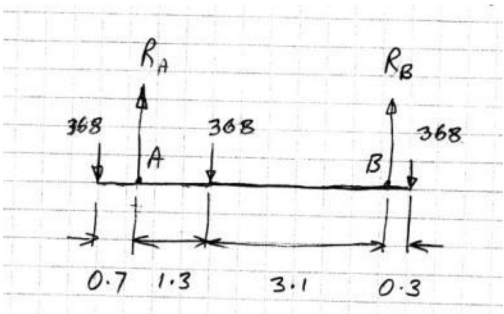
Diameter = $\sqrt{[(4 \times 1019)/\pi]}$ = 36.02 mm

say **<u>40 mm</u>**

d. Horizontal force on cable anchor = $R_H/2$ = 788/2 = **<u>394 kN</u>**

E6.4

Design variable load from one light = 2.5 x 1.5 x 9.81 = 368 N



a.

ΣM about A

$$(368 \times 1.3) + (368 \times 4.7) = (368 \times 0.7) + (R_B \times 4.4)$$

$$R_B = \underline{\mathbf{443\ N}}$$

ΣV = 0

$$R_A = (3 \times 368) - 443$$

$$= \underline{\mathbf{661\ N}}$$

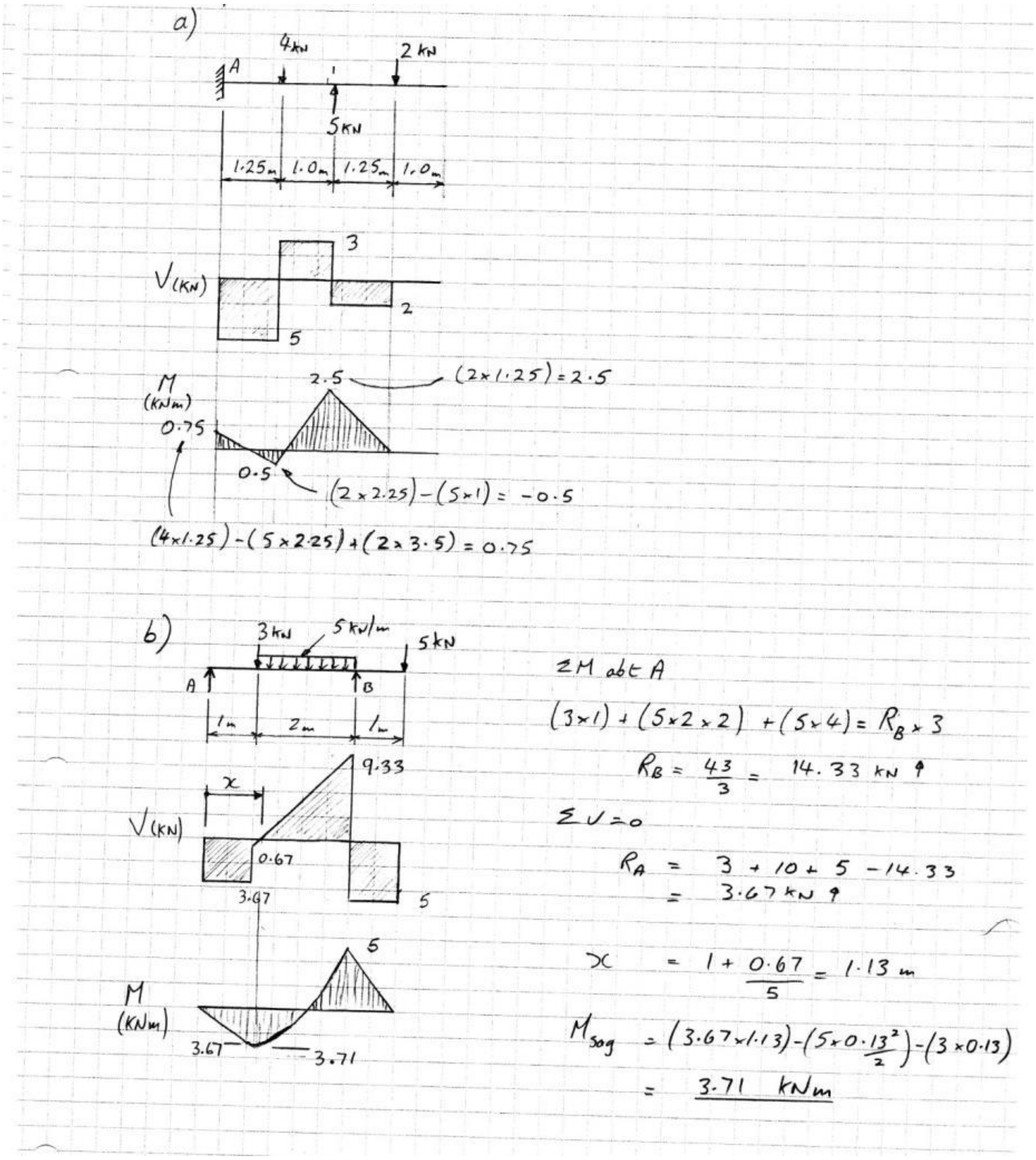
b.

Area of wire required = 661/(300/5) = 11.01 mm^2

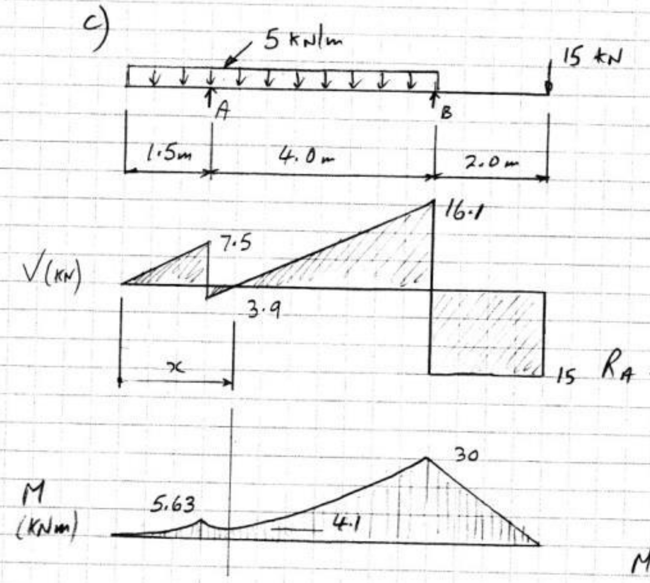
Diameter of wirte = $\sqrt{[(4 \times 11.01)/\pi]}$ = 3.75 say **4.0 mm**

Chapter 7 - Beams

E7.1



c)



ΣM about A

$(5\times5.5\times1.25)+(15\times6)=R_B\times4$

$R_B = 31.1$ kN.

ΣV = 0

$R_A = (5\times5.5)+15-31.1$

$= 11.4$ kN.

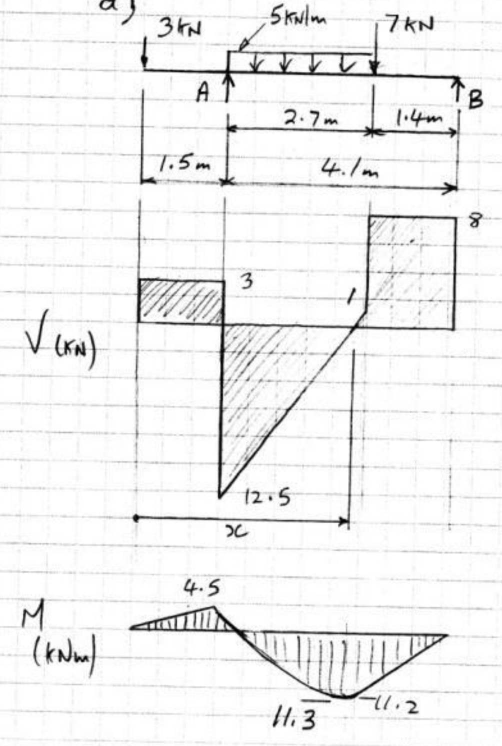
$x = 1.5 + \frac{3.9}{5} = 2.28$ m

$M_x = (11.4\times0.78)-\left(5\times2.28\times\frac{2.28}{2}\right)$

$= -4.1$

Not really relevant as it is a min. not a max.

d)



ΣM about A

$(5\times2.7\times1.35)+(7\times2.7)=(3\times1.5)+(R_B\times4.1)$

$R_B = 8.0$ kN.

ΣV = 0

$3+(5\times2.7)+7 = R_A+8.0$

$R_A = 15.5$ kN.

$x = 1.5 + \frac{12.5}{5}$

$= 4.0$ m

$M_x = (8\times1.6)-(7\times0.2)-\left(5\times\frac{0.2^2}{2}\right)$

$= 11.3$ kN m

E7.2

Bending moment = $wl^2/8$ = $22 \times 8.0^2/8$ = 176 kNm

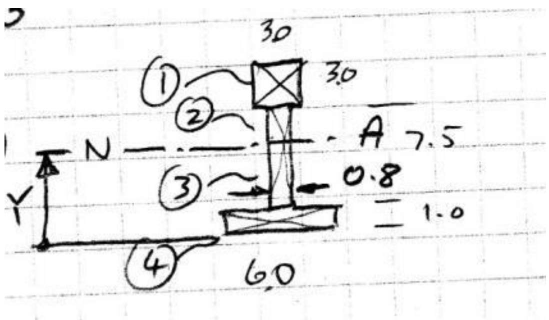
f_y = 275, γ_M = 1.0

∴ required W_{pl} = $(176 \times 10^6)/(275 \times 10^3)$ = 640 cm^3

Use 406 x 140 x 39 kg/m Universal Beam

E7.3

a.



Total area = 9 + 6 + 6
= 21cm^2

Half area = 21/2
= 10.5 cm^2

For the plastic modulus the neutral axis divides the total area into two equal parts.

Y = 1.0 + (10.5 – 6)/0.8 = 6.625 cm

Item	A (cm^2)	Y (cm)	Ay (cm^3)
1	9	3.375	30.3
2	1.5	0.938	1.4
3	4.5	2.813	12.7
4	6	6.125	36.8
			81.3

Plastic modulus, W_{pl} = 81.3 cm^3

b.

Design load = (1.35 x 0.9) + (1.5 x 3.5) = 6.47 kN/m^2

Design load/m, w = 6.47 x 0.35 = **2.26 kN/m**

c.

Design bending moment = $wl^2/8$ = $2.26l^2/8$ ①

Design resistance moment = $W_{pl} \times f_y$ = $81.3 \times 275/10^3$

= 22.36 ②

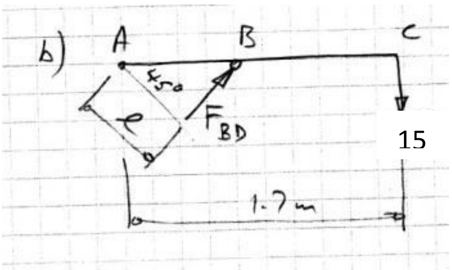
Equating ① and ②

$2.26l^2/8$ = 22.36

l_{max} = 8.9 m

E7.4

a. Design load = 10 x 1.5 = **<u>15 kN</u>**



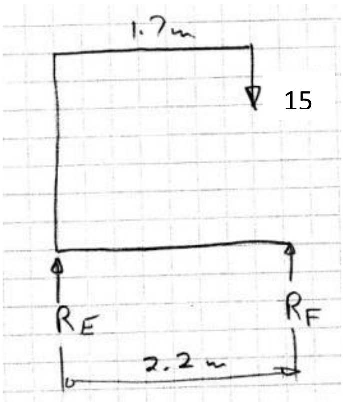
Lever arm, l = 0.8 cos 45°

= 0.566 m

ΣM about A:

15 x 1.7 = F_{BD} x 0.566

Force in ram, F_{BD} = **<u>45.1 kN</u>**



ΣM about E:

15 x 1.7 = R_F x x2.2

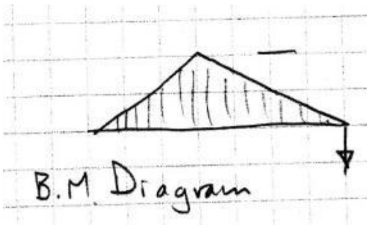
R_F = **<u>11.59 kN</u>**

ΣV = 0:

15 = 11.59 + R_E

R_E = **<u>3.41 kN</u>**

c.



M_{max} = 15 x 0.9 = 13.5 kNm

W_{pl} required = $(13.5 \times 10^6)/(275 \times 10^3)$

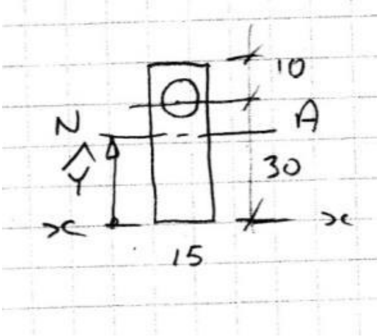
= 49.1 cm^3

From appendix:

<u>Use 100 x 100 x 4.0 square hollow section</u> (W_{pl} = 54.9 cm^3)

E7.5

Part	**Area (mm^2)**	y_n	**A y_n**	y	**Ay^2**	I_{self}
Rectangle	600	20	12000	1.51	1368	80 000
Hole	-78.5	30	-2355	11.51	-10400	-491
	521.5		9645		-9032	79 509



Y = 9645/521.5 = 18.49

I_{NA} = 79 509 – 9032 = 70 477 mm^4

$W_{el,top}$ = 70 477/ (40-18.49) = 3276 mm^3

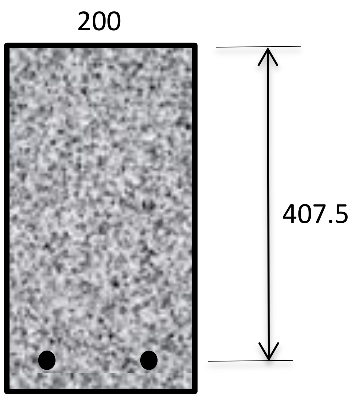
$W_{el,bottom}$= 70 477/ 18.49 = 3812 mm^3

σ_{top} = $0.5 \times 10^6/3276$ = **153 N/mm^2 compression**

σ_{bottom} = $0.5 \times 10^6/3812$ = **131 N/mm^2 tension**

E7.5

a. Effective depth, d = 450 – 30 12.5 = 407.5 mm



Based on concrete:

M_{ult} = $0.214 f_{ck}bd^2$ = $0.214 \times 30 \times 200 \times 407.5^2 \times 10^{-6}$
= 213.2 kNm

Based on reinforcement:

Table 3.1 Area of steel = 2 x 491
= 982 mm^2

M_{ult} = $0.696 A_s f_{yk} d$ = $0.696 \times 982 \times 500 \times 407.5 \times 10^{-6}$
= 139.3 kNm

But M_{Ed} = $wl^2/8$ = $40l^2/8$

$\therefore l$ = $\sqrt{}8 \times 139.3/40)$ = **5.28 m span**

b. Shear force on beam, V_{Ed} = 5.28 x 40/2 = 105.6 kN

Design shear stress, v_{Ed} = $105.6 \times 10^3/(200 \times 407.5)$ = 1.30 N/mm^2

% reinforcing steel = $100\ A_s/bd$

= 100 x 982/(200 x 407.5) = 1.2

Interpolating from *table 7.2* $v_{Rd,c}$ = 0.71 N/mm^2 < 1.30 – ∴ shear links required

New , v_{ED} = 105.6 x 10^3/(200 x 407.5 x 0.9)

= 1.44 N/mm^2

From *table 7.3* (cot θ = 2.5) $v_{Rd,max}$ = 5.28 > 1.30

∴ A_{sw} = $v_{Ed}sb$ /1087 = 1.44sb/1087

Max link spacing, *s* = 0.75 x 407.5 = 305.6

Say = 300 mm

∴ A_{sw} = *1.44 x 300 x 200*/1087

= 79.5 mm^2

From *table 3.1* **<u>Use 8 mm dia. Links at 300 mm spacing</u>** (A_{sw} = 100 mm^2)

Chapter 8 - Compression

E8.1

From *equation 8.1* $N_{Rd} = f_{ck}/\gamma_c \times b \times h_w \times (1 - 2e/h_w)$

Where: Eccentricity, $e = l_w/400 = 3750/400$
$= 9.4$ mm

$\therefore N_{Rd} = 40/1.5 \times 400 \times 400 \times (1 - 2 \times 9.4/400) \times 10^{-3}$
$=$ **4066 kN**

E8.2

For 10mm x 10mm strut, I $= bd^3/12 = 10 \times 10^3/12 = 833\ \text{mm}^4$

`For steel, $E = 210\,000\ \text{N/mm}^2$

Euler buckling load, $P_{crit} = \pi^2 EI/L^2 = \pi^2 x\ 210\,000 \times 833/900^2$

$=$ **2130 N**

E8.3

Design load $= 250/2 \times 1.5 = 187.5$ kN

Approximate area $= 187.5 \times 10^3/100 \times 10^2 = 18.7\ \text{cm}^2$

From *appendix* Try 114.3 x 6.3 circular hollow section ($A = 21.4\ \text{cm}^2$, $i = 3.82$ cm)

Effective length $= 3500 \times 2 = 7000$ mm

Non-dimensional slenderness, $\lambda = 7000/(86 \times 38.2) = 2.13$

From *fig 7.38* $\chi = 0.18$

Buckling stress $= 275 \times 0.18 = 49.5\ \text{N/mm}^2$

Actual stress $= 187.5 \times 10^3/21.4 \times 10^2 = 87.6\ \text{N/mm}^2$
– no good

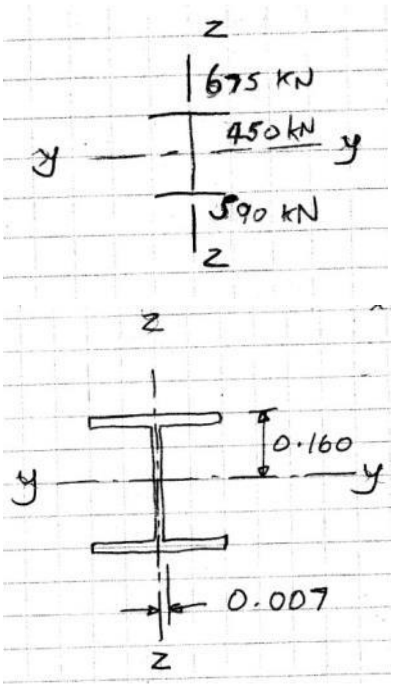
From *appendix* Try 168.3 x 5.0 circular hollow section ($A = 5.7\ \text{cm}^2$, $i = 5.78$ cm)

Non-dimensional slenderness, λ	=	7000/(86 x 57.8)	=	1.4
From *fig 7.38* χ	=	0.38		
Buckling stress	=	275 x 0.38	=	104.5 N/mm^2
Actual stress	=	$187.5 \times 10^3 / 25.7 \times 10^2$	=	73 N/mm^2 ✓

<u>Use 168.3 x 5.0 circular hollow section</u>

Chapter 9 – Combined axial and bending stresses

E9.1



Axial load, N_{Ed} = 675 + 450 + 590
= 1715 kN

Approx. area required = 1715 x 10^3/100 x 10^2
= 172 cm^2

Try 305 x 305 x 137 Universal Column
(A = 175 cm^2, $W_{pl,y}$ = 2300 cm^3, $W_{pl,z}$ = 1052 cm^3, i_z= 7.82 cm,
h = 320.5, t_f = 13.8)

Dimensions as shown

M_{yEd} = (675 – 590) x (0.160 + 0.1)
= 22.1 kNm

M_{zEd} = 450 x (0.007 + 0.1)
= 48.2 kNm

Non-dimensional slenderness ratio, λ = $L_E/(86 \times i_z)$ = 6000/(86 x 78.2)
= 0.89

From [9.2]

$$\frac{N_{Ed}}{A\chi f_y} + \frac{M_{y,Ed}}{\chi_{LT} f_y W_{pl,y}} + \frac{M_{z,Ed}}{f_y W_{pl,z}} < 1.0$$

From *fig 7.38* χ = 0.55

Buckling stress = 0.55 x 275 = 151 N/mm^2

Bending stress = 0.55 x 275 = 151 N/mm^2

$\therefore$ Axial resistance, $A\chi f_y$ = 175 x 151 x 10^{-1} = 2642 kN

y-y bending resistance, $\chi_{LT} f_y W_{pl,y}$= 151 x 2300 x 10^{-3} = 347 kNm

z-z bending resistance, $f_y W_{pl,z}$ = 275 x 1052 x 10^{-3} = 289 kNm

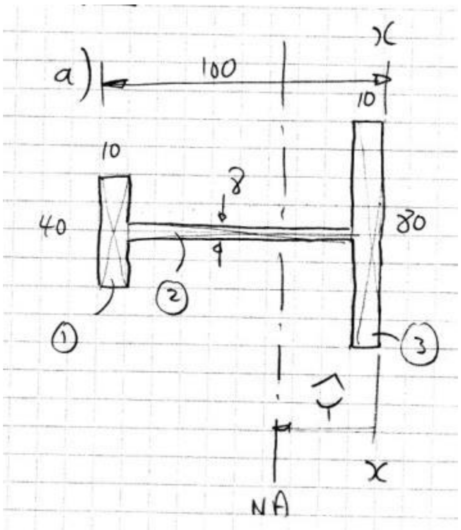
substitute into *equation [9.2]*

$$\frac{1715}{2642} + \frac{22.1}{347} + \frac{48.2}{289} = 0.963 < 1.0 ✓$$

Use 305 x 305 x 137 Universal Column

E9.2

a.



Item	Area (cm^2)	y_n (cm)	$A\ y_n$	y	Ay^2	I_{self}
1	4.0	9.5	38	5.48	120	-
2	6.4	5.0	32	0.98	6	34
3	8.0	0.5	4	3.52	99	1
	18.4		74		225	35

Y = 74/18.4 = 4.02 cm

I_{NA} = 225 + 44 = 260 cm^4

$W_{el,inner}$ = 260/4.02 = **64.7 cm^3**

$W_{elouter}$ = 260/(10-4.02) = **43.5 cm^3**

b.

Total load = (30 x 70) + 2000 = 4100 kg

Force = 4100 x 9.81/10^3 = 40.2 kN

Eccentricity,*e* = 0.26 +0.042 = 0.30 m

Moment = 40.2 x 0.30 = 12.06 kNm

$\sigma = F/A \pm M/W_{el}$

σ_{inner} = 40.2 x 10^3/18.4 x 10^2 + 12.06 x 10^6 /64.7 x 10^3
= 21.8 + 186.4 = **208.2 N/mm^2 tens.**

σ_{inner} = 40.2 x 10^3/18.4 x 10^2 - 12.06 x 10^6 /43.5 x 10^3
= 21.8 – 276.0 = **254.2 N/mm^2 comp**.

E9.3

Elastic moduli:

$W_{el,top}$ = 18 000/10 = 1800 cm^3

$W_{el,bottom}$ = 18 000/15 = 1200 cm^3

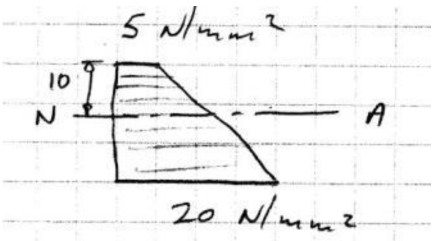
Sagging BM:

σ_{bottom} = $24 \times 10^6/1200 \times 10^3$ = 20 N/mm^2 tension

Hogging BM:

σ_{top} = $9 \times 10^6/1800 \times 10^3$ = 5 N/mm^2 tension

$\therefore$ Required pre-stress -



Stress at N.A. level = 5 + (15 x 10/25) = 11 N/mm^2

$\therefore$ pre-stressing force = $11 \times 200 \times 10^2/10^3$ = **<u>220 kN</u>**

Consider top stresses, σ_{top} = $P/A - Pe/W_{el,top}$ = 5

5 = $11 - 220 \times 10^3 \times e/1800 \times 10^3$

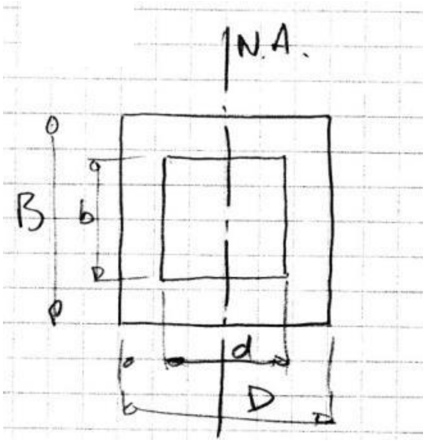
5 = $11 - 0.122e$

e = **<u>49.2 mm</u>**

Check bottom stresses, σ_{bottom} = $11 + 220 \times 10^3 \times 49.2/1200 \times 10^3$

= 11 + 9.02 = 20.02 N/mm^2 - check

E9.4



I_{NA} = $BD^3/12 - bd^3/12$

= $3 \times 3^3/12 - 1.8 \times 1.8^3/12$

= $6.75 - 0.875$

= $5.875\ m^4$

Elastic modulus, W_{el} = $5.875/1.5$

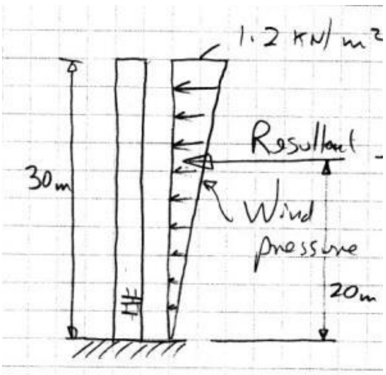
= $3.917\ m^3$

Area of section = $3^2 - 1.8^2$

= $5.76\ m^2$

Total weight = $30 \times 5.76 \times 20$

= 3456 kN



Total wind force = $30/2 \times 1.3 \times 3.0$

= 54 kN

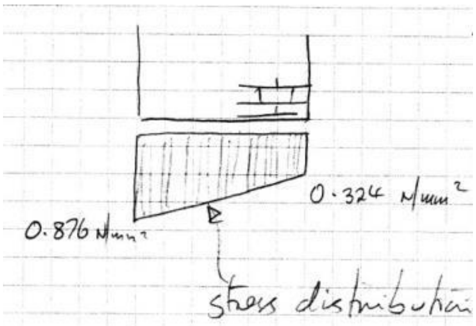
Resultant acts 2/3 of height up chimney:

= $2/3 \times 30$

= 20 m

Overturning moment = 54×20

= 1080 kNm



Stresses at base = $F/A \pm M/W_{el}$

= $3456/5.76 \pm 1080/3.917$

= $600 \pm 276\ kN/m^2$

= **<u>0.876 N/mm² Compressive</u>**

And

= **<u>0.324 N/mm² Compression</u>**

Chapter 10 - Torsion

E10.1

$$T_{ult} = 2\tau_y \pi R^3/3$$

$$= 2 \times 159 \times \pi \times 15^3/3$$

$$= 1\ 124\ 000 \text{ Nmm}$$

$$= \mathbf{1.12\ kNm}$$

$$\phi = L\tau/GR$$

$$= (3000 \times 90)/(77\ 000 \times 15)$$

$$= 0.234 \text{ rad}$$

$$= \mathbf{13.4°}$$

E10.2

a.

Area of sign	=	1.8 x 0.9	=	$1.62\ m^2$
Design load	=	1.62 x 1.1	=	1.782 kN
Bending moment	=	1.782 x 2.8	=	**4.98 kNm**
Torque	=	1.782 x (0.4 + 0.9/2)	=	**1.51 kNm**

b.

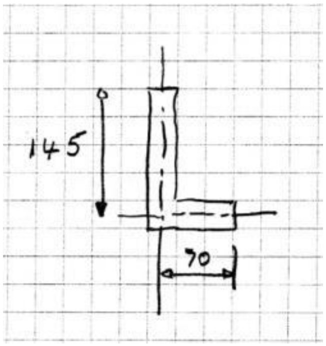
Plastic modulus for bending, W_{pl} = $4.98 \times 10^6/275 \times 10^3$ = $18.1\ cm^3$

Try 60 x 60 x 4.0 square hollow section (W_{pl} = $18.6\ cm^3$)

	$\tau_{bending\ shear}$	=	1782/(2 x 60 x 4)	=	$3.7\ N/mm^2$
	A_m	=	56 x 56	=	$3136\ mm^2$
Equation [10.6]	$\tau_{torsion\ shear}$	=	$T/2A_m t$	=	$1.51 \times 10^6/(2 \times 3136 \times 4)$
		=	$60.2\ N/mm^2$		
	τ_{total}	=	3.7 + 60.2	=	$63.9\ N/mm^2 < 159$ ✓

Use 60 x 60 x 4.0 square hollow section

E10.3



a.

Equation [10.7]	τ_{max}	=	$3T/at^2$
Where	a	=	145 + 70
		=	215 mm
	τ_{max}	=	$3 \times 1.2 \times 10^6/(215 \times 10^2)$
		=	**<u>167 N/mm²</u>**

b.

$$\theta = 3TL/at^3G = \frac{3\times1.2\times10^6\times4000}{215\times10^3\times77000}$$

$$= 0.870 \text{ rad} = \underline{\mathbf{49.8°}}$$

Solutions to the end of chapter exercises for *Understanding Structures fifth edition* by Derek Seward

Chapter 11 - Connections

E11.1

a.
based on gross area of tie – *equation [6.2]*

N_{Rd} = $70 \times 10 \times 275/10^3$ = 192.5 kN

based on net area of tie – *equation [6.3]*

N_{Rd} = $(70-24) \times 10 \times 0.9 \times 410/(1.1 \times 10^3)$ = 154.3 kN

b.
based on shear capacity - *equation [11.3]*

F_v = $2 \times 0.6 \times 400 \times \pi \times 11^2 \times 10^{-3}/1.25$ = 146.0 kN

c.
based on bearing capacity - *equation [11.4]*

F_b = $2 \times 400 \times 22 \times 10 \times 10^{-3}/1.25$ = **140.8 kN**

Design resistance, N_{Rd} = **140.8 kN**

E11.2

a.
based on gross area of tie – *equation [6.2]*- as above

N_{Rd} = $70 \times 10 \times 275/10^3$ = 192.5 kN

based on net area of tie – *equation [6.3]*

N_{Rd} = $(70-22) \times 10 \times 0.9 \times 410/(1.1 \times 10^3)$ = 161.0 kN

b.
based on slip resistance - *equation [11.5]*

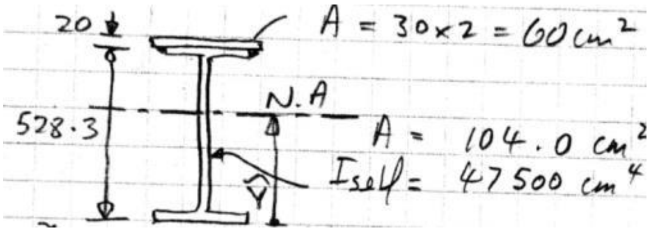
F_v = $2 \times 0.7 \times 1 \times 0.5 \times 800 \times 245 \times 10^{-3}/1.25$ = **109.80 kN**

c.
based on bearing capacity - *equation [11.4]*

F_b = $2 \times 400 \times 20 \times 10 \times 10^{-3}/1.25$ = 128 kN

Design resistance, N_{Rd} = **109.8 kN**

E11.3



Item	A(cm^2)	y_n (cm)	Ay_n(cm^3)	Y (cm)	Ay^2 (cm^4)	I_{self} (cm^4)
Plate	60	53.8	3228	17.4	18166	20
Beam	104	26.6	2746	10.0	10400	47500
	164		5974		28566	47520

$\hat{Y}$ = 5974/164 = 36.4 cm

I_{NA} = 28566 + 47520 = 76086 cm^4

Equation [11.9]

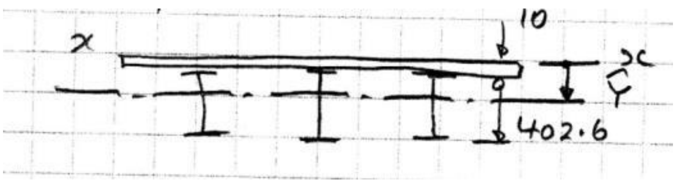
shear flow/weld, q = VQ/I = 2500 x 60 x 17.4/76086

= 17.15 kN/cm

= 1.72 kN/mm

From *table 10.1* **Use 12 mm fillet weld** (strength = 1.87 N/mm)

E11.4



Item	A(cm^2)	y_n (cm)	Ay_n(cm^3)	Y (cm)	Ay^2 (cm^4)	I_{self} (cm^4)
Plate	300	0.5	150	8.45	21421	-
Beams	205.2	21.3	4371	12.35	31298	55800
	505.2		4521		52719	55800

$\hat{Y}$ = 4521/505.2 = 8.95 cm

I_{NA} = 52 719 + 55 800 = 108 519 cm^4

a.

$W_{pl,top}$ = 108 519/8.95 = 12 125 cm^3

$W_{pl,bottom}$ = 108 519/(41.6-8.95) = 3 324 cm^3

For critical BM put load at midspan:

M_{Rd} = $PL/4$ = 150 x 8/4

= 300 kNm

σ_{comp} = 300 x 10^6/12 125 x 10^3 = **25.0 N/mm^2**

σ_{tens} = $300 \times 10^6 / 3\,324 \times 10^3$ = **90.3 N/mm^2**

b.

For critical shear put load adjacent to support:

V_{Rd} = 150 kN

shear flow, q = VQ/I = $150 \times 10^3 \times 300 \times 8.45/108\,519$

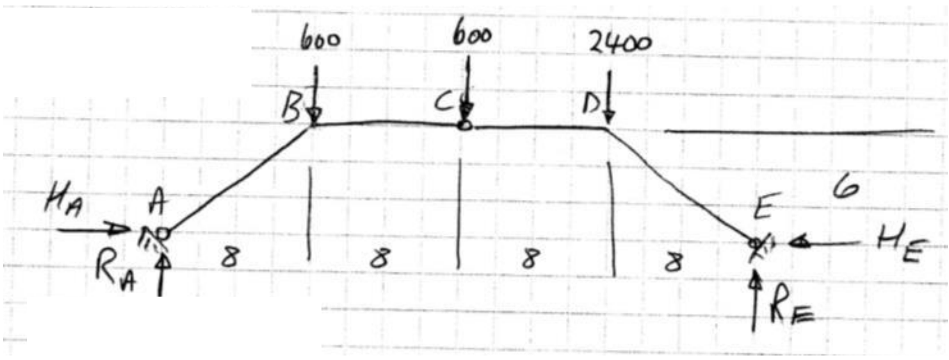
= 3504 N/cm

But this is resisted by 6 welds:

Shear flow/weld = 3504/(6 x 10) = **58.4 N/mm**

Chapter 12 – Arches and portal frames

E12.1



ΣM about A:

$(600 \times 8) + (600 \times 16) + (2400 \times 24) = R_E \times 32$

$R_E = 2250$ kN

ΣV =0:

$600 + 600 + 2400 = 2250 + R_A$

$R_A = 1350$ kN

ΣM about C:

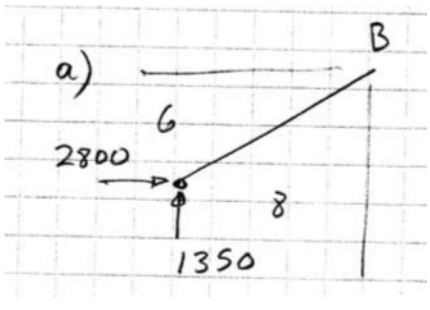
$(2400 \times 8) + (H_E \times 6) = 2250 \times 16$

$H_E = 2800$ kN

ΣH =0:

$H_A = -2800$ kN

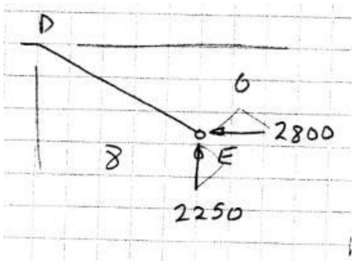
a.



$M_B = (1350 \times 8) - (2800 \times 6)$

$=$ **-6000 kNm – tension in top**

b.



$M_D = (2800 \times 6) - (2250 \times 8)$

$=$ **-1200 kNm – tension in bottom**

$L_{DE} = 5$ m

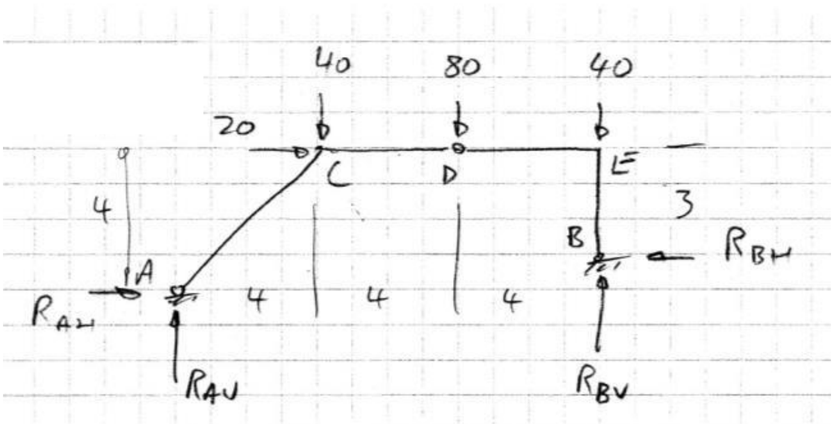
For thrust resolve forces parallel to DE:

Thrust $= 2250 \times 3/5 + 2800 \times 4/5 =$ **3590 kN**

For shear resolve forces perpendicular to DE:

Shear $= 2250 \times 4/5 - 2800 \times 3/5 =$ **120 kN**

E12.2



ΣM about A:

(20 x 4) +(40 x 4) + (80 x 8) + (40 x12)	=	$(R_{BV} \times 12) + (R_{BH} \times 1)$	
1360	=	$12R_{BV} + R_{BH}$	①

ΣM about D:

	(40 x 4) + $(R_{BH} \times 3)$	=	$R_{BV} \times 4$	
	160	=	$4R_{BV} - 3R_{BH}$	②
② x 3	480	=	$12R_{BV} - 9R_{BH}$	③
① - ③	880	=	$10R_{BH}$	
	R_{BH}	=	**<u>88 kN</u>**	
Sub in ①	1360	=	$12R_{BV}$ + 88	
	R_{BV}	=	**<u>106 kN</u>**	

ΣV =0:

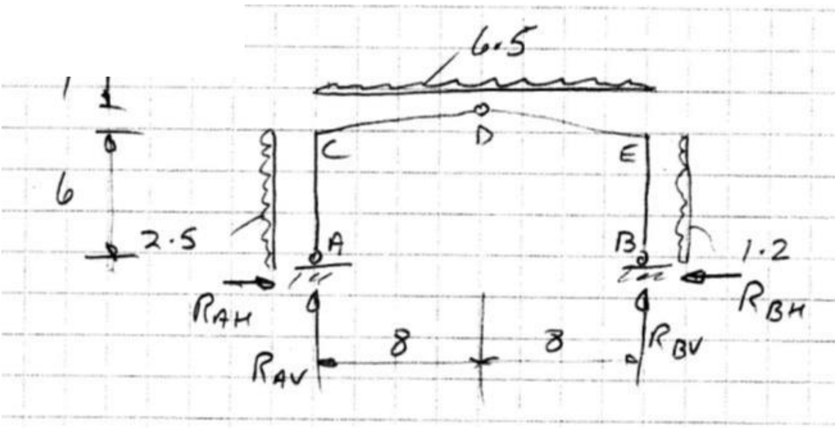
40 + 80 + 40	=	106 + R_{AV}
R_{AV}	=	**<u>54 kN</u>**

ΣH =0:

R_{AH} + 20	=	88
R_{AH}	=	**<u>68 kN</u>**

M_C	=	(63 x 4) – (68 x 4)	=	**<u>-56 kNm</u>** (tension in top)
M_E	=	(88 x 3)	=	**<u>264 kNm</u>** (tension in top)
Thrust $_{AC}$	=	54 cos 45° + 68 cos 45°	=	**<u>86.3 kN</u>** (compression)
Shear $_{AC}$	=	54 cos 45° - 68 cos 45°	=	**<u>-9.9 kN</u>**

E12.3



ΣM about A:

(2.5 x 6 x 3) +(6.5 x 16 x 8) + (1.2 x 6 x3)	=	R_{BV} x 16
R_{BV}	=	56.2 kN

M about D:

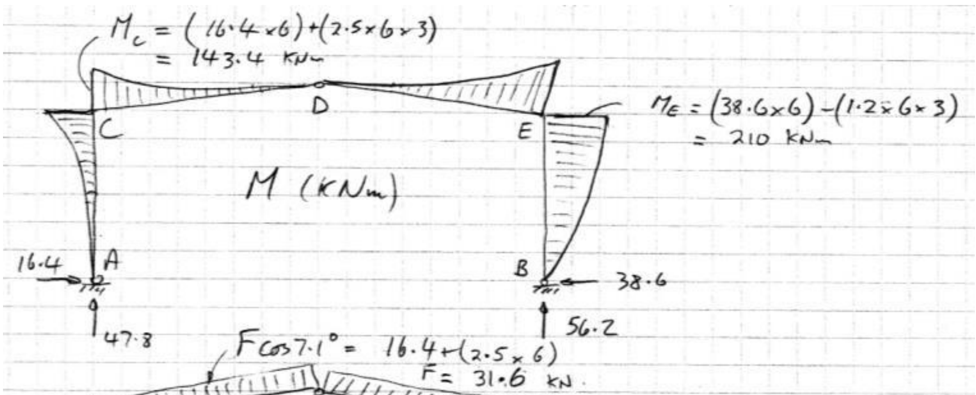
(6.5 x 8 x 4) +(R_{BH} x 7)	=	(56.2 x 8) +(1.2 x 6 x 4)
R_{BH}	=	38.6 kN

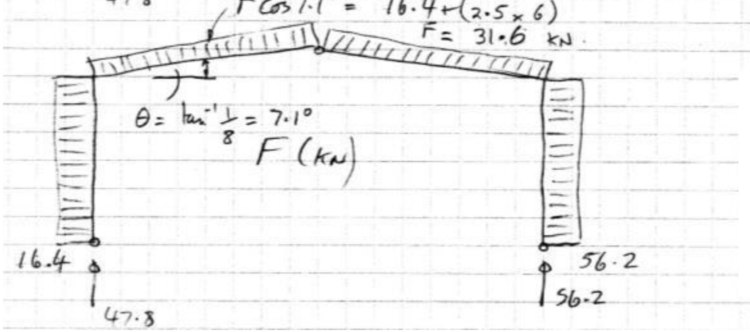
ΣV =0:

6.5 x 16	=	56.2 + R_{AV}
R_{AV}	=	47.8 kN

ΣH =0:

(2.5 x 6) +(1.2 x 6) + R_{AH}	=	*38.6*
R_{AH}	=	16.4 kN

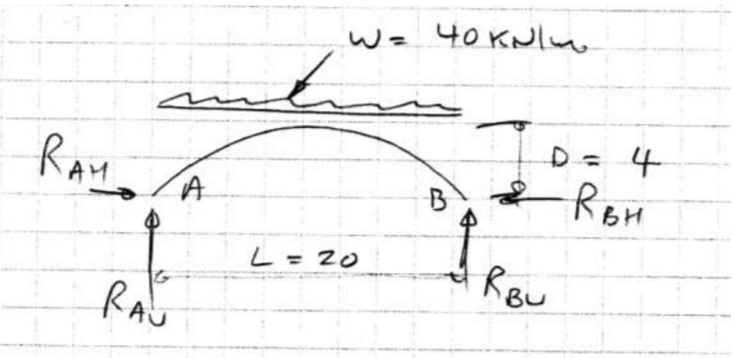




Design values:

M_{Rd}	=	**<u>210 kNm</u>**
N_{Rd}	=	**<u>56.2 kN</u>**

E12.4



Using *equations 6.1 to 6.3*

Reactions R_{AV} = R_{BV} = $wL/2$ = 40 x 20/2 = **<u>400 kN</u>**

Reactions R_{AH} = $-R_{BH}$ = $wL^2/8D$ = 40 x 20^2/(8x4) = **<u>500 kN</u>**

Maximum force in arch = $\sqrt{(400^2 + 500^2)}$ = **<u>640.3 kN</u>**

Approximate area required = 640.3 x 10^3/150 x 10^2 = 42.7 cm^2

Try 152 x 152 x 37 Universal Column (i_z= 3.87 cm, A = 47.4 cm^2)

Non-dimensional slenderness ratio, λ = 2000/(86 x 38.7) = 0.60

From *figure 7.38* χ = 0.76

N_{Rd} = 0.76 x 47.4 x 275 = 991 kN>640.3

Try a smaller size:

Try 152 x 152 x 30 Universal Column (i_z= 3.82 cm, A 38.2 cm^2)

Non-dimensional slenderness ratio, λ = 2000/(86 x 38.2) = 0.61

From *figure 7.38* χ = 0.75

N_{Rd} = 0.75 x 38.2 x 275 = 788 kN>640.3✓

<u>Use 152 x 152 x 30 Universal Column</u>

Chapter 13 – Foundations and retaining walls

E13.1

Total unfactored load = 350 + 275 = 625 kN

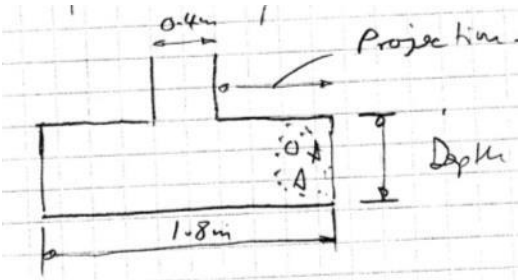
table 13.1 - stiff clay, bearing pressure, σ_{gd} = 200 kN/m^2

Area of foundation = 625/200 = 3.125 m^2

For a square, width = √3.125 = 1.77 m say 1.8 m

Projection = 0.9 -0.2 = 0.7 m

From *section 13.5.2* Depth, say = 0.75 m



E13.2

For reinforced concrete pad use same plan dimensions as above but only 300 mm deep.

Design column load, N_{Rd} = (1.35 x 350) + (1.5 x 275) = 885 kN

Ground pressure, q = 885/1.8^2 = 273 kN/m^2

Design moment, M_{Rd} = 0.5 x 273 x 1.8 (1.8/2 -0.4/2)^2 = 120 kNm

Estimate 12 mm dia. Bars
Effective depth, d = 300 – 40 -12 -6 = 242 mm

K = M/bd^2f_{ck} = 120 x 10^6/(1800 x 242^2 x 30)
= 0.04

From *figure 7.47* z/d = 0.95
z = 0.95 x 242 = 230 mm

A_s = M/f_yz = 120 x 10^6/(500/1.15 x 230)
= 1200 mm^2

Number of 12 mm bars = 1200/113 = 11

1.5(c + 3d) = 1.5(0.4 + 3 x 0.242) = 1.69 m < 1.8 m
∴ put 2/3 of bars in middle band

Width of band = $C + 3d$ = 0.4 + (3 x 0.242) = 1.13 m
Number of bars in band = 2/3 x 11 = 7
Spacing within band = 1130/6 = 188 mm

But spacing if evenly spaced = (1800-80)/10 = 172 mm
∴ Use even spacing

Shear across pad

Shear force, V_{Ed} = $q \times L \times L/2 - C/2 - d)$ = 273 x 1.8 x (0.9 – 0.2 – 0.242)
= 225 kN

Shear area = $L \times d$ = 1800 x 242 = 435 600 mm^2

Shear stress, v_{Ed} = $V_{Ed}/(L \times d)$ = 225 x 10^3/435 600 = 0.52 N/mm^2

100 A_s/bd = (100 x 1200)/(1800 x 242) = 0.28

From *table 7.2* $v_{Rd,c}$ = 0.54 > 0.52 –∴ no shear reinforcement required.

Punching shear

Check at the face of the column:

$$\text{Shear stress, } v_{Ed} = \frac{q(L^2 - C^2)}{4Cd} = \frac{273(1.8^2 - 0.4^2)\times 10^{-3}}{4\times 0.4\times 0.242}$$

= 2.17 N/mm^2

From *table 7.3* $v_{Rd,max}$ = 5.28 N/mm^2 > 2.17 N/mm^2 ✓

Check at a = d:

$$\text{Shear stress, } v_{Ed} = \frac{q(L^2 - 4Cd - \pi d^2)}{(4C + 2\pi d)d}$$

$$= \frac{273(1.8^2 - 4\times 0.4\times 0.242 - \pi\times 0.242^2)\times 10^{-3}}{(4\times 0.4 + 2\times\pi\times 0.242)\times 0.242}$$

= 0.961 N/mm^2

From shear across pad $v_{Rd,c}$ = 0.54 N/mm^2
$v_{Rd,}$ = 0.54 × 2d/d
= 1.08 N/mm^2 > 0.961 N/mm^2 ✓

***Check at a* = 2*d*:**

Shear stress, $v_{Ed} = \dfrac{q(L^2 - 8Cd - 4\pi d^2)}{4(C + \pi d)d}$

$= \dfrac{273\left(1.8^2 - 8\times0.4\times0.242 - 4\times\pi\times0.242^2\right)\times10^{-3}}{4(0.4+\pi\times0.242)\times0.242}$

$= 0.413\ \text{N/mm}^2$

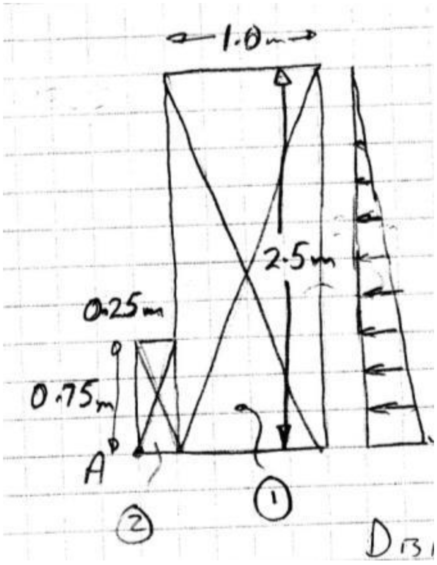
From above $v_{Rd,c} = 0.54\ \text{N/mm}^2$

$v_{Rd} = 0.54 \times 2d/2d$

$= 0.54\ \text{N/mm}^2 > 0.413\ \text{N/mm}^2$ ✓

Proposed dimensions and reinforcement are satisfactory

E13.3



ΣM about A:

Item	Weight (kN)	Distance (m)	Moment (kNm)
1	1.0 x 2.5 x 24 = 60.0	0.750	45.0
2	0.25 x 0.75 x 24 = 4.5	0.125	0.5
	64.5		45.5

Distance from A to centre of mass = 45.5/64.5
= 0.705 m

Distance from base centre = 0.705 – 0.625
= 0.080 m

Soil pressure at base = $K_a\gamma_s z$ = 0.33 x 20 x 2.5 = 16.5 kN/m^2
Active force = 16.5/2 x 2.5 = 20.63 kN
Overturning moment = 20.63 x 2.5/3 = 17.2 kNm
Eccentiicity, e_r = 17.2 – (64.5 x 0.08)/64.5 = 0.187 m
Distance to edge of mid. 1/3rd = 1.25/6
= 0.208 m>0.187 m
i.e. resultant is within middle third

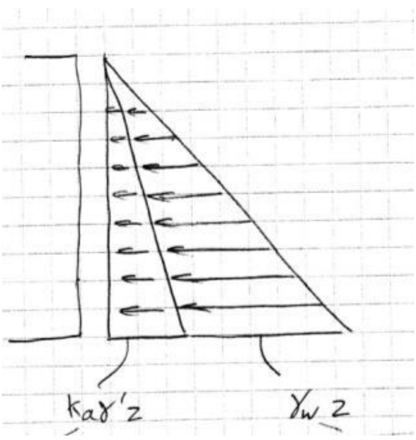
σ_{max} = $F/A + Fe_r/W_{el}$ = 64.5/1.25+64.5 x 0.187/(1 x 1.25^2/6)
= 51.6 + 37.1 = **88.7 kN/m^2**

Sliding:

Active force = 20.63 kN
Friction = 64.5 x 0.6 = 38.7 kN
Passive resistance force = $K_p\gamma_s z^2/2$ = 3 x 20 x 0.5^2/2 = 7.5 kN
Factor of safety = (38.7 + 7.5)/20.63 = **2.24**

E13.4

Pressure behind wall becomes:



$K_a\gamma' z$ = 0.33 x (20 – 9.81) x 2.5 = 3.36 kN/m^2

$\gamma_w z$ = 9.81 x 2.5 = 24.53 kN/m^2

Total pressure = 3.36 + 24.53 = 27.89 kN/m^2

Force = 27.89/2 x 2.5 = 34.9 kN

Moment = 34.9 x 2.5/3 = 29.1 kNm

E_r = 29.1 – (64.5 0.08)/64.5 = 0.37 m > 0.208 m
i.e outside middle third

$x = d/2 - e_r = 0.625 – 0.37 = 0.255$

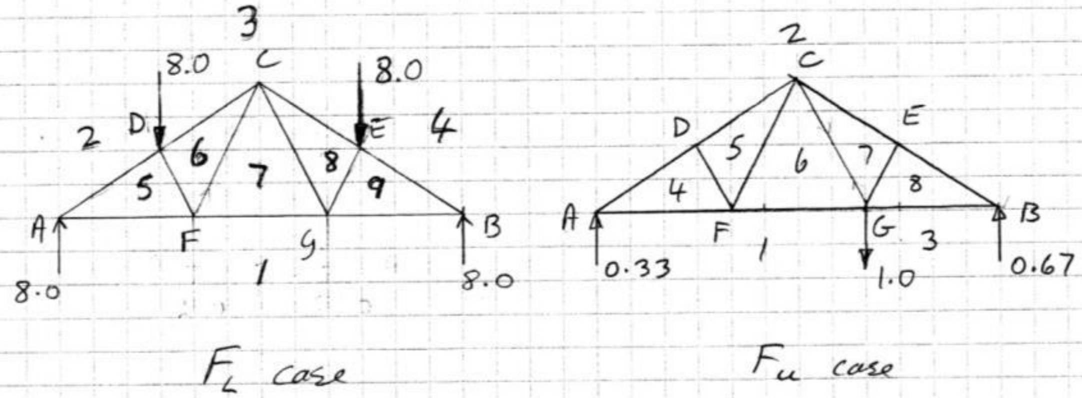
$\sigma_{max} = 2 \times 64.5/(3 \times 0.255) =$ **169 kN/m^2**

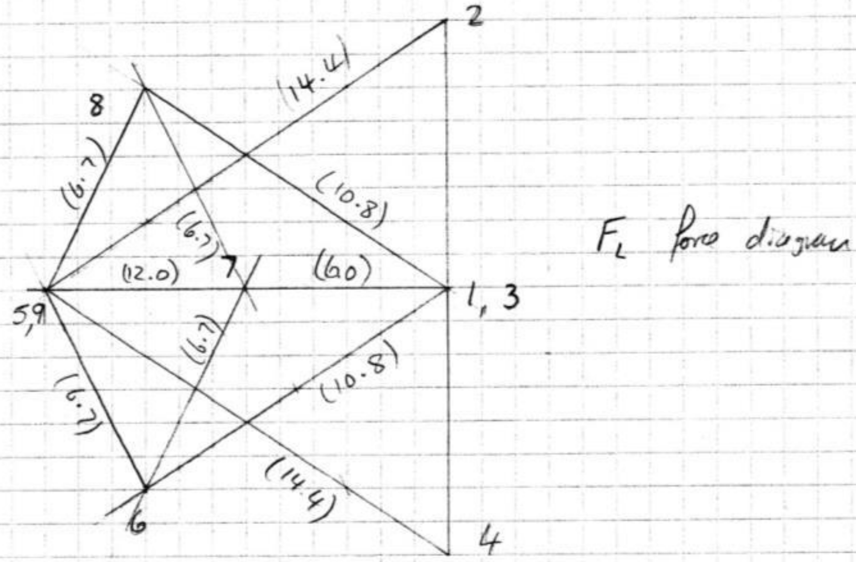
Sliding:

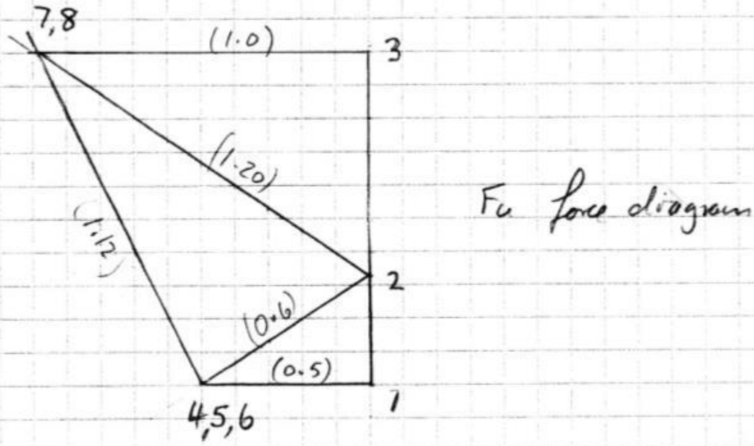
Factor of safety = (38.7 + 7.5)/34.9 = **1.32**

Chapter 14 – Deflection

E14.1





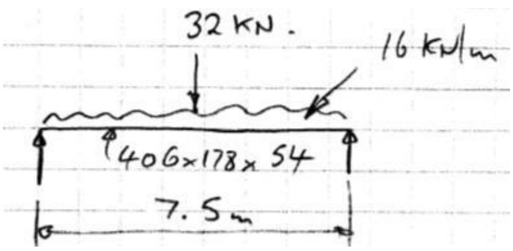


E = 7000

Member	L (mm)	A (mm^2)	F_L (N)	$\delta = \frac{F_L L}{EA}$ (mm)	F_u	$F_u\delta$
AD	3606	5000	14 400	-1.48	-0.60	0.89
DC	3606	5000	10 800	-1.11	-0.60	0.67
CE	3606	5000	10 800	-1.11	-1.20	1.33
EB	3606	5000	14 400	-1.48	-1.20	1.78
BG	4000	5000	12 000	1.37	1.00	1.37
GF	4000	5000	6 000	0.69	0.50	0.35
FA	4000	5000	12 000	1.37	0.50	0.69
DF	2236	**5000**	6 700	-0.43	0	0.00
FC	4472	5000	6 700	0.86	0	0.00
CG	4472	5000	6 700	0.86	1.12	0.96
GE	2236	5000	6 700	-0.43	0	0.00
					Σ =	**8.04**

Deflection = 8.04 mm

E14.2



from *appendix:*

I = 18 600 cm^4
E = 205 000 N/mm^2

Use **superposition:**

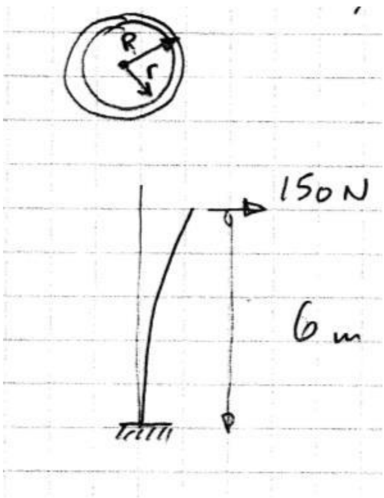
Deflection, $\delta = \frac{5wL^4}{384EI} + \frac{PL^3}{48EI}$

$= \frac{5\times16\times7500^4}{384\times205000\times18600\times10^4} + \frac{32\times7500^3}{48\times205000\times18600\times10^3}$

= 17.3 + 7.4

= **24.7 mm**

E14.3



Aluminium, $E = 70\,000$ N/mm^2 - *table 3.3*

2[nd] mom. Area, I $= \pi R^4/4 - \pi r^4/4$ - *figure 7.31*

$= \pi/4\,(60^4 - 58^4) = 1.29 \times 10^6$ mm^4

$\delta = PL^3/3EI$

$= 150 \times 6000^3/(3 \times 70\,000 \times 1.29 \times 10^6)$

$=$ **<u>120 mm</u>**

E14.4

Table 3.1 $A_s = 3 \times 491 = 1473$ mm^2

Effective depth, d = 500 – 40 - 10 – 12.5 = 437.5 mm

% reinforcement = 100 x 1473/(300 x 437.5) = 1.12%

Table 14.1 by interpolation:

Basic span/depth ratio = 6.76

Maximum span = 6.76 x 437.5 = 2960 mm say **<u>3.0 m</u>**

Chapter 15 – Indeterminate structures and computers

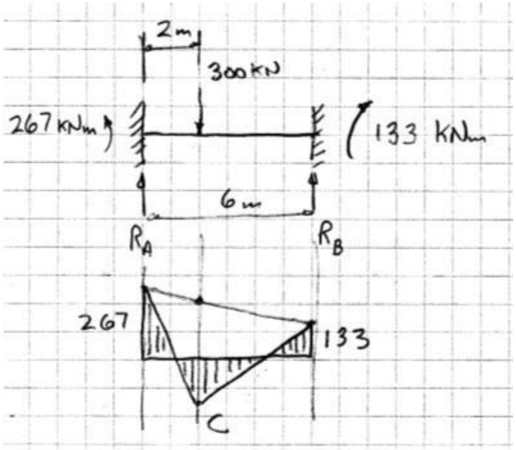
E15.1

a. Each crossed member (2) adds a redundancy plus additional internal roller supports (2) – **answer – 4.**

b. Simple cantilever is statically determinate so internal roller supports add two redundancies - **answer – 2**

c. A Three-pin portal is statically determinate so making two pins into rigid joints adds two redundancies - **answer – 2**

E15.2



ΣM about A:

$-267 + (300 \times 2) + 133 - 6R_B = 0$

$R_B = 78$ kN

`

$\Sigma V = 0$

$R_A = 300 - 78$

$= 222$ kN

$M_C = 267 - (222 \times 2)$

$= 177$ kNm

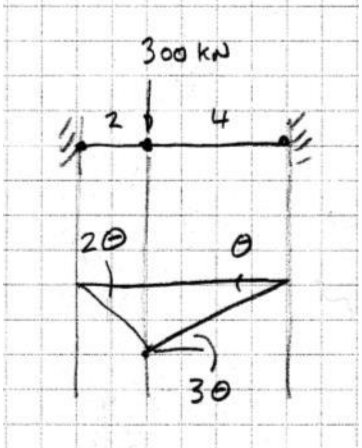
$\therefore$ Design moment, $M_{Ed} = 267$ kNm (i.e. biggest moment on beam)

$W_{pl} = 267 \times 10^6 / 275 \times 10^3$ ($\gamma_M = 1.0$)

$= 971$ cm^3

Use 406 x 178 x 54 kg/m universal beam ($W_{pl} = 1050$ cm^3)

E15.3



$M_P(2\theta + 3\theta + \theta) = 300 \times 4\theta$

$6M_P = 1200$

$M_P = 200$ kNm

$W_{pl} = 200 \times 10^6 / 275 \times 10^3$ ($\gamma_M = 1.0$)

$= 727$ cm^3

Use 406 x 140 x 39 kg/m universal beam ($W_{pl} = 724$ cm^3)

% reduction = (54-39)/54 x 100 = **27.8%**