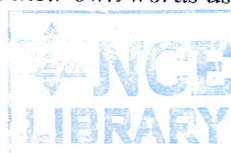


TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2083 Baishakh

Exam.	Back (New Course)		
Level	BE	Full Marks	60
Programme	All(Except BAR)	Pass Marks	24
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II (ENSH 151)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



Group A (12×2)

1. a) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.
- b) If $u = xyz, v = xy + yz + zx, w = x + y + z$, find jacobian $\frac{\partial(u, v, w)}{\partial(x, y, z)}$
2. a) Evaluate $\int_0^{2x} \int_0^1 \frac{1}{x^2 + y^2} dx dy$
- b) Find the mass of solid bounded by $z = 1 - x^2$ and planes $z = 0, y = 1, y = -1$ with $\rho(x, y, z) = z(y+2)$.
3. a) Find the directional derivatives of the $\phi = 4x^2 + 3y - 4z$ at $(1, 2, 1)$ in the direction $\vec{2i} + 2\vec{j} + \vec{k}$
- b) Prove that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is solenoidal.
- c) Find the $\nabla \times (\nabla \phi)$ where $\phi = x^3 + y^3 + z^3 - 3xyz$.
4. a) State existence criteria of the Laplace transform. Evaluate the Laplace transform of $\frac{1 - e^t}{t}$
- b) Find the inverse Laplace transform of $\frac{1}{s^2 - 3s + 2}$
5. a) Find the rank of the matrix $\begin{pmatrix} 6 & 1 & 3 \\ 4 & 2 & 6 \\ 10 & 3 & 9 \end{pmatrix}$
- b) Find the eigen values of the matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
6. Express the function $f(x) = 1 + x - x^2$ in terms of Legendre's polynomials.

Group B (9×4)

7. Find the extreme value of the function $u(x, y) = x^2 + xy + y^2 + 3z^2$ under the constraint $x+2y+4z = 60$.
8. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$ by changing the order of the integration.
9. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable t to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$
10. State Green's theorem in plane. Apply it to find the area of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$
11. Apply Gauss divergence theorem, evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = (2xy + z) \vec{i} + y^2 \vec{j} - (x + 3y) \vec{k}$ and S is region bounded by the surface of the planes: $2x+2y+z = 6, x=0, y=0, z=0$.

OR

Apply Stoke's theorem to evaluate the line integral $\oint_C (xy dx + xy^2 dy)$ where C is the boundary of the square in xy -plane with vertices $(-1,0), (1,0), (0,1)$ and $(0,-1)$.

12. Solve the differential equation $y''(t) + 4y'(t) + 4y(t) = e^{-t}; y(0) = y'(0) = 0$, using Laplace transform method.
13. Test the consistency of the system of linear equations and find their solution if found consistent.

$$2x - y + 3z = 8$$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$

14. Reduce the quadratic form $Q(X) = 5x_1^2 + 2x_2^2 + 2x_3^2 + 4x_1x_3 + 2x_2x_3 + 4x_1x_2$ into canonical form.
15. Solve $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method.

OR

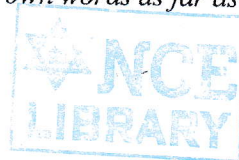
Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right\}$ where the symbols have their usual meanings.

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2082 Bhadra

Exam. Level	Regular (New Course)		
	BE	Full Marks	60
Programme	All (Except BAR)	Pass Marks	24
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II (ENSH 151)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



Group A (12x2)

1. a) Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ where $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$.
b) If $x = r \cos \theta$, $y = r \sin \theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
2. a) Find the moment about x -axis of the lamina bounded by $y = x$ and $y = x^2$ with density $\rho(x, y) = 1$.
b) Find the area of the region bounded by the lines $x = y^2$, $x = 2y - y^2$.
3. a) A particle moves along the curve $x = 3 \cos t$, $y = 3 \sin t$, $z = 6t$, then find the velocity and acceleration at time $t = \frac{\pi}{3}$.
b) If $\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$, then find $\text{div } \vec{F}$
c) Find the work done by the force $\vec{F} = x^2 \vec{i} - xy \vec{j}$ along the path C given by $x^2 + y^2 = 1$, $z = 0$.
4. a) State existence condition for Laplace transform. Obtain the Laplace transform of $t \sin 3t$.
b) Find the inverse Laplace transform of $F(s) = \cot^{-1}(s + 1)$.
5. a) Find the rank of matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$.
b) Reduce the quadratic form $Q(X) = 17x_1^2 - 30x_1x_2 + 17x_2^2$ into canonical form.
6. Express the function $f(x) = 4 + 2x + x^2$ in terms of Legendre's polynomials.

Group B (9x4)

7. Find the extreme values of the function $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$.
8. Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$.
9. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable t to have constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$.

10. State Green's theorem in plane and using it find the area of the hypocycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

11. Use Gauss-Divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ where S is the surface of parallelopiped bounded by $x = 0, x = a, y = 0, y = b$ and $z = 0, z = c$.

OR

Apply Stoke's theorem to evaluate the $\oint_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of triangle with vertice $(2,0,0), (0,3,0)$ and $(0,0,6)$.

12. Using Laplace transform, solve the initial value problem

$$x'' - 3x' + 2x = e^{-t}, x(0) = 0, x'(0) = 1.$$

13. Apply Cayley-Hamilton theorem to find the inverse of matrix $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

14. Diagonalize the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

15. Solve the differential equation $y'' + xy' + y = 0$ by using the power series method.

OR

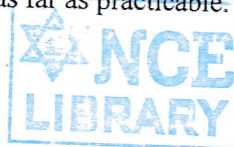
Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right\}$ where the symbols have their usual meanings.

TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2081 Ashwin

Exam.	Regular (New Course -2080 Batch)		
Level	BE	Full Marks	60
Programme	All(Except BAR)	Pass Marks	24
Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II (SH 151)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



1. a) By using Euler's theorem show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
where, $u = \tan^{-1}(x^2 + 2y^2)$ [2]
- b) If $u = x^2$ and $v = y^2$, then find $\frac{\partial(u,v)}{\partial(x,y)}$ [2]
2. a) Evaluate $\int_0^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx dz$ [2]
- b) Evaluate the mass of the solid region bounded by $z = 1 - x^2$ and planes $z = 0$, $y = 1$, $y = -1$ with $\rho(x, y, z) = z(y + 2)$ [2]
3. a) Find the directional derivative of $\Phi(x, y) = 4x^2 + 3y - 4z$ at $(1, 2, 1)$ in the direction of $2\vec{i} + 2\vec{j} + \vec{k}$ [2]
- b) A particle moves along the curve $x = \sqrt{2} \cos t$, $y = \sqrt{2} \sin t$, $z = 4t$, then find the velocity and acceleration at time $t = \frac{\pi}{4}$ [2]
- c) If $\phi = x^3 + y^3 + z^3 - 3xyz$, then find $\text{div}(\text{grad } \phi)$ at the point $(1, -1, 1)$ [2]
4. a) Find the Laplace Transform of the function: $\frac{1-e^t}{t}$ [2]
- b) Find the inverse Laplace transform of $\frac{s}{(s+2)^3}$ [2]
5. a) Find the rank of the following matrix: [2]
$$\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \end{bmatrix}$$
- b) Test whether the vectors $(1, 1, 1)$, $(1, -1, 1)$ and $(2, 0, 3)$ are linearly independent or dependent. [2]
6. Express $2x^2 - 4x + 2$ as the Legendre's polynomials. [2]
7. Find the minimum value using Lagrange multiplier method of $x^2 + xy + y^2 + 3z^2$ subject to the condition $x + 2y + 4z - 60 = 0$. [4]
8. Change the order of integration and evaluate $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} dx dy$. [4]
9. Prove that the necessary and sufficient conditions for a vector function $\vec{a}$ of a scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$ [4]

10. Find the area of asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{2/3}$ using Green's theorem. [4]
11. Apply Gauss-Divergence theorem to evaluate $\int \int_s \vec{F} \cdot \vec{n} ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ and s is the cylinder formed by the surface $z = 0$, $z = 1$, and $x^2 + y^2 = 4$ [4]
12. Using the Laplace transform technique, solve the initial value problem: [4]
 $y''(t) - y'(t) + 6y(t) = e^{-t}$, $y(0) = 0$, $y'(0) = 0$
13. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$ [4]
14. Reduce the quadratic form $Q(x) = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ into canonical form. [4]
15. Solve $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method. [4]
