

Mathematical Modeling of Multiphase Oil-Water Separation

¹Ekeng, Emmanuel E, ²Agunwamba, Jonah C

¹Department of Civil Engineering, Cross River University of Technology, Calabar,

²Department of Civil Engineering, University of Nigeria, Nsukka.

ekengemmanuel@yahoo.com¹ nwa mbaagu@yahoo.com²

Abstract.

The manuscript describes oil-water separation based model. Over the years effluent quality from oil and gas processes sent to the environment has been a major problem to stake holders. A multiple regression model and Buckingham pi- model were used for the prediction of separation efficiency of some physical models constructed to examine, improve produced water from oil and gas industry sent to the receiving waters. Also some operating variables militating against quality effluent were studied like pressure and retention time. The study intended measuring, improving separation efficiency of some physical models in a micro scale, calibrate, verify them and compare their efficiencies with conventional separators. The oil-water separation models reveal that better separation efficiency was achieved from the multiple regression models than the Buckingham pi-method. Verification of the model also gave a good coefficient of correlation of $R=84.3\%$ between the measured and calculated separation efficiency at a significant level of 0.05. Coefficient of determination showed that 71.0% of the variation of the experimental results was explained by the developed multiple regression model. The operating variables of retention time, pressure were major contributors to separation efficiency Therefore, the prediction of separation efficiency in the physical model was possible through pressure analysis of film drainage, film rupturing due to coalesces and also from the developed multiple regression model than the Buckingham pi method mode.

Keywords— Pressure, concentration, flow rate, multiple regression, separation efficiency.

1. INTRODUCTION

Produced water is the largest waste stream generated in oil and gas industries. It is a mixture of different organic and inorganic compounds (Ahmadu et al. 2009). American petroleum institute has estimated that in 2012 that the average water to oil ratio was 10 barrels of water for every 1 barrel of oil produced with a daily cost for water treatment reaching \$125,000 in certain oil field (Kelland, 2014). Modelling of oil-water separation might not be new. However, a model may be seen as a description of a system using mathematical concepts and language. Hence, this would metaphor into a statement of an equality containing one or more variables called equations. This study is expected to improve on mathematical formulation describing oil-water separation due to high cost of produced water treatment with chemicals. Model equations were developed to calibrate and verified the physical models based on Buckingham pi-method using dimensional analysis and regression of nonlinear equation. However, dimensional analysis offers a method for reducing complex physical problems to simple form (Buckingham, 1914). Also, dimensional analysis had been applied in the past in various fields of human endeavours like hydraulic, heat and mass transfer, processes (Sonin, 2001) and chemical reactions (Lokarnik, 1991) This study might also help explained liquid-liquid separation in a gravity vessel both analytically and the effect of some physiochemical variables and properties. If this is achieved, the study would impart positively or benefit oil and gas industries in terms of optimization of oil production, reduction in pollution of receiving waters, produced water re-injection, design and cost reduction in terms of produced water treatment. Utilities companies, water treatment entities including the academia and the reading public, are not left out as direct beneficiaries of this study.

1.1 Aim and Objectives

Mathematical modeling of multiphase oil-water separation. The objectives were: To determine effects of the operating variables such as, velocities, flow rates, pressures, detention time, sample concentration, geometrical factors. To calibrate and verify the models using experimental data.

2.0 Materials and Methods

A transparent model of a perpex glass was used for constructing the vessel for fluid flow studies, in which produced water from hydrocarbon processes from one of the production field OML 114 in Nigeria was used. The API gravity of the crude oil was about 28° API, viscosity 1×10^{-3} Pa.s, density 700 kg/m^3 , temperature of 44°C, Bs&w of 0.2% light crude at average operating condition. The colour of crude oil from this field was brownish. About five perpex glass horizontal separator models were constructed for the research. They were all varies in length and width whilst the height was kept constant. The internals like the weirs, baffles or momentum breakers were kept constant in term of length, width and height.

These models operated at atmospheric pressure where gauge pressure was assumed zero. The inflow into the vessel was a continuous one. The stream of liquid from a model reservoir/tank aided by force of gravity impinges upon the baffle or momentum breaker inside the models. This momentum breaker was mounted opposite the inlet. The purpose of the momentum breaker was to allowed free gas to break out of the liquid jet if any, lower flow velocities and to prevent the stream from mixing directly with the fluid undergoing gravity separation already. Some of the baffle plates were perforated to provide resistance to the fluid flow, and also functions as a manifold to distribute the flow relatively, uniformly over its surface. However, at the inlet where the momentum breaker was located, the flow rate can be read off through a meter (K24 Turbine flow meter) at the inlet and the outlet of the models.

The fluid flow rate, velocity, pressure And retention time were also determined while a constant head was maintained using a variable frequency drive control pump throughout the model simulation.

In most of the reviews from the literature no single studies have actually addressed oil-water separation perfectly. Endless efforts are till on-going in attending this feat , hence this current study is intended to further increase and expand knowledge in improving oil-water separation efficiency in terms of research methods, analysis and research variables.

Few researchers have explored the effect of pressure in oil-water separation. However, to date far too little attention has been paid to pressure as an important variable to oil-water separation and in the design of oil-water separator though,there still exist some uncertainty bothering on the relationship between pressure and oil-water efficiency. Few studies have examined a two phase oil-water separators, separation efficiency and separator design. These authors Abdulkadir and Perez (2010), Sayda and Taylor (2007) used complex mathematical-numerical models like CFD in modeling oil-water separation. Abdulkadir and Perez (2010) attempted determining the effect of different parameters like velocity, droplet diameter will have on separator geometry. The physical experiment was designed in this study to provide data in developing the model equations to simulate the physical phenomenon that occurs in the separators.

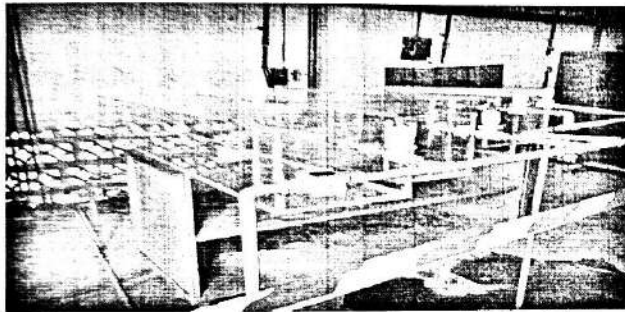


Figure 1 : Shows oil-water separation models

The pressure analysis of the fluid was considered due to gravity. The reservoir was marked at five different pressure levels: P1, P2, P3 P4 and P5. The total depth of the reservoir was 0.5056 m, and all pressure was marked at equal interval of 0.10112 m. The pressure head increases as the depth of the oil-water increases in the reservoir (steel) and the reservoir was placed at a height of about 4.36 m.

The pressures were analyzed using the hydrostatic expression

$$P = \rho gh.$$

P = Intensity of pressure (kN/m²)

ρ = density of water (1000kg/m³)

g = acceleration due to gravity (9.81m/s²)

h = hiegh of Liquid (m)

2.1 Model Equation Using Buckingham Pi-Theorem

However a model was developed in order to calibrate and verify the models using Buckingham pi-method. In developing the model it was assumed that a relationship exist between separation efficiency of a conventional separator and the separation efficiency of the models so constructed. By keeping pressure at π constant, with inlet flow rate constant, the outlet flow rate was observed to be equal to inlet flow rate while using one of the models as a control. Analysis was carried out in determining the inlet sample concentration and the outlet sample concentration in the models so as to determine the degree of separation. Applying the MLT dimensional analysis approach ,using Buckingham 's - theorem, it states that an equation involving 'n' number of physical variables which are expressible in terms of 'k' independent fundamental physical quantities can be expressed in terms of p = n-k dimensionless parameter. It is the formalization of Rayleigh's method of dimensional analysis.

Separation efficiency is a function of the following variables like flow rate Q, sample concentration C, geometric factor L, B, density ρ , velocity V, retention time T, pressure P etc.

Separation Efficiency = $\frac{C_1 - C_2}{C_1} \cdot 100$ which is equivalent to $\frac{C_2}{C_1} - 1$

Hence

$$C_2 = f(C_1, Q, P, L, B, \rho, V, R)$$

The repeating variables are:

Geometric property equals, length L ; Fluid property equals, density ρ ; Flow property equals, velocity V.

Number of variables m=9. For dimension of each variables, we have as follows: Concentration C (ML^{-3}), Flow rate Q (L^3T^{-1}), Pressure P ($ML^{-1}T^{-2}$) Density ρ (ML^{-3}), Velocity V ($L T^{-1}$), Retention Time R (T) The independent quantities m=3, hence the number of pi (π) terms= 9-3 = 6.

π terms with (m + 1) variables

$$\pi_1 = L^{a_1} \rho^{b_1} V^{c_1} \cdot C_2 \tag{1}$$

$$\pi_2 = L^{a_2} \rho^{b_2} V^{c_2} \cdot C_1 \tag{2}$$

$$\pi_3 = L^{a_3} \rho^{b_3} V^{c_3} \cdot Q \tag{3}$$

$$\pi_4 = L^{a_4} \rho^{b_4} V^{c_4} \cdot P \tag{4}$$

$$\pi_5 = L^{a_5} \rho^{b_5} V^{c_5} \cdot B \tag{5}$$

$$\pi_6 = L^{a_6} \rho^{b_6} V^{c_6} \cdot R \tag{6}$$

Hence for

$$\pi_1 = L^0 \rho^{-1} V^0 \cdot C_2 = \frac{C_2}{\rho} \tag{7}$$

$$\pi_2 = \frac{C_1}{\rho} \tag{8}$$

$$\pi_3 = L^{-2} \rho^0 V^{-1} \cdot Q$$

$$\pi_3 = \frac{Q}{L^2 V} \tag{9}$$

$$\pi_4 = L^0 \rho^{-1} V^{-2} \cdot P \tag{10}$$

$$\pi_5 = L^{-1} \rho^0 V^0 \cdot B \tag{11}$$

$$\pi_6 = L^0 \rho^0 V^0 \cdot R = R \tag{12}$$

But $f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) = 0$

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$$

But

$$\pi_1 = \frac{C_2}{\rho}$$

$$\frac{C_2}{\rho} = \left(\frac{C_1}{\rho}\right)^{a_1} \left(\frac{Q}{L^2 V}\right)^{a_2} \left(\frac{P}{\rho V^2}\right)^{a_3} \left(\frac{B}{L}\right)^{a_4} (R)^{a_5}$$

$$\frac{C_2}{\rho} = \left(\frac{C_1}{\rho}\right)^{a_1} \left(\frac{Q}{L^2 V}\right)^{a_2} \left(\frac{P}{\rho V^2}\right)^{a_3} \left(\frac{B}{L}\right)^{a_4} (R)^{a_5} \tag{13}$$

$$\text{Let } y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_0 + e$$

Where

β is the regression coefficients

Taking natural log of both side of equation (11)

$$\ln \frac{C_2}{\rho} = \ln \left[\left(\frac{C_1}{\rho}\right)^{a_1} \left(\frac{Q}{L^2 V}\right)^{a_2} \left(\frac{P}{\rho V^2}\right)^{a_3} \left(\frac{B}{L}\right)^{a_4} (R)^{a_5} \right]$$

$$\ln \frac{C_2}{\rho} = a_1 \ln \left(\frac{C_1}{\rho}\right) + a_2 \ln \left(\frac{Q}{L^2 V}\right) + a_3 \ln \left(\frac{P}{\rho V^2}\right) + a_4 \ln \left(\frac{B}{L}\right) + a_5 \ln R \tag{14}$$

$$\text{Let } a_1 \equiv \beta_1, x_1 \equiv \ln \left(\frac{C_1}{\rho}\right), x_2 \equiv \ln \left(\frac{Q}{L^2 V}\right), x_3 \equiv \ln \left(\frac{P}{\rho V^2}\right)$$

$$\begin{pmatrix} 0.1230 & -3.5927 & 3.2241 & 2.0186 & 0.634 \\ -3.5927 & 108.1207 & -96.8608 & -60.8300 & -19.0657 \\ 3.2241 & -96.8608 & 86.8036 & 54.4811 & 17.0861 \\ 2.0186 & -60.8308 & 54.4811 & 34.2318 & 10.7238 \\ 0.6346 & -19.0657 & 17.0861 & 34.2318 & 3.3631 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 4.0451 \\ 121.4967 \\ 108.8822 \\ 68.3376 \\ 21.4320 \end{pmatrix} \quad (43)$$

The above is represented by a compact matrix form as Equation (26).

The equation (26) resulted in a 5 x 5 matrix with five unknowns. Solving Equation (26) using MATLAB P2007b yields the following results for

$$a_1 = 0.3422, a_2 = -0.0260, a_3 = 0.8489, a_4 = 0.000 \text{ and } a_5 = 1.8478$$

$$E = [\Delta C^{a_1} Q_{AV}^{a_2} P^{a_3} T^{a_4} L^{a_5}] \quad (44)$$

$$E = [\Delta C^{0.3422} Q_{AV}^{-0.0260} P^{0.8489} T^{-0.000} L^{1.8478}]$$

Equation (43) was obtained as a non-linear design model for the prediction of the separation efficiency in the physical model. The model was calibrated with the first 8 data out of the 16 data obtained from model-A at pressure of 33.833kn/m².

However, due to paltriness of empirical literature, the remaining data was used in verifying the physical model. Furthermore, the verification of the model gave a good coefficient of correlation between the measured and the calculated separation efficiencies.

2.3 Statistical Evaluation of the Models

The performance indicator parameters of the developed models included the standard error of estimate, coefficient of correlation and the coefficient of determination. Theories of regression analysis had been discussed in time past (Draper and Smith, 1998).

The performance indicators are given in equations (45), (46) and (47).

$$s_{y,x} = \sqrt{\frac{\sum(Y - Y_{est})^2}{N}} \quad (45)$$

This is called the standard error of estimate of Y on X. Where $Y_{est} = a_0 + a_1X$ and N= number of sample.

Correlation Coefficient, r between Y and X is given

$$r = \frac{N \sum XY - (\sum X)(\sum Y)}{\sqrt{[N \sum X^2 - (\sum X)^2][N \sum Y^2 - (\sum Y)^2]}} \quad (46)$$

Correlation of determination r² is given by

$$r^2 = \frac{\text{explained variation}}{\text{total variation}} = \frac{\sum(Y_{est} - \bar{y})^2}{\sum(Y - \bar{Y})^2} \quad (47)$$

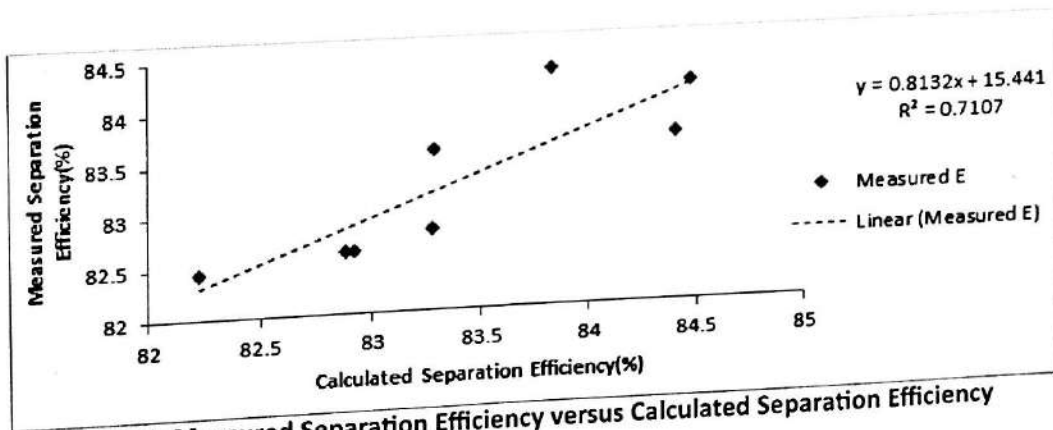


Figure 2: Measured Separation Efficiency versus Calculated Separation Efficiency

3.0 Results and Discussions

The summary of the output of linear regression analysis between the measured and calculated separation efficiency in the physical model of figure (3.1) using Spss software shows that coefficient of correlation and coefficient of determination were 84.3% and 71.1%. Also standard error of estimate was 0.4. The predicted values of separation efficiency were closed to the experimental values. The coefficient of determination (R- Squared) shows that 71.0% of the variation of the experimental results was explained by the developed model. Therefore the prediction of the separation efficiency in the model was possible from the developed model as the coefficient of correlation of 0.843 was adequate. The coefficient of correlation, regressed slope, intercept and standard error obtained were 0.843, 0.711, 15.441 and 17.670.

4.0 Conclusion

However, separation efficiency of a separator was tied to inlet concentration, flow rate, length, breath, pressure, volume, inlet velocity, retention and density.

However, separation was also aided through mechanism of coalescence as a result of film drainage and film rupturing. The film drainage occurs due to fluid flow and pressure gradient developed in the film. Also as the flow continues, the interfacial film between the droplets thins below some critical thickness where it ruptures and the capillary pressure difference causes the droplets to fuse into one droplet hence enhancing separation. The model was calibrated with the first eight data set and was verified with the remaining eight set obtained from the experiment of a total of sixteen data set from model A, since model A shows a better separation efficiency from other physical models.

The verification of the model gave a good coefficient of correlation of R=84.3% between the measured and calculated separation efficiency at a significant level of 0.05. The coefficient of determination (R- square) showed that 71.0% of the variation of the experimental results was explained by the developed model. The coefficient of correlation, regressed slope, intercept and standard error obtained were 0.843, 0.711, 15.441 and 17.670.

Therefore, the prediction of separation efficiency in the physical model was possible from the developed model concerning an effective separation.

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