 VIT-AP UNIVERSITY	Final Assessment Test – Winter (2024-25) Freshers - May 2025		
	Maximum Marks: 100		Duration: 3 Hours
Course Code: MAT1002	Course Title: Applications of differential and Difference Equations		
Set No: 4	Exam Type : Closed Book		School: SAS
Date: 20/05/2025	Slot: A2		Session: FN
Keeping mobile phone/smart watch, even in 'off' position is treated as exam malpractice			
General Instructions if any:			
1. "fx series" - non Programmable calculator are permitted : YES			
2. Reference tables permitted : NO (if Yes, Please specify:)			

Answer any TEN Questions, Each Question Carries 10 Marks ($10 \times 10 = 100$ Marks)

1. Suppose a frozen package of processed meat is removed from a freezer that is set at 0°F and placed in a refrigerator that is set at 38°F . Six hours after being placed in the refrigerator, the temperature of the meat is 12°F .

a) Derive a function for the temperature of the meat, T in Fahrenheit, as a function of time, t in hours. (5 M)

b) How long will it take for the meat to thaw (reach a temperature above 32°F)? Give answer to 2 decimal places. (5 M)

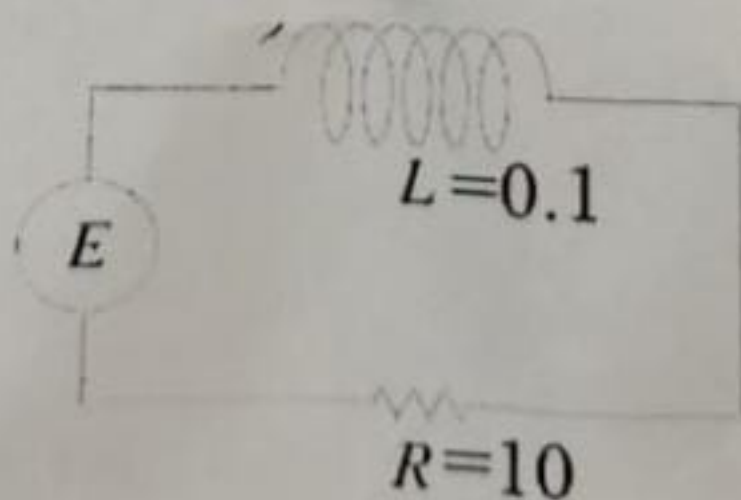
2. A circuit consist of an inductance of 0.05H , a resistance 5 Ohms , and a condenser of capacitance $4 \times 10^{-4}\text{F}$. If $Q = I = 0$ at $t = 0$, Find $Q(t)$ and $I(t)$ when, there is an alternating emf $E(t) = 200\cos(100t)$, find the steady state solution. (Use the mathematical model $LQ'' + RQ' + Q/C = E(t)$) (10 M)

3. A spring with a mass of 2 kg has natural length 0.5m . Assuming there is no damping force, a force of 25.6 N is required to maintain it stretched to a length of 0.7m . If the spring is stretched to a length of 0.7m and then released with initial velocity 0 , find the position of the mass at any time t . Consider the Mathematical model of the Mass Spring system as follows: $my'' + cy' + ky = 0$. (10 M)

4. The applied voltage of an electrical circuit is given as $f(t) = \begin{cases} 0, & t \leq \pi \\ t - \pi, & \pi < t < 2\pi \\ 0, & t > 2\pi \end{cases}$

Compute the Laplace transform of the function $f(t)$. Can we have a function $f(t)$, whose Laplace transform is $F(s) = s^2$? Justify your answer. (10 M)

5. Consider the LCR circuit given below:



if the applied voltage is given as $f(t) = \begin{cases} 0, & t \leq 5 \\ 2, & t > 5 \end{cases}$. Assuming the initial charge and voltage to be zero, compute the current at any time t using Laplace transform method. (Use the mathematical model $Lq'' + Rq' + q/c = f(t)$) (10 M)

6. Use Cayley-Hamilton method to compute A^{20} of the matrix $A = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$ and then compute the eigen values of A^8 using property of eigen values. (10 M)

7. An undamped mass spring system is modelled by the differential equation $y'' + 16y = 0$.

- Convert the above differential equation into a system of first order differential equation. (5 M)
- If initial displacement and velocity are given as $y(0) = 1$ and $y'(0) = 0$, find the displacement and velocity using matrix diagonalization method. (5 M)

8. Let the Potential energy of a system is given as a quadratic form $V = 4x^2 + 4xy + y^2 + 15z^2$. Express V into canonical form using orthogonal reduction. Can the potential energy be negative for any value of x and y . (10 M)

9. Find the series solution of the Airy's equation $y'' + xy = 0$, about the ordinary point $x = 0$. (10 M)

10. Expand the function $f(x) = 1 - x$, in terms of eigen functions of Sturm-Liouville problem $y'' + \lambda y = 0, y'(0) = y'(\frac{\pi}{2}) = 0$. (10 M)

11. a) If the increase in population at $(n + 1)$ th year of certain species in a culture is equals to the average population of the previous two years plus n th power of 2. If initial two years of the population of species are 0 and 1, find the explicit solution for the population at n th year. (5 M)

b) Using convolution theorem, find the inverse Z-transform of the function $F(z) = \frac{z^2}{(z-1)^3}$ (5 M)

12. Consider the first order difference equation $y_n - 6y_{n-1} + 5y_{n-2} = 4, n = 1, 2, 3 \dots$ with $y(0) = 1$. Find the solution of the difference equation using Z-transforms. (10 M)