VIT-AP UNIVERSITY	Final Assessment Test – Winter (2024-25) Freshers - May 2025	
	Maximum Marks: 100	Duration: 3 Hours
Course Code: MAT1002	Course Title: Applications of Differential and Difference Equations	
Set No: 9	Exam Type : Closed Book	School: SAS
Date: 19/05/2025	Slot: A1	Session: FN

Keeping mobile phone/smart watch, even in 'off' position is treated as exam malpractice

General Instructions if any:

1. **"fx series" – non-Programmable calculator is permitted: YES**
2. **Reference tables permitted: NO**

PART – A: Answer any TEN Questions, Each Question Carries 10 Marks (10×10=100 Marks)

1. A young person with no initial capital ($S(0) = 0$) invests k dollars per year at an annual rate of return r. Assume that investments are made continuously and that the return is compounded continuously. The governing equation is $\frac{dS}{dt} = rS + k$. **(10 M)**
   (a) Determine the sum $S(t)$ accumulated at any time t.
   (b) If $r = 75\%$, determine k so that \$1 million will be available for retirement in 40 years.

2. Suppose that the motion of a certain spring-mass system satisfies the differential equation $x'' + 3x' + 2x = 3\cos(t)$ and the initial conditions $x(0) = 2, x'(0) = 3$. **(10 M)**
   Find the displacement of the system and describe the behaviour of the displacement for large t.

3. The initial-value problem governing the motion of a certain mass-spring system is given by
$$y'' + 4y = 5\,\delta(t-3),\ \ y(0) = 1, y'(0) = 0,$$
   where $\delta(t-3)$ describes the impulsive force that acts on the mass at $t = 3$. Determine the motion of the mass. **(10 M)**

4. (a) Verify the initial value theorem for the function $f(t) = e^{-t}(t+2)^2$ **(10 M)**
   (b) Find the transfer function of the system $y'' + 7y' + 12y = f(t)$, with input $f(t)$ and output $y(t)$. Hence, find the impulse response of the system.

5. Consider the currents $i_1(t)$ and $i_2(t)$ in a certain network containing an inductor, a resistor, and a capacitor shown in governed by the system of first-order differential equations **(10 M)**
$$L\,i_1' - Ri_2 = E(t)$$
$$RC\,i_2' + i_2 - i_1 = 0.$$
   Solve the system using the matrix method under the conditions $E(t) = 60$ V, $L = 1$ h, $R = 50\,\Omega$, $C = 10^{-4}$ f, and the currents i_1 and i_2 are **initially zero.**

6. Consider a potential energy of a certain system $Q(x_1, x_2, x_3) = 4x_1^2 + 4x_1x_2 + 4x_2^2 + 4x_2x_3 + 4x_3^2 + 4x_1x_3$. Express $Q(x_1, x_2, x_3)$ without cross-product terms. Also, find the orthogonal matrix P. Further, find a relation between old and new coordinate systems. **(10 M)**

7. (a) Reduce the higher-order differential equation $3y''' + 5y'' - y' + 7y = 0$ into a system of first-order differential equations and express the system into the matrix equation form.
   (b) Use the Cayley-Hamilton theorem to find A^{10} for the matrix $A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$ **(5+5 = 10 M)**

8. Consider the Sturm-Liouville problem $y'' + \lambda y = 0$ with the boundary conditions $y(0) = 0$ and $y(\pi) = 0$. Find the corresponding eigenfunctions. Express the function $f(x) = 5$ as a linear combination of the obtained eigenfunctions.

(10 M)

9. Find the power series solution of the Airy's equation of the form $y'' - x\,y = 0$ about an ordinary point $x_0 = -2$.

(10 M)

10. Express the function $f(x) = x^3 + x$ in terms of Fourier-Legendre series (up to 4 terms). (hint: $c_n = \frac{2n+1}{n}\int_{-1}^{1} f(x)P_n(x)dx$, $P_0(x) = 1; P_2(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1);\ P_3(x) = \frac{1}{2}(5x^3 - 3x)$)

(10 M)

11. Assume that the rabbit population in an island is 3 at time $n = 0$ and 13 at time $n = 1$. The rabbit population is given by the difference equation $a_n = 5a_{n-1} + 4a_{n-2} + 5n + 3$ for $n > 2$. Solve the recurrence relation.

(10 M)

12. The dynamics of a discrete-time system are determined by the difference equation

$y_{k+2} + 5y_{k+1} + 6y_k = u_k$ with initial conditions $y_0 = y_1 = 1$.

Determine the response of the system to the unit step input:

(10 M)

$$u_k = \begin{cases} 0 & k < 0 \\ -1 & k \geq 0 \end{cases}$$