TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2082 Chaitra

Exam.	Regular (New Course)		
Level	BE	Full Marks	60
Programme	BCE	Pass Marks	24
Year / Part	II / I	Time	3 hrs.

Subject: - Numerical Methods (*ENSH 202*)

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

LIBRARY

1. Approximate a real root of the non-liner equation $3x + \sin x - e^x = 0$ correct to three decimal places using the Bisection Method.

2. Solve the following system of non-linear equations using Newton-Raphson Method for a precision of 3 decimal places by taking (1.8, 1.8) as the initial guess values.

$$x^4 - \cos y = 12$$
$$\sin x + y^4 = 15$$

3. Write a python program to solve a system of linear equations using the Gauss-Jordan method.

4. Find the dominant eigen value and corresponding eigen vector of the following matrix using the Power method.

$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 2 & 6 \\ 6 & 6 & 2 \end{pmatrix}$$

5. Using the method of least squares, fit the following data to a second-degree polynomial of the form $y = ax^2 + bx + c$.

x	2	4	6	8	10	12
y	2.5	2.7	3.5	6.5	5.4	4.8

6. Construct divided difference table from the following data and use Newton's divided difference interpolation technique to approximate the value of $y(9)$.

x	2	5	7	8	10
y	8.10	6.72	6.70	8.04	15.22

7. Derive expressions for evaluating the first and second order derivatives using Newton's forward interpolation formula.

8. Approximate the following integral using Gauss-Legendre 3-point formula.

$$\int_1^2 e^{\sin x} dx$$

9. Given that $y' = \sin(\pi x) + \cos(y)$ and $y(0) = 2$, find the values of y in $0 \le x \le 1$, taking a step-size of $h = 0.5$ using the Runge-Kutta fourth order method.

10. The results of two initial attempts while solving the two-point boundary value problem $y'' = 0.5\,x - y + y' + 3,\ y(0) = 5,\ y(2) = 8$ using the shooting method with Euler's formula taking a step-size of 0.5 is given below.

Initial Slope (y') guessed	Final y obtained
$m_1 = 1.0$	4.7187
$m_2 = 3.0$	11.5938

Using this information, solve the given problem.

11. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ on a squaregrid with $0 \le x \le 2$ and $0 \le y \le 2$ with step-size $h = 2/3$ in both directions having the following boundary conditions.

$$u(x,0) = 240x, \qquad u(0,y) = 180y,$$
$$u(x,2) = 180(2-x), \qquad u(2,y) = 240(2-y)$$

12. Solve the wave equation $u_{tt} = 4u_{xx}$ for 5 time steps taking $\Delta x = h = 0.4$ with the following boundary and initial conditions:

$$u(0,t) = u(2,t) = 0$$
$$u(x,0) = 5x(2-x) \text{and } u_t(x,0) = 0.$$

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1. Discuss the major difference between Secant method and Regula Falsi method. Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to four decimal places using the secant method.

2. Write Jacobian matrix for the following system of non-linear equations and use Newton Raphson method to approximate the solution correct to two decimal places taking $(x_0, y_0) = (0.9, 0.5)$ as initial guess values.
$$x^2 + y^2 = 1$$
$$x^3 - y = 0$$

3. Find the inverse of the following matrix using Gauss Jordan method.
$$\begin{pmatrix} -2 & 6 & 4 \\ 4 & -6 & 0 \\ -2 & 4 & 2 \end{pmatrix}$$

4. Write a python program to determine the dominant eigen value and corresponding eigen vector of a square matrix using the power method.

5. Using the least square method, fit the following data to a curve of the form $y = ab^x$.

x	5	15	25	35	45	55
y	1.8	7.1	16.6	30.2	47.5	68.5

6. Estimate $y(5)$ and $y(22)$ from the following data using appropriate polynomial interpolation technique used for equally spaced intervals.

x	4	8	12	16	20	24
y	15.4	16.5	14.2	10.8	8.6	9.9

7. Evaluate the following integral with an accuracy of three decimal places using Romberg's method starting from $h = 0.5$.
$$\int_{-1}^{1} \frac{2}{1+x^2}\,dx$$

8. Evaluate the following integral using Simpson's three-eighth rule and Weddle's rule by dividing the range into six equally spaced intervals and compare the results with the exact solution.
$$\int_{0}^{3\pi} (\sin x + 5)\,dx$$

9. Solve the following initial value problem for $y(0.2)$ and $y(0.5)$ using Runge-Kutta fourth order method (take $h = 0.2$ and $h = 0.3$ respectively).
$$y' = \sqrt{x+y} - 1, \qquad y(0) = 1$$

10. Using finite difference method, solve the following boundary value problem for three internal nodes: $y'' - 3y' + 2y = 4x - 8,\ y(2) = 3,\ y(4) = 5$

11. Write recurrence relation for solving Poisson's equation $u_{xx} + u_{yy} = f(x, y)$ and hence use it to solve partial differential equation $u_{xx} + u_{yy} = 243x^2y$ on domain $0 \le x \le 1$ and $0 \le y \le 1$ with $u = 30$ on the boundary and mesh length $h = 1/3$.

12. Derive the recurrence relation for solving one dimensional heat equation using Bender-Schmidt method and use it to solve the following problem for five time steps taking $\Delta x = h = 1$.

$$u_t = 0.25\, u_{xx}, \qquad u(0,t) = u(5,t) = 0, \qquad u(x,0) = x(5-x)$$

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11. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ for a square mesh with the following boundary conditions using the finite difference method employing Gauss-Seidal iteration method to solve the equations.

$$0 \le x \le 1, \qquad 0 \le y \le 1,$$
$$\Delta x = \Delta y = h = 1/3,$$
$$u(x,0) = 360x, \qquad u(x,1) = 240(1-x),$$
$$u(0,y) = 240y, \qquad u(1,y) = 360(1-y)$$

12. Derive the recurrence relation for solving one dimensional heat equation using Bender-Schmidt method and use it to solve the following problem for five time steps taking $\Delta x = h = 1$.

$$u_t = 0.5\, u_{xx}, \qquad u(0,t) = u(4,t) = 0, \qquad u(x,0) = 2x(4-x)$$

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TRIBHUVAN UNIVERSITY
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1. Find a real root of $e^{0.5x+3} + \cos(0.4x) - 50 = 0$ precise to 4 significant figures using the bisection method.

2. Write Jacobian matrix for the following system of non-linear equations and use Newton-Raphson method to approximate a solution correct to two decimal places taking $(x_0, y_0) = (2, 1)$ as initial guess values.
$$x^3 - y^2 = 8$$
$$x^2 + y^3 = 6$$

3. Solve the following system of linear equations using Do-Little's Factorization method.
$$\begin{bmatrix} 2 & 5 & 4 \\ 3 & 4 & 5 \\ 5 & 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ 28 \\ 32 \end{bmatrix}$$

4. Find dominant eigen value and corresponding vector of the given matrix using the power method.
$$\begin{bmatrix} 2 & 4 & 1 \\ 4 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

5. Construct divided difference table from the following data and use Newton's divided difference interpolation technique to approximate y (9).

x	0	2	5	7	8	10
y	4.72	7.1	5.72	5.7	7.04	14.22

6. Using the least square method, fit the following data to a curve of the form $y = ae^{bx}$.

x	2	4	6	8	10
y	4.1	11.2	30.2	81.9	204.6

7. Evaluate the following integral using Gauss-Legendre 3-point formula and compare the result with exact value of the integral.
$$\int_0^4 \left(\cos\left(\frac{\pi x}{2}\right) + 5\right) dx$$

8. Derive the expressions to evaluate first and second order derivatives from a set of tabulated data using Newton's forward interpolation formula. Also derive the expressions for the initial point.

9. Write a python program to solve an initial value problem involving a first order ordinary differential equation using Runge-Kutta fourth order method. The program should also show the results graphically.

10. Using finite difference method, solve the following boundary value problem for three internal nodes.
$$y'' - 5y' + 4y = 5x, \quad y(2) = 4, \quad y(4) = 5$$