TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2082 Chaitra

Exam.	Regular (New Course)		
Level	BE	**Full Marks**	60
Programme	All except BAR	**Pass Marks**	24
Year / Part	II / I	**Time**	3 hrs.

Subject: - Engineering Mathematics (*ENSH 201*)

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) In the series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, find the coefficient b_n for the function $f(x) = e^{-x}$ where $x \in (0, 2\pi)$. [2]

   b) Find the Fourier integral representation of $f(x) = \begin{cases} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$ [2]

   c) Solve the integral equation $\int_0^{\infty} f(x)\cos \alpha x\, dx = e^{-\alpha}, \alpha > 0$. [2]

2. a) Define harmonic Function. Show that the function $u(x,y) = e^x cosy$ is harmonic. [2]

   b) Find the bilinear transformation which maps the points $z = 0, -1, i$ $into$ $w = i, 0, \infty$. [2]

   c) Evaluate $\int_c \frac{5z-2}{z-1} dz$, C being the circle $|z| = 2$. [2]

3. a) Find the residue of $f(z) = \frac{2z+1}{(z-3)(z+5)}$ at each of the singularities. [2]

   b) Find the Taylor series expansion of $f(z) = \frac{1}{z-3}$ centered at the point $z = 4i$ [2]

4. a) Solve the partial differential equation
   $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given that $\frac{\partial z}{\partial y} = -2\sin y$; when $x = 0$, $z = 0$; $y = \frac{\pi}{2}$ [2]

   b) Write down the classification rules for general second order partial differential equation and hence use it to classify the equation $u_{xx} + x^2 u_{yy} = 0$. [2]

5. a) Find x (0) and x (1) of Z-transform $x(t) = 1 - e^{-t}$. [2]

   b) Find inverse z-transform of $\frac{z}{(z-1)(z-2)}$ [2]

6. If $f(x) = x + x^2$ for $-\pi \le x \le \pi$, then find the Fourier series of $f(x)$. [4]

7. Find Fourier cosine integral of $f(x) = e^{-kx}, k > 0, x > 0$ and use it to evaluate $\int_0^{\infty} \frac{\cos \omega x}{k^2+\omega^2} d\omega$ [4]

8. Find Fourier sine transform of $f(x) = e^{-x}$ and hence use the result in Parseval's identity to show $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)^2} = \frac{\pi}{4}$. [4]

9. Express the function $\frac{1}{z^2 - 3z + 2}$ in Laurent series form in the region $1 < |z| < 2$. [4]

10. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\sin\theta} d\theta$ using method of contour integration in the complex plane. [4]

11. Apply Charpit's method to solve the PDE: $2zx - px^2 - 2xy + pq = 0$, where the symbols have their usual meanings. [4]

OR

Solve the PDE: $z(p-q) = z^2 + (x+y)^2$, using Lagrange's method where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

12. Using Z-transform technique, solve the difference equation

$x(k+2) - 4x(k+1) + 4x(k) = 0$ under $x(0) = 0$, $x(1) = 1$. [4]

13. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ completely. [4]

OR

Solve the wave equation $u_{tt} = c^2 u_{xx}$ under boundary conditions

$u(0,t) = 0, u(\pi,t) = 0$ and initial conditions $\frac{\partial u}{\partial t} = 0$ at $t = 0$ and $u(x, 0) = x$ for $0 < x < \pi$.

14. Derive Navier-Stoke's equation of motion for fluid flow. [4]

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TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
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2082 Kartik

Exam.	Back (New Course)			
Level	BE		Full Marks	60
Programme	All	(Except BAR)	Pass Marks	24
Year / Part	II / I		Time	3 hrs.

Subject: - Engineering Mathematics III *(ENSH 201)*

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Find half range sine series for the function $f(x) = \pi - x$ in $0 < x < \pi$. [2]
   b) Find Fourier integral representation of $f(x) = e^{-|x|}$. [2]
   c) Determine Fourier cosine transform of $f(x) = \begin{cases} 1 \ ; 0 < x < a \\ 0 \ ; \text{otherwise} \end{cases}$. [2]
2. a) Show that the function $u(x, y) = 2x + x^3 - 3xy^2$ is harmonic. [2]
   b) What will be the image of circle $|z - i| = 1$ under the mapping $w = f(z) = \frac{1}{z}$ in complex plane? [2]
   c) Expand $f(z) = e^{-z}$ in a Taylor's series about $z = 0$. [2]
3. a) Find the residue of $f(z) = \frac{3z-4}{(z-1)(z-2)}$ at each of its singularities. [2]
   b) Evaluate : $\oint_C \frac{e^z}{(z-1)(z-2)} dz$ over the circle $C: |z| = 3$. [2]
4. a) Classify the PDE: : $x^2\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + x\frac{\partial^2 u}{\partial y^2} + 17\frac{\partial u}{\partial t} = 100u$. [2]
   b) Solve the PDE : $y^2u_x - x^2u_y = 0$ by the method of separation of the variables. [2]
5. a) Find $x(\infty)$ using the final value theorem of z- transform when $x(t) = 1 - e^{-at}$. [2]
   b) Apply Z- transform to find the sum of infinite series: $\sum_{k=0}^{\infty} e^{ak}$ for a > 0. [2]
6. Find Fourier series representation of $f(x) = x - x^2$ for $-1 \leq x \leq 1$. [4]
7. Obtain Fourier integral of the function $f(x) = \begin{cases} e^{-x}; & x > 0 \\ 0; & x < 0 \end{cases}$. Hence evaluate $\int_0^{\infty} \frac{1}{1+x^2} dx$. [4]
8. Solve the integral equation $\int_0^{\infty} f(x) \sin \omega x \, dx = \begin{cases} 1; & 0 \leq \omega < 1 \\ 2; & 1 \leq \omega < 2 \\ 0; & \omega \geq 2 \end{cases}$, using Fourier transform. [4]
9. An electrostatic field in a plane is given by the potential function $\phi(x, y) = 3x^2y - y^3$. Find the corresponding stream function and complex potential $f(z)$. [4]

   OR

   Find bilinear transform that maps the points $1, i, -1$ in z-plane into the points $0, 1, \infty$ into w plane.
10. Evaluate the real integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ using Cauchy's residue theorem. [4]
11. Apply Charpit's method to find complete integral of $2xz - px^2 - 2qxy + pq = 0$, where the symbols have their usual meanings. [4]

    OR

    Apply Lagrange's method to solve the PDE: $y^2p - xyq = x(z - 2y)$, where the symbols have their usual meanings.

12. Derive two dimensional heat equation for steady state heat flow. [4]

OR

Derive Navier-Stoke equation of fluid flow.

13. A tightly stretched string fixed at two ends $x = 0$ and $x = \pi$ is initially at rest in equilibrium position. If it is set in vibration by giving to each of its points a velocity $\frac{\partial u}{\partial t} = 0.03 \sin x - 0.04 \sin 3x$, find the deflection $u(x, t)$ at any point of the string at any time. [4]

14. Apply Z-transform to solve the difference equation $x(k + 2) - x(k + 1) - 2x(k) = 0$, given that $x(0) = 0, x(1) = 1$. [4]

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INSTITUTE OF ENGINEERING

Examination Control Division

2081 Chaitra

Exam.	**Regular (New Course)**		
Level	BE	**Full Marks**	60
Programme	All(Except BAR)	**Pass Marks**	24
Year / Part	II / I	**Time**	3 hrs.

Subject: - Engineering Mathematics III *(ENSH 201)*

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks****.*
- ✓ *Assume suitable data if necessary.*

1. a) Find the half range Fourier sine series of the function $f(x) = x$ in the interval $(0,2)$. [2]

   b) Find the Fourier cosine integral of $f(x) = e^{-ax}$ for $x > 0$, $a > 0$. [2]

   c) Find the Fourier transform of $f(x) = \begin{cases} 1 \text{ if } |x| < 1 \\ 0 \text{ if } |x| > 1 \end{cases}$ [2]

2. a) For a complex variable , show that $f(z) = \sin z$ is analytic. [2]

   b) Find the image of the circle $|z - i| = 1$ under the mapping $w = \frac{1}{z}$. [2]

   c) Show that $\int_C \frac{1}{z-a}\, dz = 2\pi i$ where C is a unit circle $|z - a| = 1$. [2]

3. a) Expand the function $f(z) = \frac{1}{z}$ about $z = i$ in Taylor's series. [2]

   b) Find the residue of $f(z) = \frac{3z-4}{(z-2)(z-3)}$ at each of the singularities. [2]

4. a) Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 12x^2(t+1)$ subject to $u(0,t) = \cos 2t$ and $u_x(0,t) = \sin t$ [2]

   b) Find the solution of the equation $3u_x + 2u_y = 0$, by the method of the separation of the variables. [2]

5. a) If $Z[x(k)] = X(z)$, show that $x(1) = \lim_{z\to\infty} z[X(z) - x(0)]$, where the symbols have their usual meaning. [2]

   b) If $Z[x(k)] = X(z)$ and $y(k) = \sum_{m=0}^{k} x(m)$, then show that $Y(z) = \frac{z}{z-1} X(z)$. [2]

6. Find the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the relation. [4]

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

7. Find the Fourier integral of $f(x) = \begin{cases} e^{-x}; x > 0 \\ 0; x < 0 \end{cases}$ and hence show that [3+1]

$$\int_0^\infty \frac{\cos \omega x + \omega \sin \omega x}{1+\omega^2}\, d\omega = \begin{cases} \pi e^{-x} & ; x > 0 \\ \frac{\pi}{2} & ; x = 0 \\ 0 & ; x < 0 \end{cases}$$

8. Find the Fourier transform of $f(x) = e^{-ax}$ and use the result to show

$$\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)} = \frac{\pi}{2ab(a+b)}$$ [2+2]

9. Prove that $u = y^3 - 3x^2y$ is harmonic function. Find its harmonic conjugate and hence construct corresponding analytic function $w = f(z)$. [2+2]

10. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$, using method of contour integration in the complex plane. [4]

OR

Evaluate $\oint_C \frac{\sin z}{z^2} dz$, where C is any closed path not passing through 0.

11. Apply Charpit's method to solve the PDE: $p^2x + q^2y = z$, where the symbols have their usual meanings. [4]

OR

Solve the partial differential equation $(y - z)p + (x - y)q = (z - x)$, using Lagrange's method.

12. Derive one dimensional wave equation representing the vibration of the particles a string. [4]

OR

Derive Navier-Stoke equation of fluid flow.

13. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to $u(0,t) = u(\ell,t) = 0$ and $u(x,0) = u_0 \sin\frac{\pi x}{\ell}$ [4]

14. Apply Z-transform to solve the difference equation $x(k+2) - 4x(k+1) - 5x(k) = 0$, given that $x(0) = 0, x(1) = 1$. [4]

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