Bar Model





Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.

Bead Strings

-000-000-000-000-

 $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 3 = 5$

 $5 \times 3 = 15$ $3 \times 5 = 15$ $15 \div 5 = 3$

-0000-0000-0000-0000-

$$4 \times 5 = 20$$

 $5 \times 4 = 20$ $20 \div 4 = 5$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$$6 \times 3 = 18$$

 $3 \times 6 = 18$



 $18 \div 3 = 6$

Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)





$$4 \times 5 = 20$$

 $5 \times 4 = 20$



Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach O.

Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

 $20 \div 4 = 5$

Number Lines (blank)



A red car travels 3 miles. A blue car 4 times further. How far does the blue car travel?



Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)



24 × ₁3 72



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Base 10/Dienes (division)



$$68 \div 2 = 34$$

Tens	Ones

$$72 \div 3 = 24$$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the partwhole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)







Х	40	4
30	1200	120
2	80	8



1408

Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)







Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

Times Tables

Skill	Year	Representatio	ns and models
Recall and use	4	Bar model	Ten frames
multiplication and		Number shapes	Bead strings
division facts for the		Counters	Number lines
2-times table		Money	Everyday objects
Recall and use	4	Bar model	Ten frames
multiplication and		Number shapes	Bead strings
division facts for the		Counters	Number lines
5-times table		Money	Everyday objects
Recall and use	4	Hundred square	Ten frames
multiplication and		Number shapes	Bead strings
division facts for the		Counters	Number lines
10-times table		Money	Base 10

Skill	Year	Representation	Representations and models		
Recall and use multiplication and division facts for the 3-times table	4	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects		
Recall and use multiplication and division facts for the 4-times table	4	Hundred squareBead string4Number shapesNumber lingCountersEveryday obj			
Recall and use multiplication and division facts for the 8-times table	5	Hundred square Number shapes	Bead strings Number tracks Everyday objects		
Recall and use multiplication and division facts for the 6-times table	5	Hundred square Number shapes	Bead strings Number tracks Everyday objects		

Skill	Year	Representations and models			
Recall and use multiplication and division facts for the 7-times table	5	Hundred square Number shapes	Bead strings Number lines		
Recall and use multiplication and division facts for the 9-times table	5	Hundred square Number shapes	Bead strings Number lines		
Recall and use multiplication and division facts for the 11-times table	5	Hundred square Base 10	Place value counters Number lines		
Recall and use multiplication and division facts for the 12-times table	5	Hundred square Base 10	Place value counters Number lines		









Primary 4

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.



							1	
							11	
							21	
ļ		6		\bigcirc	\bigcirc		31	(
0	()	00	2	$(\cdot \cdot)$	()		41	
S	25	Sp	33	1 A	Eng	5	51	
		10	v c		ر » ح		61	
	8	IC)	24	52		71	(
а					r	1	81	
	8	16	24	32	40		91	
	48	56	64	72	80			

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Skill: 8 times table



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Primary 5

Skill: 6 times table

Primary 5

|--|--|--|

6	12	18	24	3 <mark>0</mark>
3 <mark>6</mark>	42	4 <mark>8</mark>	5 4	6 <mark>0</mark>
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	64	55	56	57	58	59	60
51 61	52 62	53 63	6 4	55 65	56 66	57 67	58 68	59 69	60 70
51 61 71	52 62 72	53 63 73	6 4 74	55 65 75	56 66 76	57 67 77	58 68 78	59 69 79	60 70 80
51 61 71 81	52 62 72 82	53 63 73 83	647484	55 65 75 85	56 66 76 86	57 67 77 87	58 68 78 88	59 69 79 89	60 70 80 90





Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

9	18	27	36	45
54	63	72	81	90

Skill: 9 times table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	5 <u>3</u>	64	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

Primary 5

2				
7	14	21	28	35
42	49	56	63	70

Skill: 7 times table	
----------------------	--

								-	
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
2	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	<u>49</u>	50
51	52	53	54	55	<u>56</u>	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100





Primary 5

Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

	Primary 5		
11 22 33 44 77 88 99 110 10 1 10 1 10 1 10 1 10 1 10 1 10 1 10 1	55 66 121 132 10 1 10 1 10 1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 83 89 90 91 92 93 94 95 96 97 98 99 100	Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the eleven times
	44 55 66	77 88 99 110 121 132	table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100



Primary 5

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

Multiplication

Skill	Year	Representations and models			
Solve one-step problems with multiplication	2/3	Bar model Number shapes Counters	Ten frames Bead strings Number lines		
Multiply 2-digit by 1- digit numbers	4	Place value counters Base 10	Short written method Expanded written method		
Multiply 3-digit by 1- digit numbers	5	Place value counters Base 10	Short written method		
Multiply 4-digit by 1- digit numbers	6	Place value counters	Short written method		

Skill	Year	Representations and models			
Multiply 2-digit by 2- digit numbers	6	Place value counters Short written method Base 10 Grid method			
Multiply 2-digit by 3- digit numbers	7	Place value counters Short written method Grid method			
Multiply 2-digit by 4- digit numbers	7	Formal written method			







Skill: Multiply 4-digit numbers by 1-digit numbers



1,826 × 3 = 5,478

	Th	Н	Т	0
	1	8	2	6
×		2	1	3
	5	4	7	8

Primary 6

When multiplying 4digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.



Primary 6

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10 The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers



	100	100	10	10	10		Tł
10	1.000	1.000	100	100	100	10 10 10 10	
•	1000	1.000	100	100	100	10 10 10 10	×
•	1.000	1.000	100	100	100	10 10 10 10	
	100	100	10	10	10		7
	100	100			10		7

Children can continue to use the area model when multiplying 3digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

234	X	32	=	7,488
-----	---	----	---	-------

×	200	30	4
30	6,000	900	120
2	400	60	8

Н

2

4

0

4

Т

3

3

6

2

8

0

4

2

8

0

8

l: Multiply 4-c	numbers Primary 7				
TTh	Th	Н	Т	0	When multiplying 4- digits by 2-digits, children should be
	2	7	3	9	confident in the written method.
×	5	3	2	87	If they are still struggling with times
2	1	9	1	2	tables, provide multiplication grids to
5	1 4	7	1 8	0	are focusing on the use of the method.
7	6	6	9	2	Consider where
					exchanged digits are

placed and make sure this is consistent.

2,739 × 28 = 76,692



Skill	Year	Representations and models			
Solve one-step problems with division (sharing)	2/3	Bar model Real life objects	Arrays Counters		
Solve one-step problems with division (grouping)	3	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters		
Divide 2-digits by 1- digit (no exchange sharing)	4	Straws Base 10 Bar model	Place value counters Part-whole model		
Divide 2-digits by 1- digit (sharing with exchange)	Divide 2-digits by 1- digit (sharing with 4 Base exchange) Bar mo		Place value counters Part-whole model		

Skill	Year	Representations and models	
Divide 2-digits by 1- digit (sharing with remainders)	4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1- digit (grouping)	4	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1- digit (sharing with exchange)	5	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1- digit (grouping)	5	Place value counters Counters	Place value grid Written short division

Skill	Year	Representations and models	
Divide 4-digits by 1- digit (grouping)	6	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	7	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	7	Written long division	List of multiples

Skill: Solve 1-step problems using multiplication (sharing) Primary 2/3 I have 10 cubes, Children solve 20 can you share them equally in 2 problems by sharing groups? amounts into equal ? ? ? ? ? groups. Children use concrete and pictorial representations to There are 20 apples altogether. solve problems. They They are shared equally between 5 bags. are not expected to How many apples are in each bag? record division formally.

















Skill: Divide 3-digits by 1-digit (grouping)

Primary 5

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

1

5

4

1₆

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.





Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient - The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor