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Introduction

You may be wondering what mathematics has to do with economic analysis. Like many students you have a serious interest in studying economics and understanding how economics and economic analysis contribute to both microeconomic and macroeconomic activities. As we shall see throughout this text, serious students of modern economic analysis need a number of essential mathematical skills and techniques. Such skills and techniques are necessary to allow you to properly understand economic theory, economic behaviour and modern economic analysis. Let's consider the following scenarios.

Scenario 1

Following the banking crisis that began in 2008, particularly in the US and the UK, a number of governments had to put considerable emergency funding into their banking system to support banks and other financial institutions that were close to collapse. As a result, government expenditure and therefore borrowing increased dramatically. The Finance Minister has now decided that the government deficit (the difference between what the government collects in taxes and what it spends) needs to be cut back significantly to bring the government budget more into balance. However, the Minister is concerned about the impact that reducing government spending will have on particular sectors of the economy. Reducing government expenditure has consequences for the firms who supply the government with goods and services, for their employees, for their shareholders and often for the wider community as well. One sector in particular is under serious scrutiny: the government is thinking of cancelling a couple of major naval shipbuilding contracts. These defence cuts would initially save the government a good deal of money. However, the shipbuilding companies would be badly affected and would have to reduce their workforce considerably. In turn this would lead to a loss in tax revenue for the government and increased welfare payments for those who lost their jobs. The Minister has asked for your economic analysis of the overall impact of such a decision.

Scenario 2

You have been approached by both easyJet and Ryanair, two highly successful budget airlines operating in Europe. They're concerned that the European Union is considering imposing additional taxes on passengers who book short-haul flights by adding the tax to the ticket price charged by the airline. The declared purpose behind such taxes is to encourage passengers to switch to more environmentally friendly transport (such as electric trains) by making short-haul air travel more expensive, and thereby to reduce the carbon footprint of travel and contribute to a reduction in global warming. Once again, you've been asked to undertake an economic analysis to assess the impact that such a tax would have on their businesses. How will such a tax affect demand for airline travel? How will it affect airline revenue? How will it affect their profitability?

Scenario 3

The government is looking for ways of increasing both its tax revenue and its popularity with the public – not a combination that's easy to achieve. One option is to capitalize on the current unpopularity of senior executives in the banking sector among the public. There was considerable criticism that senior executives were awarded very large performance bonuses at the time when banks were struggling financially and had to be financially supported by the government. The government is now thinking of introducing a special tax on bankers' bonuses. It's looking for your economic advice as to what level of tax it should introduce in order to maximize the amount of tax revenue it collects in this way.

Scenario 4

As part of its economic growth strategy, the South African government is looking for ways to help small businesses start up and expand. It's thinking of encouraging the central bank (the South African Reserve Bank) to increase the money supply in the hope that this will reduce the interest rate that businesses borrowing money have to pay. Generally, an increase in the money supply in an economy makes it cheaper and easier to borrow money. The proposed measure is expected to increase the demand from firms for investment funding and so stimulate economic growth as firms borrow more money to help them expand their economic activities. What economic advice can you give on the impact such a policy would have on the economy?

These scenarios are all realistic – and real. They illustrate the situations that economists and economic analysis are frequently involved in. Some of the scenarios involve analysing and assessing what will happen at a *microeconomic* level – at the level of individual markets, organizations or people. Some of the scenarios involve analysing and assessing at the *macroeconomic* level – at the level of the whole economy or some part of it. At both the micro- and macro-levels it's likely that you will need to do a number of things. First, you will need to analyse each scenario in order to establish the general economic impact that we would expect to happen – that is, using economic theory to predict in general what economic changes are likely to occur. For example, in the easyJet/Ryanair scenario you want to be able to explain in general the effects that introducing a tax would have regardless of the precise value of the tax. Second, you want to quantify the exact effect at the micro- or macroeconomic level – in other words, be able to accurately predict for easyJet/Ryanair the impact of a specific tax level on their business. This combination of understanding the *general* effects of an economic change as well as being able to quantify the *exact* effects of a specific change is important in economics.

Through our economic analysis we want to be able to understand the general principles at work in a given scenario as well as the specific details. For example, you wouldn't want to be in a situation where you analysed the impact of, say, a €10 tax on each flight and then have to repeat the analysis if the tax changed to €15 or then do it all again because the tax was now going to be €20. What you do need to be able to do is to assess the impact no matter what the exact tax might be. And this is where mathematics comes into economic analysis.

1.1 The need for mathematics in economics

We begin with a bold statement: in order to develop a comprehensive understanding of both economic theory and economic analysis you need a detailed understanding of key mathematical principles and of the role that mathematics can play in the study of economics. You may have opened this text on mathematics and economics with a degree of uncertainty, being unsure what to expect, and possibly even some concern. As part of your studies of economics you may well have been surprised to realize that it is necessary to undertake a formal course in mathematical economic analysis (often under the name of applied economics, quantitative economics or similar) and that the use of mathematics in economics is more widespread than you realized. It may also be the case that the prospect of having to recollect and use key mathematical principles and skills acquired at school is not one that fills you with much enthusiasm. Mathematics in general has a poor reputation with many students, who simply cannot see its relevance in the real world.

However, it is an inescapable feature of the serious study of economics that you need to be familiar and comfortable with key mathematical methods and you need to develop the skills necessary to apply such methods to the economic models that you will gradually build and explore. But it's important for us to stress from the very beginning that this text is *not* a text on mathematical economics as such but rather on the *use* of mathematics in economic analysis. You may be forgiven at this stage in your studies for wondering what the difference is and whether it really matters. In this textbook our main focus is on:

- Seeing how mathematics is used and the value it adds to economic analysis
- Helping you develop your own skills in using such mathematics to improve your own economic analysis
- Increasing the level of your own mathematical confidence
- Developing your awareness of the widespread and typical uses of mathematics in economics.

As you work through the material in this book you will gradually recognize that mathematics need not be viewed as a discipline separate from economics but, rather, one that can be used in an integrated way to help develop economic models and economic theory. We stress again, however, that the purpose of this text is not to turn you into a mathematician but to allow you to develop the mathematical skills and knowledge that you will require as an economist.

There are a number of reasons why the use of mathematics in economics has steadily increased over the years. One important reason is that mathematics is a useful tool in the study of economics. While it is possible to undertake some limited economic analysis by relying on verbal analysis and logic without much use of mathematics, an appropriate use of mathematical notation and solution methods can make life much, much easier for the economist.

Let's return to the easyJet scenario. As a first step in trying to assess the impact that an EU tax might have on passenger numbers, we might consider thinking about the factors that would affect the number of people flying on one of easyJet's routes, say from Edinburgh to London. At this stage we do not need to know the precise effects, only to grasp what might influence the number of passengers choosing to fly this route. In standard economic terms, we'd say we want to identify the key factors affecting the demand for seats on this route.

Progress Check 1.1

Before reading on, take a few minutes to list the main factors you think could affect the demand for seats on this route.

Applying basic common sense, and possibly some personal experience, you may have said that the demand for airline seats on this route will depend on a variety of factors: the price charged by easyJet for the seat, the price charged by its competitors, such as British Airways, who also fly this route, the cost of alternative travel such as the rail fare from Edinburgh to London, or people's income levels. You might have suggested other factors as well. In other words, we can build up a *verbal* picture of the economic situation. But, as we all know, words can sometimes get in the way of understanding, with different people reading different things into a particular phrase, and it can sometimes take a lot of words to describe a relatively straightforward situation. A much more concise and unambiguous way of summarizing such an economic situation is provided through simple mathematical notation, such as:

$$Q_d = b_1P + b_2P_C + b_3P_A + b_4Y \tag{1.1}$$

where we use letters to stand for some of the factors we've thought of. Here we use:

Q_d = the total number of easyJet seats people want to buy on this route (often referred to as the *quantity demanded* and pronounced 'queue dee')
P = the price of easyJet's airline seats on this route
P_C = the price of competitor airlines' seats on this route
P_A = the price of alternative travel such as railways
Y = level of consumer income.

In economics we refer to these factors as *variables* – since they vary or change according to the economic situation under consideration. We use other letters (b_1, b_2, b_3, b_4) to represent what are called the *parameters* of the relationship (frequently these are the specific numerical values that are appropriate to the particular economic relationship). Notice also that we show Eq. 1.1 using *italics*. There is no particular reason for this other than the fact that it makes them stand out in the text and so helps you to realize that they are referring to parts of a mathematical expression.

Such a mathematical presentation offers a number of advantages to the economic analyst, once you get used to them. First, to those who understand the mathematical symbols used, the use of mathematical notation to describe such economic relationships provides a definitive and unambiguous statement of the relationship. A purely verbal description of an economic relationship is more prone to misinterpretation and confusion than a mathematical one. It is for this reason that relationships such as the example above are shown in mathematical terms. However, not only can we use mathematics to describe such a relationship, but we can also apply mathematical reasoning and logic. Mathematics is a particularly powerful tool in enabling us to make logical *deductions* about economic behaviour patterns. In the above example an economist with the appropriate mathematical understanding can work out the effect of, say, a change in consumer income on the quantity demanded of the product under the critical assumption that the other factors in the equation remain unchanged. This is a very common assumption used in economic analysis and one that we'll use often. If we wanted to work out the general effect on Q_d (demand for easyJet seats on this route) if Y (consumer income) changed, then the only way we can do this is to make the assumption that all the other factors stay exactly as they are. If we did not make this assumption but allowed other factors to change at the same time, it would be impossible to work out what was causing Q_d to change. Economists refer to this assumption using the phrase 'other things being equal' or with the Latin expression '*ceteris paribus*' (pronounced 'ketter-iss parry-bus'), which literally means 'with other things the same' or 'other things being equal'.

Progress Check 1.2

Suppose British Airways lowers its prices on this route. Other things being equal, what effect would you expect this to have on demand for easyJet seats?

Given that the expression b_2P_C in (1.1) is used to show the effect of competition on Q_d, what numerical value would you expect b_2 to take: negative, positive or zero?

Other things being equal, it seems reasonable to assume that if BA increase their prices, easyJet's prices will appear cheaper and so more attractive to the customer. In other words, we would expect an increase in demand for easyJet seats as a result of an increase in BA prices. This suggests that the numerical value for b_2 would be positive – an increase in a competitor's price, P_C, would lead to an increase in Q_d. This is our first mathematical economic analysis. You may also have worked out that we'd expect b_1 to be negative – if easyJet themselves charge a higher price then we'd expect this to have a negative effect on demand; we'd expect b_3 to be positive since, again, an increase in the prices of alternative forms of travel is likely to boost demand for air travel; b_4 is slightly less clear but we'd probably conclude that, if people have more income to spend, they'd probably travel more so we might think that b_4 would be positive also.

These examples illustrate how mathematical economic analysis can help us in the scenario outlined at the beginning of the module where the EU was considering imposing a tax on the price charged for short-haul flights. Although we've only just started looking at mathematical economic analysis we can use expression (1.1) to work out that such a tax would affect P and P_C but not P_A and Y. In other words, the effect of the tax on passenger demand, Q_d, would come through the impact of a higher price that easyJet would have to charge, P, and through the impact of the higher price its competitors would also have to charge, P_C – assuming of course that all airlines were charged the same tax. But we also see that the two effects might counterbalance each other to some extent. The effect on Q of a higher value for P will be negative (through b_1), so easyJet will lose passengers thanks to the higher price they have to charge because of the tax imposed. On the other hand, airline competitors will also have to increase their prices and, as we already know through b_2, this would have a positive effect on easyJet demand – increasing passenger numbers, other things being equal. So, on the one hand easyJet would lose passengers and on the other would gain passengers. What would the net effect be? In part this would depend on the exact numerical values taken by b_1 and

b_2 (which of course we don't know). Although we don't have exact numerical values for the two parameters it's clear that there are three possibilities if we ignore the positive and negative signs:

- b_1 is bigger than b_2
- b_1 is the same as b_2
- b_1 is less than b_2.

Progress Check 1.3

Look at each of the three possibilities in turn. Overall, would easyJet lose passengers or gain passengers for each of these possibilities?

The three possibilities effectively show how competitive easyJet's prices are relative to those of its airline competitors. In the first possibility it would lose more customers than it gains. In the second possibility the gains and losses would leave it as it is. In the third possibility it would win more than it lost. You have now completed your second mathematical economic analysis.

An important point to note at this stage is that the relationship we've been looking in Eq. 1 is expressed mathematically but contains no actual numbers. This is a common misunderstanding of the role of mathematics in economics. Of course, there are frequent occasions when we wish to use specific numerical values in such an equation. A business organization – easyJet for example – would wish to obtain precise and accurate forecasts of quantity demanded given specific values for the other variables in the equation. From the viewpoint of studying economic principles and theory, however, such number values are frequently irrelevant. Economists are often concerned with establishing key principles of economic behaviour – independently of whatever specific numbers happen to be appropriate. They might wish to work out, for example, the general principles of individual consumer behaviour if income changes. They might want to understand how firms would react if their labour costs increased. They might want to work out how both consumers and firms would react if interest rates increased. Accordingly, in this book we shall frequently be using general mathematical notation to establish general conclusions about economic behaviour. Naturally, we shall also be illustrating such important deductions with specific numerical values, although these are generally used primarily as an aid to understanding. In the real world considerable effort and attention is paid to obtaining and using such numerical values. This is the area known as *econometrics* or *econometric analysis* – another important and related area of economics.

1.2 Economic theory, economic models and mathematics

This leads us to another important area: the link between economic theory, economic models and mathematics. In economics we typically begin by observing something that's happening in a certain section of the economy. How much are consumers paying for the latest Apple smartphone? How are firms responding to the changes in the currency exchange rate? How is the energy industry responding to the latest government incentives to invest in green energy production? Using the example we have been using so far, we might observe that a particular level of demand for easyJet flights occurs. The economist will ask why this product was purchased by consumers and why this particular quantity of the product at this price. Typically, we will then try to develop a theoretical explanation of this observed economic behaviour (which is what we provided in Eq. 1.1). Such a theoretical explanation will generally involve the construction of an economic *model* (again, as we have provided in Eq. (1.1)). Other professions use models in their work. An architect may create a scale model of a new building so that people can see what it will look like. An engineer may use a model to help with the design of a new aeroplane so that he can see how changes in design may affect the aeroplane's performance.

In economics, we use models to help us understand various aspects of economic behaviour. There is no particular reason why an economic model has to be mathematical or why the underpinning theory needs to be expressed in mathematical terms. Indeed, much early economic thinking did not make use of mathematics as such. However, as we have seen, there are factors that may strongly encourage us to make use of mathematics in the model-building process. In addition, if the model is mathematical it will involve an equation (or equations) linking certain economic variables together. Typically, we will then wish to examine the model in a mathematical manner. This will involve:

- Setting out the key assumptions on which the model is built
- Using these assumptions to examine the logical deductions to be obtained from the model
- Reaching conclusions about predicted economic behaviour
- Comparing our conclusions with actual economic behaviour.

Naturally, such a process is not usually as simple as it first appears. The whole process, in fact, will be iterative: we specify key assumptions, make logical deductions, reach conclusions and then we may find that the conclusions derived from the model are inconsistent with observed economic behaviour. We then have to return to the model for further development and refinement until we are satisfied that the model provides a reasonable explanation of the observed economic phenomenon (or until we abandon this theory because of its repeated failure to provide such an explanation). Mathematics in economics, therefore, is primarily concerned with the application of mathematical principles and logic to the theoretical aspects of economic analysis. Frequently, the next stage is a rigorous empirical investigation of the theory that has been developed thus far.

At this stage, econometrics comes into play. Econometrics is primarily concerned with the measurement of economic data and economic relationships. Using both mathematics and the principles of statistical inference, econometrics seeks to empirically evaluate a theoretical economic model. In this book we are not concerned with econometrics or indeed with empirical evaluation of economic models as such, although we do need to be aware of its critical role in the process of economic analysis. Figure I.1 illustrates the process.

We must remember, however, that any economic model – whether mathematical or not – is a simplified representation of a far more complex real-world situation. The purpose of models in this context is to reduce these real-world complexities to a level that can be understood and analysed. By definition, a model restricts its attention to

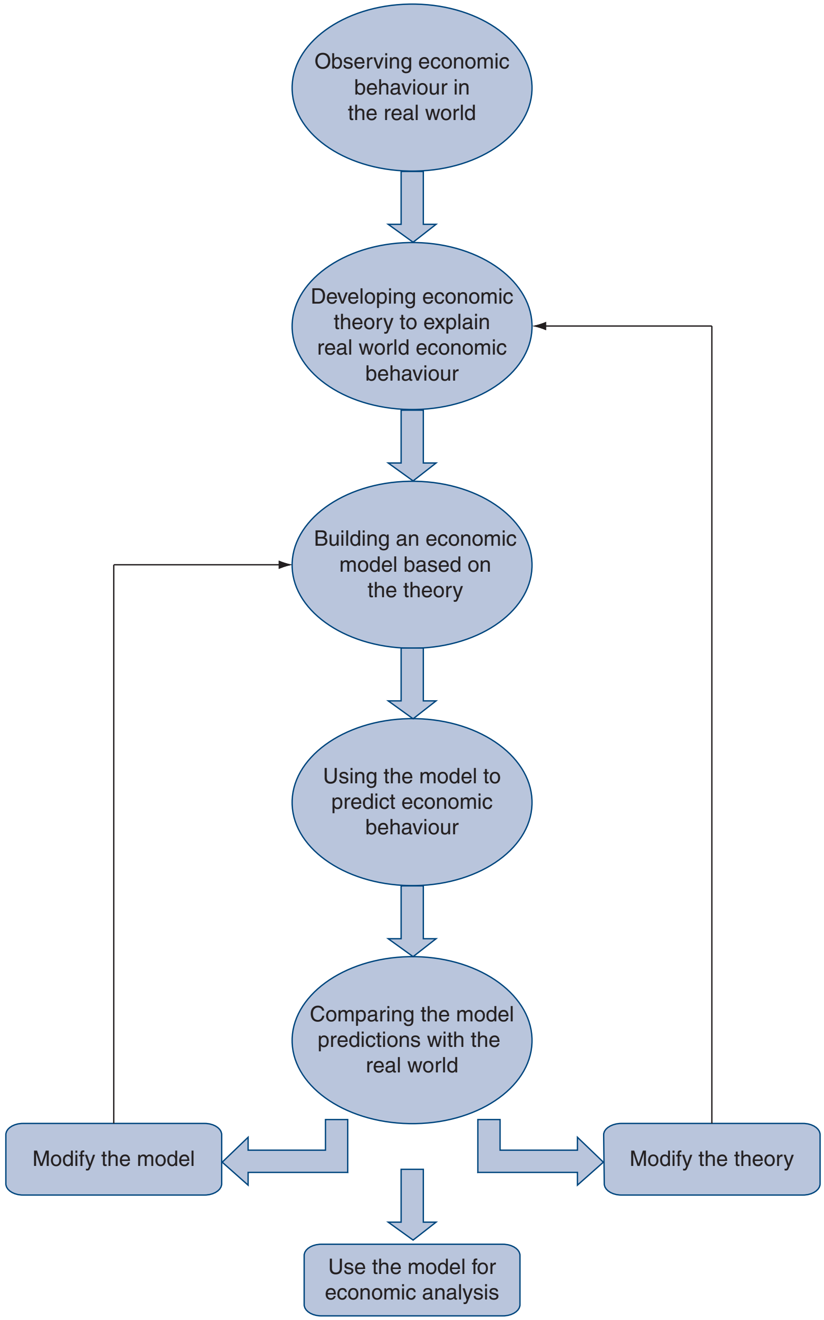

Figure I.1 Theory and models in economic analysis

what are seen to be the key features of the situation under investigation. So, in the context of our earlier example, there will be numerous factors influencing the quantity of a good that is purchased. An economic model will, however, focus on only a few of these factors – naturally, the ones thought to be most important in the context of the analysis. We did this earlier with Eq. 1.1 for easyJet.

1.3 Summary

We are now in a position to begin our investigation into the uses of mathematics in the study of economic analysis. Mathematics plays a critical role in providing economists with the logic and analytical tools needed to develop and investigate economic theories which are at the heart of economics and the study of economic behaviour. Without an adequate understanding of mathematics and its role in economics your career in this subject area will be severely curtailed. By the time you reach the end of this text we're convinced that your knowledge and appreciation of the usefulness of mathematics to the economist will have undergone a fundamental change.

Exercises

1.1 Earlier in this module we looked at the idea of a demand situation – considering the factors that will affect demand for a product. Consider the other side of the picture: the quantity of the good supplied by the individual firm. What variables do you think we would wish to link with the quantity supplied? What numerical values do you think each of these would take? Try using easyJet as an example again.

1.2 How do you think such numerical values could be obtained in practice?

1.3 Consider a variable, C, which represents the annual expenditure (consumption) of a particular individual. What variables do you think would influence consumption? Assess whether you would expect each variable to have a positive or a negative influence.

1.4 Consider an individual's consumption of a particular good – coffee. Identify a set of variables that you feel would influence such consumption and develop a simple model for determining such consumption. Consider the assumptions you are making – explicitly and implicitly – in your model. Practically, how could you test how good your model was?

1.5 In the context of Exercise 1.4 consider the annual national import of coffee. What variables do you think would influence imports of this good? How does your list of variables compare with that of Exercise 1.4 and how do you explain the differences?

Section A
The building blocks of economic analysis

In the introductory module we hope we convinced you that being able to use mathematics in economic analysis is essential if you want to study economics seriously. In this section we provide the necessary building blocks to enable you to understand and to start using mathematics and economic analysis effectively together. The section is divided into three modules.

Module A1 Tools of the trade: the basics of algebra

This module provides a refresher and reminder of the key principles of mathematics, in particular algebra and algebraic calculations, which are essential for the topics that follow.

Module A2 Linear relationships in economic analysis

This module looks at how we can express economic relationships using the simplest form of mathematics, which involves linear, or straight-line, equations. We'll look at the idea of functions and show how to use graphs to illustrate linear economic relationships.

Module A3 Non-linear relationships in economic analysis

Finally, in this section we look at non-linear relationships in economics. While linear equations are useful and easy to use, they're often restrictive in terms of building an accurate economic model. Frequently, economic relationships need to be modelled in a non-linear way. In this module we look at non-linear equations and equations involving several variables.

You may already be familiar and comfortable with the material in some of these modules. To help you check whether or not it's worth your while reading a particular module, we've included a Knowledge Check activity towards the start of each module. This is a short activity that will help you work out if you already know about the material of that module. If you answer the Knowledge Check correctly we suggest you don't need to read that module but move straight on to the exercises at the end of the module as they'll give you extra practice at seeing the connection between maths and economics. If you find any of the exercises especially challenging you can always go back and read the relevant part of that module.

Module A1

Tools of the trade: the basics of algebra

This module reviews a number of the basic principles in algebra. As we work through the text, you will see that economic analysis makes a lot of use of algebra to support and develop economic theory and to reach conclusions about economic behaviour: how firms will respond to a tax change; how consumers will respond to a change in interest rates; how government will respond to a change in exchange rates and so on. As mentioned in the Introduction, one of the main benefits of using mathematics in economic analysis is to help us to deduce general economic conclusions without having to resort to specific numerical values. Algebra allows us to do this, and although some algebra procedures may at first seem more like black magic than reasoned economic logic, you will find that, with practice, such manipulations begin to make sense. It may be some time since you last had to use algebra, so this module is intended to refresh your memory. The material that follows will allow you to gradually develop your own skills with algebra. However, if you find that you are unable to follow some of the algebraic manipulations that take place later in the text, you can return to the appropriate part of this module to help you.

Learning Objectives

By the end of this module you should be able to:

- Use algebraic notation to show economic relationships
- Work with brackets
- Work with inequalities
- Work with fractions
- Transpose an algebraic expression.

Knowledge Check A1

To check how comfortable you are with algebra already, try solving these:

(i) $y = \dfrac{3x+3}{2x+5}$

Find an expression for x

Knowledge Check A1 *(Continued)*

(ii) Simplify the expression $7x/4x^2 - 8x/2x^3$

Check your answers in Appendix 2. If you got the correct answers try out some of the exercises at the end of the module for extra practice and then move to module A2.

And, if you've no idea what to do, read on. You will be able to do these by the time you complete the end of this module.

If you want to find out more about where algebra came from try http://en.wikipedia.org/wiki/Algebra

A1.1 Algebraic notation

We start by looking at how algebra can be used to show a simple economic situation. For example, individual consumers would normally distinguish between their *gross* income and their *disposable* income. Gross income would mean all the income that they had: what they earned in wages or salaries; what they received in interest on their savings; or dividend payments from shares they had bought in companies. In most economies, though, your gross income is not what you actually have to spend. Typically, there are compulsory deductions from your gross income, such as government tax on your salary/interest/dividends, compulsory payments into a health insurance scheme in case you fall ill and compulsory payments into a pension scheme for when you retire. Disposable income is that income the consumer has left to spend after any deductions (such as tax) have been taken from their gross income. We've already seen in the Introduction how we can use mathematical notation to help illustrate simple economic models, and we will do the same here. We'll use Y_g to refer to gross income and D to refer to all deductions and we'll use Y_{d} for disposable income. Using algebraic notation we would then write:

$$Y_d = Y_g - D \tag{A1.1}$$

That is, disposable income, Y_d, is simply gross income, Y_g, less deductions, D.

As a slight digression, it is worth knowing that certain economic variables, like income, conventionally tend to be shown algebraically using specific letters. For example:

Y is used for income
P for price
C for consumption
G for government spending

and so on. We'll do the same throughout this book. And in case you're wondering why we use Y for income and not I, it is because I is used to refer to investment. It is also common practice to use subscripts with a variable when there may be different versions of that variable. That's why we have Y_g and Y_{d}.

A1.2 Arithmetic in algebra

Even with such a simple expression as Eq. A1.1 it is clear that we can obtain two related expressions:

$$D = Y_g - Y_d \tag{A1.2}$$

$$Y_g = Y_d + D \tag{A1.3}$$

Eq. A1.2 indicates that deductions, D, are simply the difference between gross income, Y_g, and disposable income, Y_d, and Eq. A1.3 indicates that gross income is equal to disposable income plus deductions. While Eqs A1.2 and A1.3 are easily obtained using some simple logic it will also be worth exploring the algebraic arithmetic. These principles will be useful when we look at more complex expressions.

To find an expression from Eq. A1.1 where D equals some combination of the other two variables, we can rearrange Eq. A1.1 (or any other algebraic expression) by understanding that if *one side* of an algebraic expression is altered we keep the algebraic relationship exactly the same *as long as* we alter the *other side* of the expression in exactly the same way. This is an important rule in algebra and one that we will use a lot. From Eq. A1.1 we have:

$$Y_d = Y_g - D$$

The rule says that the algebraic expression remains unchanged in terms of the underlying relationship if we alter both sides of the expression in the same way. If we add D to each side we have:

$$D + Y_d = Y_g - D + D$$

and by simple inspection we see that the two Ds on the right-hand side will cancel each other out to give:

$$D + Y_d = Y_g$$

If we now subtract Y_d from both sides (which again leaves the relationship unchanged as both sides of the equation are treated in the same way) this gives:

$$D + Y_d - Y_d = Y_g - Y_d$$

where, again, the two Y_d terms on the left-hand side cancel each other out to give

$$D = Y_g - Y_d$$

It is important to realize that this equation and Eq. A1.1 are identical.

We've deliberately taken a detailed, step-by-step approach, but you won't always need to be as methodical because it soon becomes obvious how to use this type of arithmetic with algebraic expressions.

Progress Check A1.1

Using the algebra we've just shown, try the following examples yourself and then carry on reading the text.

Rearrange each of the following expressions so that you have an expression in the form $Y =$

(i) $5Y + 3X - 10 = 25$

(ii) $A - C = Y + 10 - B$

(iii) $6A = 4Y - 5C$

(iv) $0.2X - 0.75Z = 0.3Y + 1512$

We'll work through each of these in the next section, but try them yourself first.

Taking each in turn, we have:

(i) $5Y + 3X - 10 = 25$

Using the rule from earlier, we can add 10 to both sides to give:

$$5Y + 3X - 10 + 10 = 25 + 10$$

with the two 10s on the left cancelling each other out to give

$$5Y + 3X = 35$$

Next we can subtract $3X$ from both sides to get:

$$5Y + 3X - 3X = 35 - 3X$$

Again, the two $3X$s on the left cancel each other out, giving:

$$5Y = 35 - 3X$$

Finally, we can divide both sides by 5:

$$\frac{5Y}{5} = \frac{35}{5} - \frac{3X}{5}$$

Which, if we do the maths, gives:

$$Y = 7 - \frac{3X}{5}$$

Once again, it is important to remember that, although this equation and the one we started with look very different, they are in fact identical.

(ii) $A - C = Y + 10 - B$

We want to rearrange this to get Y onto the left-hand side of the equation and everything else on the right-hand side. We can do this in several different ways, but let's first add B to both sides to get:

$$A - C + B = Y + 10 - B + B$$

And, with the two Bs on the right-hand side cancelling each other out, we get:

$$A - C + B = Y + 10$$

Now we can subtract 10 from both sides, giving:

$$A - C + B - 10 = Y + 10 - 10$$

or

$$A - C + B - 10 = Y$$

If we now simply swap over the left-hand and right-hand sides we get:

$$Y = A - C + B - 10$$

You might have done this in a different order, but should still have reached the same result.

(iii) $6A = 4Y - 5C$

Add $5C$ to both sides:

$$6A + 5C = 4Y - 5C + 5C$$

$$6A + 5C = 4Y$$

We now divide both sides by 4:

$$\frac{6A}{4} + \frac{5C}{4} = \frac{4Y}{4}$$

Simplifying and switching both sides gives:

$$Y = 1.5A + 1.25C$$

(iv) Finally, we had $0.2X - 0.75Z = 0.3Y + 1512$

Subtracting 1512 we obtain:

$$0.2X - 0.75Z - 1512 = 0.3Y$$

Dividing through by 0.3 we obtain:

$$\frac{0.2X}{0.3} - \frac{0.75Z}{0.3} - \frac{1512}{0.3} = Y$$

Rearranging and simplifying gives:

$$Y = 0.67X - 2.5Z - 5040$$

The last example did not show all the detailed steps and calculations, but you should be able to follow what's happening.

A1.3 Brackets in algebra

The use of brackets in algebra is quite common, and we need to be familiar with how to use them. Let's go back to Eq. A1.1 where we had:

$$Y = S - D$$

Let's now define D, deductions, as:

$$D = f + tY_g \qquad \text{(A1.4)}$$

where f is a fixed amount deducted from each person's gross income while t is a proportionate tax (expressed as a decimal) deducted from gross income Y_g. For example, suppose the government taxes everyone €100 and also sets income tax at 25% of gross income; f would be 100 and t would be 0.25, implying that deductions would be a fixed sum of €100 regardless of actual income plus 25% of gross income earned. We can now substitute Eq. A1.4 into Eq. A1.1:

$$Y_d = Y_g - D$$

$$Y_d = Y_g - (f + tY_g) \qquad \text{(A1.5)}$$

Eq. A1.5 could be simplified by removing the brackets and rearranging the expression. However, we must remember that we cannot simply remove the brackets from the expression to give:

$$Y_d = Y_g - f + tY_g$$

You should be able to see what is wrong with this expression. We should subtract *both f and Y_g* and not just f. This gives a simple rule that, if we wish to remove brackets from an expression, then *all* the terms within the brackets must have the same arithmetical operation performed on them. In Eq. A1.5, for example, we must multiply each term within the brackets by a negative sign (since this is the mathematical operator immediately before the bracket expression). This then gives:

$$Y_d = Y_g - f - tY_g$$

We then collect all the Y_g terms together (collecting the common terms together in this way is something we will frequently want to do in economics):

$$Y_d = Y_g - tY_g - f$$

We now have two Y_g terms. We can now rewrite this equation as:

$$Y_d = 1Y_g - tY_g - f$$

or rearrange it as

$$Y_d = (1 - t)Y_g - f \quad \text{(A1.6)}$$

If you look carefully at Eq. A1.6 you will note that it is the same as the previous equation. It may seem odd that we want to remove brackets first and then reintroduce them, but what we have been able to do with Eq. A1.5 is to derive an expression where similar terms appear together to help interpretation and evaluation of the expression. We can generalize the approach by saying that:

$$ab + ac = a(b + c)$$

where a is a term common to both parts. To see how this works let's go back to where we had:

$$Y_g - tY_g$$

or, as we wrote it

$$1Y_g - tY_g$$

The common term here is Y_g, so we have:

$$ab + ac = a(b + c)$$

In other words:

$a = Y_g$
$b = 1$
$c = -t$ (remember the minus sign)

so

In Eq. A1.6 we showed this as $(1 - t)Y_g$, which is the same

$$a(b + c) = Y_g(1 - t)$$

We could just as well have had three, four or more terms inside the brackets and the same approach would be appropriate. Similarly, we could have had more than one term, a, before the bracket. For example:

$$(a + b)(c + d)$$

would give

$$ac + bc + ad + bd$$

and you can see that each term within the first set of brackets has, in turn, been multiplied by each term within the second set of brackets. Notice, though, that the order in which we multiply does not matter. This principle is readily extended to more than two sets of brackets or to brackets containing more than two expressions.

Progress Check A1.2

For each of the following expressions multiply out the brackets and, where relevant, simplify the expressions.

(i) $10x(3a - c)$

(ii) $(5x - 3y)(2x + 4y)$

(iii) $3(x + y - z) - (4y + 2)x$

Try these first before reading on.

Taking each in turn we have:

(i) $10x(3a - c)$

And we can multiply the two terms inside the brackets by $10x$ to give:

$$30ax - 10cx$$

Notice that it doesn't matter whether we write $30ax$ or $30xa$.

(ii) $(5x - 3y)(2x + 4y)$

We multiply each of the two terms in the second bracket first by $5x$ and then by $-3y$ (remember the minus sign):

Multiply by $5x$: $5x2x + 5x4y$
Multiply by $-3y$: $-3y2x - 3y4y$

When we multiply a variable like x by itself we get x^2 (x squared). (If you don't remember how to work with powers like x^2 you might want to read through the short appendix to this module on p. 31.) This would give us:

$$5x2x + 5x4y = 10x^2 + 20xy$$

and

$$-3y2x - 3y4y = -6xy - 12y^2$$

and combining these two expressions together we get

$$10x^2 + 20xy - 6xy - 12y^2$$

or

$$10x^2 + 14xy - 12y^2$$

(iii) $3(x + y - z) - (4y + 2)x$

If you think this appears complicated, remember to break it into parts and then add the parts together at the end. Let's take the first part of this and multiply the brackets through by 3:

$$3(x + y - z) = 3x + 3y - 3z$$

and now the second part (remembering the minus sign)

$$-(4y + 2)x = -4xy - 2x$$

and if we now collect both parts together we have

$$3x + 3y - 3z - 4xy - 2x$$

and collecting common terms together we have

$$x + 3y - 3z - 4xy$$

Multiple brackets

We have seen how we can multiply out brackets in an expression. There are times when we have multiple sets of brackets. For example:

$$3x(4 - y(15 - x))$$

Again, this looks complicated but if we do it part by part it's straightforward. We multiply this out in much the same way, but making sure that we start with the *inside* set of brackets first – those around $(15 - x)$ – and then gradually work outwards. So, multiplying out the inside set first we have:

$$-y(15 - x) = -15y + xy$$

and then

$$3x(4 - 15y + xy)$$

and then

$$12x - 45xy + 3x^2y$$

Expressions involving multiple sets of brackets can be simplified using this approach: find the innermost set of brackets, work out that expression, find the next innermost set of brackets, work that out – and so on.

Progress Check A1.3

Simplify each of the following expressions:

(i) $15x(3x - 2y(y - x))$

(ii) $(4x - 3y(4x + 3y)(5x))$

(iii) $(2x - 3y + 4z(2x + 3(15y)))$

For (i) we have:

$$15x(3x - 2y(y - x)) = 15x(3x - 2y^2 + 2xy)$$

(multiplying the $(y - x)$ term by $2y$ and remembering the change in sign when we have two negatives multiplied). This then becomes:

$$15x(3x - 2y^2 + 2xy) = 45x^2 - 30xy^2 + 30x^2y$$

Note that we cannot simplify further: the last two terms are not identical.

(ii) $(4x - 3y(4x + 3y)(5x))$

Multiplying together the two bracket terms inside the outside bracket, $(4x + 3y)$ and $(5x)$, we have:

$$(4x - 3y(20x^2 + 15xy))$$

Multiply through by $-3y$:

$$4x - 60x^2y + 45xy^2$$

(iii) $(2x - 3y + 4z(2x + 3(15y)))$

Multiply through the two terms on the right of the expression $3(15y)$:

$$(2x - 3y + 4z(2x + 45y))$$

Multiply through by $4z$:

$$2x - 3y + 8xz + 180yz$$

A1.4 Inequalities

So far we have explored algebraic expressions in the form of *equations*, where an expression on the left-hand side is set exactly equal to another expression on the right-hand side. Occasionally we will wish to explore relationships that are expressed in the form of an *inequality*. For example, we may have:

$$x > y$$

which is read as 'x is greater than y' and where the symbol $>$ indicates that x must take values greater than y at all times. Similarly, we may have:

$x < y$ — x always takes a value less than y
$x \geq y$ — x always takes a value which is greater than or equal to y (i.e. x values cannot be less than y but they could be the same as y or greater than y)
$x \leq y$ — x always takes a value which is less than or equal to y (i.e. x values cannot be greater than y).

Let's go back to Eq. A1.4 where we had:

$$D = f + tY_g$$

where t is a tax imposed on gross income, Y_g. If we express the tax as a decimal (e.g. a tax that took 25% of income would be shown as 0.25) then we would have:

$t \geq 0$ — i.e. the tax rate could not be negative
$t < 1$ — the tax rate must be less than 1 (or less than 100%).

The first inequality could be rewritten instead as:

$$0 \leq t$$

so we could merge the two inequalities together to give

$$0 \leq t < 1$$

That is, t must lie within a range between 0 but less than 1.

It will also be worth exploring how inequalities are affected if we manipulate them using the algebraic principles developed earlier. We have already seen that we can manipulate equations in any way we wish as long as we alter both sides of the equation in the same way. Let us see if the same principle applies to inequalities. Consider:

$$x < y$$

where $x = 2$ and $y = 10$. Then:

$$2 < 10$$

which is clearly correct. Suppose we add 4 to both sides:

$$2 + 4 < 10 + 4$$
$$6 < 14$$

which is still correct. Suppose we now subtract 20 from both sides:

$$6 - 20 < 14 - 20$$
$$-14 < -6$$

which is still correct (although you may have to think about this one: -14 is lower (less) on the negative scale than -6 so the inequality holds true).

So addition and subtraction do not affect the inequality. What about multiplication and division? We had:

$$2 < 10$$

If we multiply both sides by 5:

$$2 \times 5 < 10 \times 5$$
$$10 < 50$$

which is correct. Suppose we now multiply by −2:

$$10 \times -2 < 50 \times -2$$
$$-20 < -100$$

which is clearly *incorrect* since −100 is a larger negative number and is less than −20. This leads us to a simple manipulation rule when dealing with inequalities: *if both sides of an inequality are multiplied/divided by a negative number, the direction of the inequality is reversed.*

So, if we had:

$$x < y$$

and multiplied through by $-n$, we would have

$$-nx > -ny$$

Progress Check A1.4

Simplify the following expressions by collecting all variable terms on one side and all numerical values on the other:

(i) $4x + 7 < 3x - 5$

(ii) $4x - 3 > 6x + 2$

(iii) $-4x + 5 \geq 6 - 3x$

For (i) we have:

$$4x + 7 < 3x - 5$$

Subtracting 7 gives:

$$4x + 7 - 7 < 3x - 5 - 7$$
$$4x < 3x - 12$$

Subtracting $3x$:

$$4x - 3x < 3x - 12 - 3x$$
$$x < -12$$

That is, x must always take values that are less than −12.

(ii) $4x - 3 > 6x + 2$

Add 3 to give:

$$4x > 6x + 5$$

Subtract $6x$:

$$4x - 6x > 5$$

$$-2x > 5$$

Divide through by -2:

$$x < -2.5$$

remembering that as we divide through by a negative value we must reverse the inequality sign. That is, x is less than or equal to -2.5.

(iii) $-4x + 5 \geq 6 - 3x$

Add $3x$:

$$-4x + 3x + 5 \geq 6$$

$$-x + 5 \geq 6$$

Subtract 5:

$$-x \geq 6 - 5$$

$$-x \geq 1$$

Divide through by -1:

$$x \leq -1$$

again remembering to reverse the direction of the inequality.

A1.5 Fractions

We now look at the use of fractions in algebra. You will already be familiar with numerical fractions such as:

$$\frac{2}{3} \text{ or } \frac{1}{10} \text{ or } \frac{72}{100}$$

You may also remember that the number on the top of the fraction expression is referred to as the *numerator* and the one on the bottom as the *denominator*. In algebra we may have fractions such as:

$$\frac{a}{b} \text{ or } \frac{a^2 - 3b}{2a - b^2} \text{ or } \frac{15 - b}{3a^2 - 2ab}$$

The rules for manipulation of algebraic fractions are virtually the same as those for numerical fractions.

Multiplication

To multiply two or more fractions we multiply the numerator terms together and then multiply the denominator terms together. For example:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

Division

To divide one fraction by another, we invert (turn upside down) the fraction we are dividing by and then multiply the two fractions together:

$$\frac{a}{b} / \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Addition/subtraction

To add or subtract two fractions, we put them over a *common denominator* and add/subtract the numerators. We will illustrate this with a numerical example first. Suppose we want:

$$\frac{3}{4}+\frac{1}{2}$$

A common denominator is a number of which 4 and 2 (the two original denominators) are exact multiples. In this case one common denominator would be 4 since the denominator 4 goes into this exactly once and the other denominator 2 goes into this exactly twice. We then use these multiples (1 and 2) to multiply the respective numerators. That is:

$$\frac{3}{4}+\frac{1}{2}=\frac{(1\times 3)+(2\times 1)}{4}=\frac{3+2}{4}=\frac{5}{4}$$

Note that we have multiplied the first numerator, 3, by 1 since its denominator (4) goes into the common denominator exactly once. We have multiplied the second numerator (1) by 2 since its denominator goes into the common denominator exactly twice.

Choosing a common denominator

When deciding which common denominator to use there's frequently an obvious number that will be exactly divisible by each of the two fraction denominators. There are times, however, when such a number is not immediately obvious. In such a case an easy approach is simply to use a common denominator that is the result of multiplying the two fraction denominators together. For example:

$$\frac{3}{7}+\frac{2}{3}$$

As there is no obvious common denominator that springs to mind, we choose 21 (7×3). The arithmetic would then be:

$$\frac{(3\times 3)+(2\times 7)}{21}=\frac{9+14}{21}=\frac{23}{21}$$

Progress Check A1.5

Simplify each of the following expressions:

(i) $3/5+8/4$

(ii) $2/6+3/7$

(iii) $4/5+2/3+6/8$

For (i) we use a common denominator of 20 (5×4) to give:

$$\frac{(3\times 4)+(8\times 5)}{20}=\frac{12+40}{20}=\frac{52}{20}$$

Notice that while we can leave the result as 52/20 we can simplify further since both the numerator and denominator can be divided through by a common factor.

For example, divide both through by 2 (and since we are applying the same arithmetic to top and bottom it will leave the expression unchanged):

$$\frac{52}{20} = \frac{26}{10}$$

26/10 can be further simplified, again dividing by 2 to give:

$$\frac{26}{10} = \frac{13}{5}$$

Of course, we could have divided the top and bottom of 52/20 through by 4 straightaway. This type of simplification is quite common in economics, and so it is worthwhile familiarizing yourself with it, particularly with algebraic rather than arithmetic examples.

(ii) $2/6 + 3/7$

A common denominator of 42 (6×7) gives:

$$\frac{(2 \times 7) + (3 \times 6)}{42} = \frac{14 + 18}{42} = \frac{32}{42}$$

This can be simplified again by dividing through by 2:

$$\frac{32}{42} = \frac{16}{21}$$

(iii) $4/5 + 2/3 + 6/8$

Although we have not explicitly looked at three fractions being added together, we can simply add the first two and then add this product to the third (although with practice we might be able to perform the arithmetic in one step rather than two). We have:

$$\frac{(4 \times 3) + (2 \times 5)}{15} = \frac{22}{15}$$

We use 5×3 as a common denominator for the first two fractions

and then

$$\frac{22}{15} + \frac{6}{8} = \frac{(22 \times 8) + (6 \times 15)}{120} = \frac{176 + 90}{120} = \frac{266}{120}$$

We use 15×8 as a common denominator for the next two fractions

Simplifying gives:

$$\frac{266}{120} = \frac{133}{60} \text{ (dividing through by 2).}$$

We could have performed the arithmetic in one step as:

$$\frac{4}{5} + \frac{2}{3} + \frac{6}{8}$$
$$\frac{4(3 \times 8) + 2(5 \times 8) + 6(5 \times 3)}{(5 \times 3 \times 8)} = \frac{96 + 80 + 90}{120} = \frac{266}{120}$$

Subtraction

So far we have looked only at the addition of fractions; but exactly the same approach applies to subtraction. For example:

$$\frac{3}{5} - \frac{2}{3} = \frac{(3 \times 3) - (2 \times 5)}{15} = \frac{9 - 10}{15} = \frac{-1}{15}$$

Progress Check A1.6

Simplify each of the following expressions:

(i) 3/4 – 2/3

(ii) 2/5 – 1/9

(iii) 2/5 – 4/7 + 1/8

For (i) we have:

$$\frac{3}{4} - \frac{2}{3} = \frac{(3 \times 3) - (2 \times 4)}{4 \times 3} = \frac{9-8}{12} = \frac{1}{12}$$

For (ii):

$$\frac{2}{5} - \frac{1}{9} = \frac{(2 \times 9) - (1 \times 5)}{45} = \frac{18-5}{45} = \frac{13}{45}$$

For (iii):

$$\frac{2}{5} - \frac{4}{7} + \frac{1}{8}$$

$$\frac{2(7 \times 8) - 4(5 \times 8) + 1(5 \times 7)}{(5 \times 7 \times 8)} = \frac{112 - 160 + 35}{280} = \frac{-13}{280}$$

Fractions with algebraic expressions

The same principles apply to algebraic expressions. For example:

$$\frac{x}{x+2} \times \frac{3x}{2x^2}$$

Multiplying the two denominators gives:

$$(x+2)(2x^2) = 2x^3 + 4x^2$$

and then multiplying the two numerators gives

$$\frac{3x^2}{2x^3 + 4x^2}$$

However, if we divide both the numerator and denominator by x^2 we have:

$$\frac{3}{2x+4}$$

Note that when we are cancelling out terms in an algebraic expression we must be careful to ensure that the term being used appears in all parts of the expression, as in this case.

Progress Check A1.7

Simplify each of the following expressions:

(i) $(3x - 6)/x^2$ divided by $5x/3x^2$

(ii) $5x/4x^2 + 3x^3/5x$

(iii) $7x/4x^2 - 8x/2x^3$

Taking each in turn, for (i) we have:

$$\frac{(3x-6)}{x^2} \bigg/ \frac{5x}{3x^2}$$

Recollecting that if we invert the second term and then multiply we have:

$$\frac{(3x-6)}{x^2} \times \frac{3x^2}{5x}$$

Notice that we can cancel the x^2 term on both top and bottom to give:

$$\frac{(3x-6)}{1} \times \frac{3}{5x} = \frac{9x-18}{5x}$$

We cannot cancel the x terms since they do not appear in each part of the final expression (the -18 term does not have an x attached to it).

For (ii):

$$\frac{5x}{4x^2} + \frac{3x^3}{5x}$$

we have a common denominator of $(4x^2)(5x)$ which is $20x^3$:

$$\frac{5x(5x) + 3x^3(4x^2)}{20x^3} = \frac{25x^2 + 12x^5}{20x^3}$$

If we wished we could simplify further as:

$$\frac{25 + 12x^3}{20x}$$

We can divide through by the common term x^2

One useful way of checking whether we can simplify by cancelling a common term is to break the fraction into its component parts:

$$\frac{25x^2 + 12x^5}{20x^3} = \frac{25x^2}{20x^3} + \frac{12x^5}{20x^3}$$

It will then be apparent that both parts of the expression have a common term which can be cancelled (x^2 in this case).

For (iii):

$$\frac{7x}{4x^2} - \frac{8x}{2x^3}$$

we have a common denominator of $(4x^2)(2x^3)$ or $(8x^5)$ giving:

$$\frac{7x(2x^3) - 8x(4x^2)}{8x^5} = \frac{14x^4 - 32x^3}{8x^5}$$

Cancelling through by $2x^3$ we have:

$$\frac{7x-16}{4x^2}$$

A1.6 Transposing an expression

The last aspect of algebra that we shall examine relates to the *transposition* of an expression (basically, rearranging it into another form). For example, consider the expression:

$$-ax = bx - cy + d \tag{A1.7}$$

We wish to rearrange this into an expression such that:

$x =$ an expression involving all other terms.

The first step is to collect x terms together. From Eq. A1.7 we can subtract bx from both sides to give:

$$-ax - bx = -cy + d$$

The two terms on the left-hand side have an x term in common, so we have:

$$x(-a - b) = cy + d$$

Dividing both sides through by $(-a - b)$ gives:

$$x = \frac{cy + d}{-a - b}$$

Progress Check A1.8

From Eq. A1.7 derive an expression for y.

We have:

$$-ax = bx - cy + d$$

Subtracting bx gives:

$$-ax - bx = -cy + d$$

Subtracting d:

$$-ax - bx - d = -cy$$

Multiplying through by -1:

$$ax + bx + d = cy$$

Dividing through by c:

$$y = \frac{ax + bx + d}{c}$$

We may also apply these principles to a more complex expression. Suppose we wish to derive an expression for x from:

$$y = \frac{x + 2}{x - 4}$$

Multiplying through by $(x - 4)$:

$$y(x - 4) = x + 2$$

Multiplying out the left-hand side:

$$yx - 4y = x + 2$$

Adding $4y$ to both sides:

$$yx = x + 2 + 4y$$

Subtracting x:

$$yx - x = 2 + 4y$$

The left-hand side terms have x in common, so:

$$x(y-1)=2+4y$$

and dividing through by $(y-1)$:

$$x=\frac{2+4y}{y-1}$$

Although this type of manipulation looks complicated it is simply a matter of practice and applying a few basic rules. Use the algebraic principles we have developed to:

- Remove any fractions by cross-multiplication
- Multiply out any brackets
- Collect x terms on one side
- Find any factors/multiples of x
- Divide through by the x coefficient.

If you're in any doubt as to whether you've applied these principles correctly, choose a couple of numerical values for x and solve for y using the original expression. Then use these y values in your transposed result and see whether you get the same x values (which, of course, you will if you've not made a mistake anywhere).

Progress Check A1.9

Find an expression for x from:

(i) $y=\dfrac{x-5}{x+3}$

(ii) $y=\dfrac{3x+3}{2x-5}$

For (i), using the steps above:

$$y(x+3)=x-5$$
$$yx+3y=x-5$$
$$yx-x=-3y-5$$
$$x(y-1)=-3y-5$$
$$x=\frac{-3y-5}{y-1}$$

For (ii):

$$y=\frac{3x+3}{2x-5}$$
$$y(2x-5)=3x+3$$
$$2yx-5y=3x+3$$
$$2yx-3x=5y+3$$
$$x(2y-3)=5y+3$$
$$x=\frac{5y+3}{2y-3}$$

A1.7 Summary

This brings us to the end of this module on basic algebra and, although at times it might have looked complicated, algebra follows a set of basic rules. As long as you know what the rules are and have a steady, methodical approach to working with algebraic expressions, you will soon see how it works. If you've been able to follow what we've been doing in this module then you're ready to move on to where we can really start seeing how mathematics can be used in economic analysis. If, at any stage in the text, you have difficulty following the algebraic manipulations, return to the relevant part of this module and re-read that section.

Learning Check

Having read this module you should have learned that:

- A basic rule in algebra is that if both sides of an expression are changed in the same way the expression remains unchanged
- When you're working with multiple brackets, start with the ones on the inside and work outwards
- If both sides of an expression are multiplied or divided by a negative value, the direction of an inequality is reversed
- To divide by a fraction, turn it upside down and multiply
- To add/subtract fractions, put them over a common denominator and add/subtract the numerators.

Exercises

A1.1 For each of the following equations find the simplest form:

i) $y = \dfrac{3x+3}{2x-5}$

ii) $7x/4x^2 - 8x/2x^3$

(These were in the Knowledge Check at the start of this module.)

A1.2 For each of the following expressions find the simplest form:

i) $5(x-y)+2(y-3x)$

ii) $4x(3x-2)+0.5(x-4y)$

iii) $(x-2y)(3y-5x)$

iv) $z(2x-y)-z(5x-2)$

v) $3x(5-2x(y-x(3x-6)))$

vi) $0.4y(3x(2-4y)+2y(5-3x(x-10)))$

A1.3 For each of the following expressions find the simplest form:

i) $x/(x-1) \times 2/x(x-4)$

ii) $7x/2x^3 + 5x/2x$

iii) $15x^3/3x^2 - 0.5x^2/3x$

iv) $6x/3x(5x-10) + 2x^3/4(3-10x)$

Exercises *(Continued)*

A1.4 A company selling a particular product knows that the quantity of the product demanded by customers is given by the expression:

$$Q_d = 100 - 5P$$

where Q_d is the quantity of the product demanded and P is the price charged. Similarly, the quantity that the company is willing to supply is given by:

$$Q_s = -100 + 20P$$

where Q_s is the quantity supplied and P is the price charged. Equilibrium is defined as the price charged so that $Q_d = Q_s$. Find the price that will give equilibrium. What quantity will be demanded/supplied at this price?

A1.5 For the firm in A1.4, we now have:

$$Q_d = a - bP$$

$$Q_s = c + dP$$

Find an algebraic expression that will allow you to determine equilibrium price. Check this using the parameters in A1.4.

Appendix A1 Powers and exponents

When using mathematics in economic analysis we frequently come across terms such as x^2 or x^5 or $x^{-0.5}$. You will need to be able to use expressions like these. This is relatively straightforward once you understand that such notation is in fact a form of mathematical shorthand. Suppose we want to show some simple arithmetic:

$$10 \times 10 = 100$$

$$10 \times 10 \times 10 = 1000$$

$$10 \times 10 \times 10 \times 10 = 10{,}000$$

and so on. There's nothing wrong with showing such arithmetic in this way. However, it can be more convenient at times to use mathematical shorthand:

$$10 \times 10 = 10^2$$

where we say that the result of multiplying 10 by itself is 10^2 where the term ^{2} is known as the *power* or *exponent*. The power/exponent simply shows how many times we multiply a number/variable by itself. So, from above, we have:

$$10 \times 10 = 100 = 10^2$$

$$10 \times 10 \times 10 = 1000 = 10^3$$

$$10 \times 10 \times 10 \times 10 = 10{,}000 = 10^4$$

If we were using variables rather than numbers we'd have:

$$a^2 = a \times a$$

$$a^3 = a \times a \times a$$

$$a^4 = a \times a \times a \times a$$

Sometimes we'll come across exponents that appear a little odd, for example a^{-1}. This looks like a multiplied by itself –1 times. Earlier we saw that:

$$10^4 = 10{,}000$$
$$10^3 = 1000$$
$$10^2 = 100$$

Clearly there is a pattern here. As the exponent drops from 4 to 3 to 2 a zero is 'lost' from the actual number on the right. So, if we continue this pattern, we obtain:

$$10^4 = 10{,}000$$
$$10^3 = 1000$$
$$10^2 = 100$$
$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = 0.1$$
$$10^{-2} = 0.01 \text{ and so on.}$$

Now let's consider the items we have added. Normally we wouldn't bother writing 10 as 10^1 but write just 10 instead. $10^0 = 1$ might seem odd at first but it follows from the logic of the sequence. In fact, we'll state without proof that any number/variable to the power 0 equals 1: this is worth remembering. The negative exponents are just as logical. Negative exponents show decimals in exponent form. Sometimes these are written in a different way. Recollect that:

$$0.1 = \frac{1}{10}$$

and

$$10 = 10^1$$

so

$$0.1 = \frac{1}{10} = \frac{1}{10^1}$$

So:

$$10^{-1} = 0.1 = 1/10^1$$

Similarly:

$$10^{-2} = 1/10^2$$

and a^{-3} would be $1/a^3$.

Just as we can carry out algebraic arithmetic on ordinary numbers or variables, so we can do much the same when dealing with exponents. There are four simple rules for doing algebraic arithmetic with exponents.

Rule 1

$$a^n \times a^m = a^{n+m}$$

For example:

$$10^2 \times 10^4 = 10^6$$
$$a^5 \times a^3 = a^8$$

Here $n = 2$ and $m = 4$
Try it out using the actual numbers if you're not sure

Rule 2

$$a^n/a^m = a^{n-m}$$

For example:

$$10^6/10^3 = 10^3$$

$$a^5/a^4 = a^1 = a$$

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Rule 3

$$(a^n)^m = a^{nm}$$

For example:

$$(10^3)^2 = 10^6$$

$$(a^2)^4 = a^8$$

We're squaring the number 10^3 so this is actually Rule 1: $10^3 \times 10^3$

Rule 4

$$(ab)^n = a^n b^n$$

For example:

$$(3 \times 10)^3 = 3^3 10^3$$

This may be seen more clearly if we write:

$$(3 \times 10)^3 = (3 \times 10) \times (3 \times 10) \times (3 \times 10)$$

The order in which we multiply is unimportant, so we can rearrange this as:

$$(3 \times 10)^3 = (3 \times 3 \times 3) \times (10 \times 10 \times 10) = 3^3 10^3$$

An example involving variables rather than numbers is:

$$(ab)^4 = a^4 b^4$$

Powers and exponents are easy to work with if you remember the rules.

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