VIT-AP UNIVERSITY	Final Assessment Test – Winter (2024-25) Freshers - May 2025	
	Maximum Marks: 100	Duration: 3 Hours
Course Code: MAT1002	Course Title: Applications of Differential and Difference Equations	School: SAS
Set No: 8	Exam Type : **Closed Book**	Session: FN
Date: 23/05/2025	Slot: C1	

Keeping mobile phone/smart watch, even in 'off' position is treated as exam malpractice

General Instructions if any:

1. **"*fx* series" - non-Programmable calculator is permitted: YES**
2. **Reference tables permitted: NO**

Answer any TEN Questions, Each Question Carries 10 Marks (10×10=100 Marks)

1. An electric circuit consists of a resistor R ohms and an inductor L henrys connected in series, along with a capacitor of capacitance C farads and an electromotive force (e.m.f.) of E volts. The current $i(t)$ at time t is governed by the equation: $L\frac{di}{dt} + Ri + \frac{q}{C} = E$, where $q(t)$ is the charge at time t. If $E = 10$ volts, $R = 10$ ohms, $C = 0.001$ farads, $L = 0.25$ henrys, with the initial conditions: $q(0) = 0$ coulombs, $i(0) = 0$ amperes, find both the charge $q(t)$ and the current $i(t)$ at time t, where $i(t) = \frac{dq}{dt}$. **(10 M)**

2. The radial displacement in a rotating disc at a distance r from the axis is given by
$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = -r^3$$
with the boundary conditions $u(0) = 0$ and $u(a) = 0$, find the displacement $u(r)$. **(10 M)**

3. Consider a mass-spring system governed by the second order differential equation of the form $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = u(t)$, where $u(t)$ is the unit step function defined as $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$. Find the motion of the spring using the Laplace transforms method with $x(0) = 1, \frac{dx(0)}{dt} = 0$. Also find $x(\infty)$, using final value theorem. **(10 M)**

4. Consider a linear time-invariant system in the convolution form $y(t) = e^{-2t} * f(t)$, where $f(t)$ is the input, and $y(t)$ is the output. (i) Find the transfer function of the system, (ii) Find the output $y(t)$ corresponding to the unit step input function. **(10 M)**

5. A 1 kg mass is suspended from a spring with a spring constant of 6 N/m. The system is subjected to a damping force proportional to the 5 times of its velocity, and an external constant force of 10 N. The motion of the damped spring-mass system is governed by the second-order differential equation:
$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$
(i) Convert the second order differential equation into two first order differential equations.
(ii) Find the displacement and velocity using matrix methods. **(10 M)**

6. Find out what type of conic section, the following quadratic form represents and transform using orthogonal transformation: $Q(x_1, x_2) = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$. **(10 M)**

7. Consider a vibrating system governed by a second order coupled differential equations of the form: $\frac{d^2x_1}{dt^2} = -5x_1 + 2x_2$ and $\frac{d^2x_2}{dt^2} = 2x_1 - 2x_2$, with the initial conditions $x_1(0) = 1, x_2(0) = 0, x_1'(0) = 0, x_2'(0) = 0$. Find the displacements of the system using matrix methods. **(10 M)**

8. Consider the vibration of a string fixed at both ends. The transverse displacement of the string, $y(x)$, at a point x along the string satisfies the second-order ordinary differential equation:

$$\frac{d^2y}{dx^2} + \lambda y = 0, 0 < x < L$$

with boundary conditions: $y(0) = 0$ and $y(L) = 0$ where λ is a constant and L is the length of the string. Find the eigenvalues λ and the corresponding eigenfunctions $y(x)$. Also write orthogonality relation. **(10 M)**

9. The displacement function for a vibrating rod is defined as: $f(x) = \begin{cases} 0, -1 < x \leq 0 \\ x, 0 < x < 1 \end{cases}$.

(a) Write down the first four Legendre polynomials $P_0(x), P_1(x), P_2(x), P_3(x)$, using Rodrigues' formula: $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$.

(b) Using the Fourier–Legendre expansion formula, $f(x) = \sum_{n=0}^{\infty} a_n P_n(x)$, where $a_n = \frac{2n+1}{2}\int_{-1}^{1} f(x)P_n(x)dx$, find the Fourier–Legendre series expansion for $f(x)$ up to 4 terms. **(10 M)**

10. (a) For a discrete-time system modelling the voltage response in an electrical circuit, the Z-transfer function is given by: $H(z) = \frac{z}{z^2-2z+1}$. If the input signal is a unit impulse $\delta[n]$, use the Initial Value Theorem of the Z-transform to find the initial value $y(0)$.

(b) Find the sequence u_n whose Z-transform is given by: $U(z) = \frac{1}{(z-1)(z-2)}$. **(10 M)**

11. A cup of coffee initially at $180^0 F$ cools to $170^0 F$ after one minute when placed in a room with a constant temperature of $50^0 F$. Using Newton's Law of Cooling in the form of a difference equation: $T_{n+1} - T_n = -k(T_n - T_s)$, where T_n is the temperature at the nth minute, T_s is the surrounding temperature, and k is a constant. Find the temperature of the coffee after 20 minutes and 40 minutes. **(10 M)**

12. A discrete-time control system models the temperature $y(n)$ in a room where a heater supplies energy growing exponentially as 2^n. The system dynamics are governed by the second-order linear difference equation: $y(n+2) + 6y(n+1) + 9y(n) = 2^n$, with initial conditions $y(0) = 0$, $y(1) = 0$. Find the complete solution $y(n)$ satisfying the initial conditions. **(10 M)**