VIT-AP UNIVERSITY	Final Assessment Test – Winter Semester Freshers 2024-25 - May - 2025	
	Maximum Marks: 100	Duration: 3 Hours
Course Code: MAT1002	Course Title: Applications of Differential and Difference Equations	
Set No: 06	Exam Type : Closed Book	School: SAS
Date: 24/05/2025	Slot: C2	Session: FN

Keeping mobile phone/smart watch, even in 'off' position is treated as exam malpractice

General Instructions if any:

1. **"fx series" - non Programmable calculator are permitted: YES**
2. **Reference tables permitted: NO**

Answer any TEN Questions, Each Question Carries 10 Marks (10×10=100 Marks)

1. We want to study the motion of a charged particle in the presence of magnetic field. The force acting on a charged particle of mass m in the presence of a uniform magnetic field $\vec{B}=B_0\hat{k}$ may be written as $\vec{F}=-q\left(\vec{V}\times\vec{B}\right)$ where $\vec{V}(t)=u(t)\hat{i}+v(t)\hat{j}$ is the velocity of the charged particle.
   i) Write the momentum equations. (Hint: Substitute $\vec{F}$ and $\vec{V}$ in Newton's law $m\frac{d\vec{V}}{dt}=\vec{F}$ and then write the $\hat{i}$ and $\hat{j}$ components)
   ii) Assume $u(0)=u_0$ and $v(0)=0$. Find the velocity expressions $u(t)$ and $v(t)$. Why speed of the particle is constant? Interpret your result.
   iii) Assume that $x(0)=0$ and $y(0)=0$. Find the displacements $x(t)$ and $y(t)$. What is the trajectory of the particle? Interpret your result.

2. Consider a simple harmonic oscillator of the form $\frac{d^2y}{dt^2}+4y=0$. Find the free motion of the system by assuming the initial conditions $y(0)=2$ and $\left(\frac{dy}{dt}\right)_{t=0}=0$. If the oscillator is driven by an input of the form $f(t)=\sin 2t$, find the forced motion of the system. Explain the result. What happened if you include the damping force to the system.

3. Consider a linear oscillator of the form $\frac{d^2y}{dt^2}+10\frac{dy}{dt}+16y=f(t)$, where $f(t)=\begin{cases}-2; & 0<t<4\\ 4; & t\geq 4\end{cases}$ and the initial conditions $y(0)=y'(0)=0$.
   i) Find the displacement of the system using Laplace transform.
   ii) Find $y(t)$ as $t\to\infty$. Obtain the same result using Final value theorem in Laplace transforms.
   iii) Write the transfer function of the system.

4. (A) The equation governing the build-up of charge $q(t)$ on the capacitor of an RC circuit is $R\frac{dq}{dt}+\frac{q}{c}=v_0$, where v_0 is the constant d.c voltage. Initially, the circuit is relaxed and the circuit is then closed at $t=0$ and so $q(0)=0$ is the initial condition for the charge. Use Laplace transform method to find the charge $q(t)$. Hence, write the charge $q(t)$ as $t\to\infty$.

(B) Consider response of certain falling body motion is given in terms of the convolution of the form $y(t)=\int_0^t \sin(t-\tau)\cos\tau d\tau$. Find the motion of the system.

5. Consider the mass-spring system with two masses governed by a system of DE of the form:

$$m_1\frac{d^2y_1}{dt^2}=-k_1y_1+k_2(y_2-y_1);\; m_2\frac{d^2y_2}{dt^2}=-k_2(y_2-y_1)$$

i) Write the matrix differential equation of the form $\frac{d^2Y}{dt^2}=AY$.

ii) If $m_1=m_2=1$ and $k_1=6;\ k_2=4$, find the frequencies and displacements of the system. Explain your results.

6. Consider a system governed by the coupled equations $\frac{dy_1}{dt}=-6y_1-2y_2$, $\frac{dy_2}{dt}=-2y_1-6y_2$ with the initial conditions $y_1(0)=2$ and $y_2(0)=0$. Solve the system using matrix diagonalization method. Explain your result with reference to any physical situation.

7. Consider a harmonic oscillator in quantum mechanics of the form $\frac{d^2u}{dx^2}-2x\frac{du}{dx}+2nu=0$.

i) Find the series solution about an ordinary point $x=0$.

ii) Write the first three Hermite polynomials $H_0(x), H_1(x)$ and $H_2(x)$. Note that, for integer values of $n=0,1,2,....$, one of the two series terminates, becoming a polynomial of degree n and the other series remains on infinite series. The arbitrary multiplicative constant chosen so that the coefficient of the term x^n is 2^n, the resulting solution is the Hermite polynomial $H_n(x)$).

8. Consider a saw-tooth wave of the form $f(x)=x$, where $0<x<1$.

i) Find the eigen function of $f(x)=x$ in terms of eigen functions $u_n(x)=\sin(n\pi x)$

ii) Find the eigen function of $f(x)=x$ in terms of eigen functions $u_n(x)=\cos(n\pi x)$

9. The dynamics of a certain LTI discrete system is determined by the second order linear difference equation of the form $y_{n+2}-\frac{3}{4}y_{n+1}+\frac{1}{8}y_n=u(n)$ with $y_0=y_1=0$, where $u(n)$ is a unit step

function. Find the output y_n using $Z-$transforms. Hence, find y_∞ using final value theorem. Is the system stable? Explain.

10. A body at a temperature of 80^0C is placed in a room of constant temperature 40^0C. If after 5 minutes the temperature of the body has decreased to 70^0C. When will the body be at a temperature of 50^0C as a difference equation formulation.

11. The potential energy of certain mass-spring system is represented by a quadratic form $Q=6x_1^2+6x_2^2+4x_1x_2$. Express this form into the linear combination of sum of the squares (Canonical form). What is the angle of rotation to get the required canonical form? Is the quadratic form positive definite? Write the relation between the old and new variables.

12. (A) The impulse response for a discrete system is given by $h_n=\begin{cases}1/2\,;\ for\ n=0,1\\ 0;otherwise\end{cases}$. Find the output. Also, find the Transfer function. Comment on the stability of the system.

(B) For a ramp function $f(x)=xH(x)$, where $H(x)$ is Heaviside unit step function. Find the first **THREE** coefficients in the Fourier Legendre series $f(x)=\sum_{n=0}^{\infty}a_nP_n(x)$, where $P_0(x)=1$, $P_1(x)=x$, $P_2(x)=\frac{3x^2-1}{2}$ and $a_n=\frac{2n+1}{2}\int_{-1}^{1}f(x)P_n(x)dx$