TRIBHUVAN UNIVERSITY	Exam.	Back (New Course)		
INSTITUTE OF ENGINEERING	Level	BE	Full Marks	60
Examination Control Division	Programme	All(Except BAR)	Pass Marks	24
2083 Baishakh	Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(ENSH 151)*

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

Group A (12×2)

1. a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

   b) If $u = xyz, v = xy + yz + zx, w = x + y + z$, find jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

2. a) Evaluate $\int_1^2 \int_0^x \frac{1}{x^2+y^2}\,dxdy$

   b) Find the mass of solid bounded by $z = 1-x^2$ and planes $z = 0$, $y = 1$, $y = -1$ with $\rho(x,y,z) = z(y+2)$.

3. a) Find the directional derivatives of the $\phi = 4x^2+3y-4z$ at (1,2,1) in the direction $2\vec{i}+2\vec{j}+\vec{k}$

   b) Prove that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is solenoidal.

   c) Find the $\nabla \times (\nabla\phi)$ where $\phi = x^3 + y^3 + z^3 - 3xyz$.

4. a) State existence criteria of the Laplace transform. Evaluate the Laplace transform of $\frac{1-e^t}{t}$

   b) Find the inverse Laplace transform of $\frac{1}{s^2 - 3s + 2}$

5. a) Find the rank of the matrix $\begin{pmatrix} 6 & 1 & 3 \\ 4 & 2 & 6 \\ 10 & 3 & 9 \end{pmatrix}$

   b) Find the eigen values of the matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

6. Express the function $f(x) = 1 + x - x^2$ in terms of Lagendre's polynomials.

Group B (9×4)

7. Find the extreme value of the function $u(x,y) = x^2 + xy + y^2 + 3z^2$ under the constraint $x+2y+4z = 60$.

8. Evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dxdy$ by changing the order of the integration.

9. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable t to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$

10. State Green's theorem in plane. Apply it to find the area of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

11. Apply Gauss divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$, where $\vec{F} = (2xy + z)\,\vec{i} + y^2\,\vec{j} - (x + 3y)\,\vec{k}$ and S is region bounded by the surface of the planes: $2x+2y+z = 6$, $x=0$, $y = 0$, $z = 0$.

OR

Apply Stoke's theorem to evaluate the line integral $\phi_c(xydx + xy^2dy)$ where C is the boundary of the square in xy-plane with vertices (-1,0), (1,0), (0,1) and (0,-1).

12. Solve the differential equation $y''(t) + 4y'(t) + 4y(t) = e^{-t}; y(0) = y'(0) = 0,$ using Laplace transform method.

13. Test the consistency of the system of linear equations and find their solution if found consistent.

$$2x-y+3z = 8$$
$$-x+2y+z = 4$$
$$3x+y-4z = 0$$

14. Reduce the quadratic form Q (X) = $5x_1^2 + 2x_2^2 + 2x_3^2 + 4x_1x_3 + 2x_2x_3 + 4x_1x_3$ into canonical form.

15. Solve $y''-4xy'+(4x^2 - 2)y = 0$ by power series method.

OR

Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$ where the symbols have their usual meanings.

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2082 Bhadra

Exam.	**Regular (New Course)**			
Level	BE		**Full Marks**	60
Programme	All	(Except BAR)	**Pass Marks**	24
Year / Part	I / II		**Time**	3 hrs.

Subject: - Engineering Mathematics II *(ENSH 151)*

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

Group A (12×2)

1. a) Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ where $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$.

   b) If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

2. a) Find the moment about x-axis of the lamina bounded by $y = x$ and $y = x^2$ with density $\rho(x,y) = 1$.

   b) Find the area of the region bounded by the lines $x = y^2, x = 2y - y^2$.

3. a) A particle moves along the curve $x = 3\cos t$, $y = 3\sin t$, $z = 6t$, then find the velocity and acceleration at time $t = \frac{\pi}{3}$.

   b) If $\vec{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$, then find $div\,\vec{F}$

   c) Find the work done by the force $\vec{F} = x^2\vec{\imath} - xy\vec{\jmath}$ along the path C given by $x^2 + y^2 = 1,\ z = 0$.

4. a) State existence condition for Laplace transform. Obtain the Laplace transform of $t\sin 3t$.

   b) Find the inverse Laplace transform of $F(s) = \cot^{-1}(s+1)$.

5. a) Find the rank of matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$.

   b) Reduce the quadratic form $Q(X) = 17x_1^2 - 30x_1x_2 + 17x_2^2$ into canonical form.

6. Express the function $f(x) = 4 + 2x + x^2$ in terms of Legendre's polynomials.

Group B (9×4)

7. Find the extreme values of the function $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$.

8. Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$.

9. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable t to have constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$.

10. State Green's theorem in plane and using it find the area of the hypocycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1.$

11. Use Gauss-Divergence theorem to evaluate $\iint_S \vec{F}\,.\,\hat{n}ds$, where $\vec{F}=(x^2-yz)\vec{i}+(y^2-zx)\vec{j}+(z^2-xy)\vec{k}$ where S is the surface of parallelopiped bounded by $x=0\,,x=a\,,\ y=0\,,y=b$ and $z=0\,,z=c$.

OR

Apply Stoke's theorem to evaluate the $\oint_C (x+y)dx+(2x-z)dy+(y+z)dz$ where C is the boundary of triangle with vertice $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$.

12. Using Laplace transform, solve the initial value problem

$$x''-3x'+2x=e^{-t},\, x(0)=0, x'(0)=1.$$

13. Apply Cayley-Hamilton theorem to find the inverse of matrix $\begin{pmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 2\end{pmatrix}$

14. Diagonalize the matrix $\begin{bmatrix}1 & 4\\ 3 & 2\end{bmatrix}$

15. Solve the differential equation $y''+xy'+y=0$ by using the power series method.

OR

Prove that $J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right\}$ where the symbols have their usual mearings.

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2081 Ashwin

Exam.	**Regular (New Course -2080 Batch)**		
Level	BE	**Full Marks**	60
Programme	All(Except BAR)	**Pass Marks**	24
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II *(SH 151)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) By using Euler's theorem show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ where, $u = \tan^{-1}(x^2 + 2y^2)$ [2]

   b) If $u = x^2$ and $v = y^2$, then find $\frac{\partial(u,v)}{\partial(x,y)}$ [2]

2. a) Evaluate $\int_0^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$ [2]

   b) Evaluate the mass of the solid region bounded by $z = 1 - x^2$ and planes $z = 0$, $y = 1, y = -1$ with $\rho(x, y, z) = z(y + 2)$ [2]

3. a) Find the directional derivative of $\Phi(x, y) = 4x^2 + 3y - 4z$ at $(1, 2, 1)$ in the direction of $2\vec{i} + 2\vec{j} + \vec{k}$ [2]

   b) A particle moves along the curve $x = \sqrt{2}\cos t,\ y = \sqrt{2}\sin t,\ z = 4t$, then find the velocity and acceleration at time $t = \frac{\pi}{4}$ [2]

   c) If $\phi = x^3 + y^3 + z^3 - 3xyz$, then find *div (grad* ϕ*)* at the point $(1, -1, 1)$ [2]

4. a) Find the Laplace Transform of the function: $\frac{1-e^t}{t}$ [2]

   b) Find the inverse Laplace transform of $\frac{s}{(s+2)^3}$ [2]

5. a) Find the rank of the following matrix: [2]

$$\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \end{bmatrix}$$

   b) Test whether the vectors $(1, 1, 1), (1, -1, 1)$ and $(2, 0, 3)$ are linearly independent or dependent. [2]

6. Express $2x^2 - 4x + 2$ as the Legendre's polynomials. [2]

7. Find the minimum value using Lagrange multiplier method of $x^2 + xy + y^2 + 3z^2$ subject to the condition $x + 2y + 4z - 60 = 0$. [4]

8. Change the order of integration and evaluate $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} dxdy$. [4]

9. Prove that the necessary and sufficient conditions for a vector function $\vec{a}$ of a scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$ [4]

10. Find the area of asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{2/3}$ using Green's theorem. [4]

11. Apply Gauss-Divergence theorem to evaluate $\int\int_s \vec{F}.\vec{n}ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ and s is the cylinder formed by the surface $z = 0,\ z = 1$, and $x^2 + y^2 = 4$ [4]

12. Using the Laplace transform technique, solve the initial value problem: [4]

$$y''(t) - y'(t) + 6y(t) = e^{-t},\ \ y(0) = 0, y'(0) = 0$$

13. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$ [4]

14. Reduce the quadratic form $Q(x) = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4\,x_1x_3$ into canonical form. [4]

15. Solve $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method. [4]

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