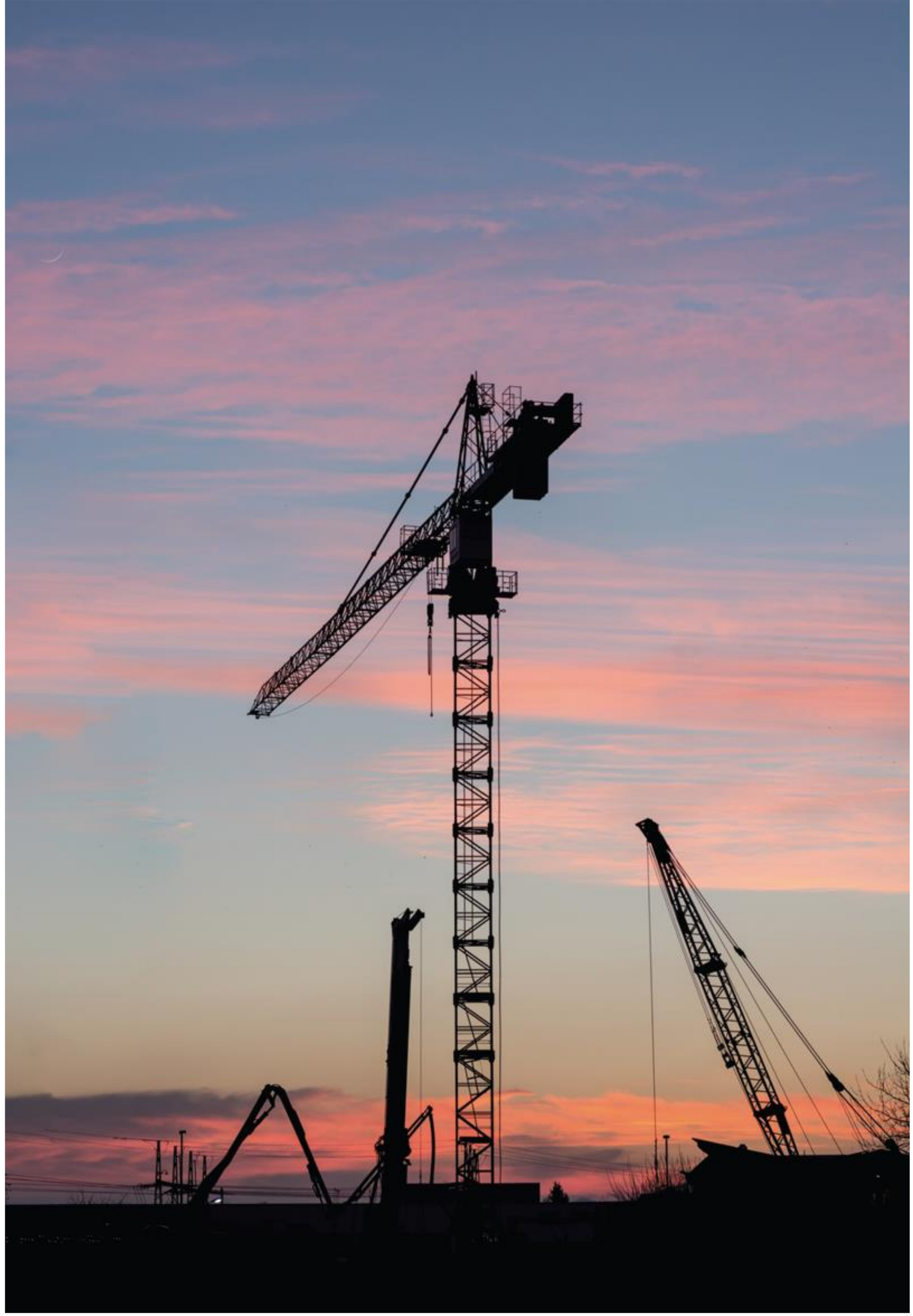
# CHAPTER 2
# FORCE SYSTEMS

CHAPTER OUTLINE





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# Article 2/1 Introduction

- Chapter Purpose

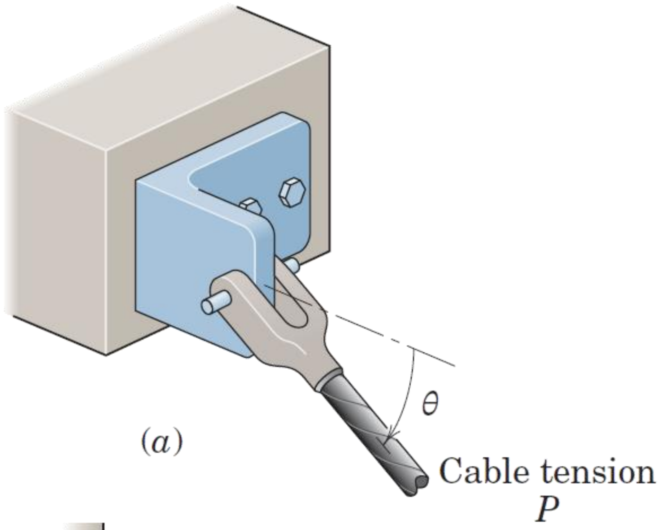
  We study the effects of forces which act on engineering structures and mechanisms. The experience gained here will help you in the study of mechanics and in other subjects such as stress analysis, design of structures and machines, and fluid flow. This chapter lays the foundation for a basic understanding not only of statics but also entire subjects of mechanics, and you should master of this material thoroughly.

- Importance of Chapter 2 Concepts

  We will deal with two- and three-dimensional forces and their moments at a point.

# Article 2/2 Force

- A force has been defined in Chapter 1 as ***an action of one body on another***.
- A force is a **vector quantity**, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.
- We can separate the action of a force on a body into two effects, *external* and *internal.*



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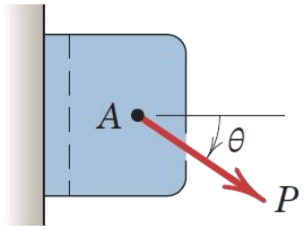


(b)

# Article 2/2 Force

- For the bracket in the figure the *external effects* of **P** to the bracket

are:

✓the reactive forces (not shown) exerted on the bracket by the

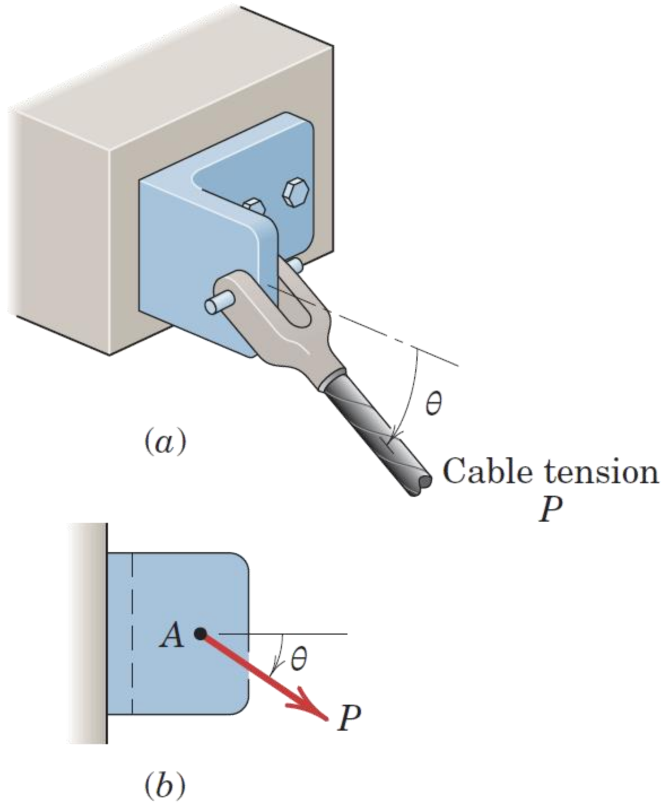
foundation and bolts because of the action of **P**.

- Forces external to a body can be either *applied forces* or *reactive forces*.
- The internal effects of **P** to the bracket are:

✓the resulting internal forces and deformations distributed

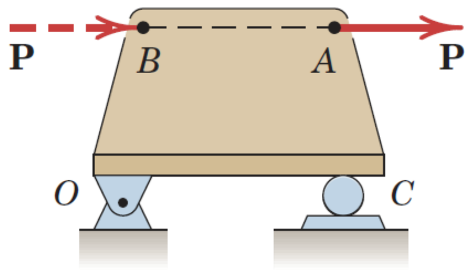
throughout the material of the bracket.

- The relation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.



# Article 2/2 – Principle of Transmissibility

- A force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts.




When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point.

- Whenever we are interested in only the resultant external effects of a force, the force may be treated as a sliding vector, and we need specify only the magnitude, direction, and line of action of the force, and not its point of application.

# Article 2/2 – Force Classification

- **Contact Force:** A contact force is produced by direct physical contact
- **Body Force:** A body force is generated by virtue of  the position of a body within the force field such as gravitational field.
- **Concentrated Force:** Every contact force is actually applied over a finite area and is a distributed force. But considering the application area of force is so small than the other dimensions of body. So that, we may consider the force to be concentrated at a point with negligible loss of accuracy.
- **Distributed Force:** Force can be distributed over an area, as in the case of mechanical contact, over a volume when a body force such as weight is acting, or over a line, as in the case of the weight of a suspended cable.
- **Action and Reaction Pairs :** According to Newton's third law, the action of a force is always ac-companied by an equal and opposite reaction.

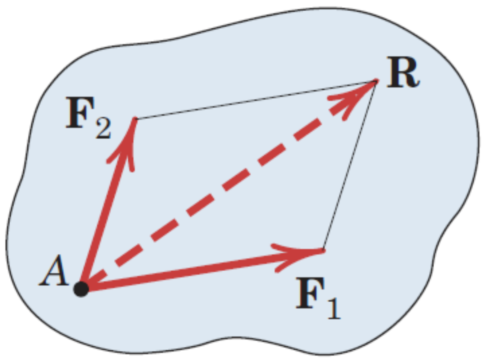
# Article 2/2 – Concurrent Forces (1 of 2)

- Two or more forces are said to be concurrent at

a point if their lines of action intersect at that point.

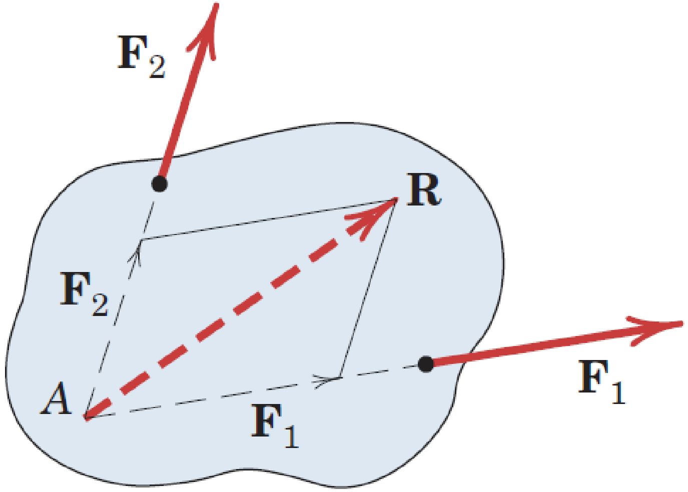
- The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in figure, have a common

point of application and are concurrent at the point A.

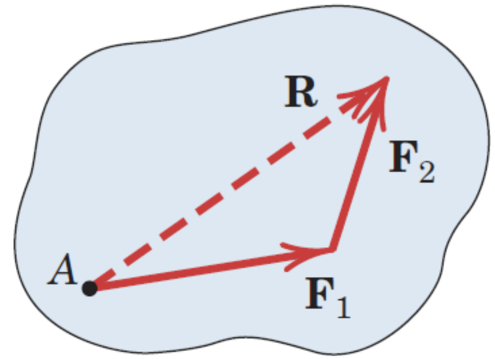



- Thus, they can be added using parallelogram law in their common plane to obrain their sum or resultant R.

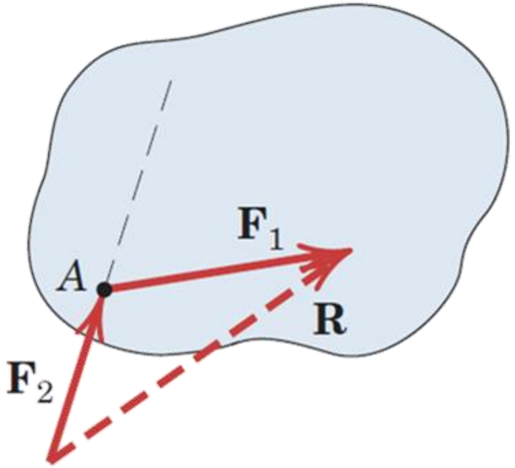
- Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Figure below.



# Article 2/2 – Concurrent Forces (2 of 2)



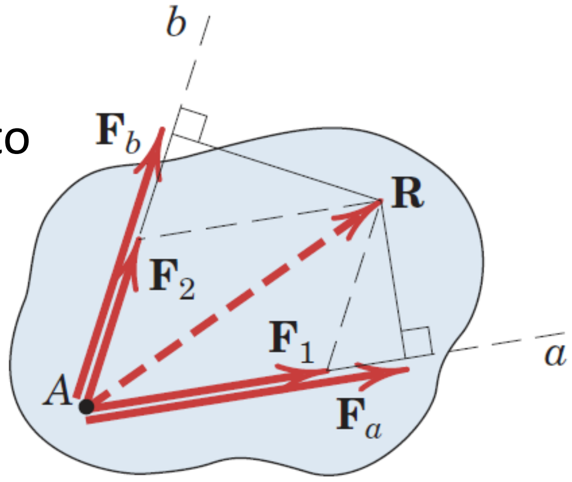

- By the principle of transmissibility, we may move them along their lines of action and complete their vector sum **R** at the point of concurrancy.
- $\mathbf{F}_1$ and $\mathbf{F}_2$ forces can be replaced with resultant **R** without altering the external effects on the body.




- If we add two forces as shown in figure below, we correctly preserve the magnitude and direction of the resultant **R**, but we lose the correct line of action, Because the obtained **R** in this way does not pass thorough the point A.
- Therefore, this type of combination should be avoided.
- We can express the sum of the two forces mathematically by the vector equation

$$\mathbf{R}=\mathbf{F}_1+\mathbf{F}_2$$

# Article 2/2 – Vector Components




- In addition to combining forces to obtain their resultant, we often need to replace a force by its vector components in directions which are convenient for a given application.
- The vector sum of the components must be equal the original vector.

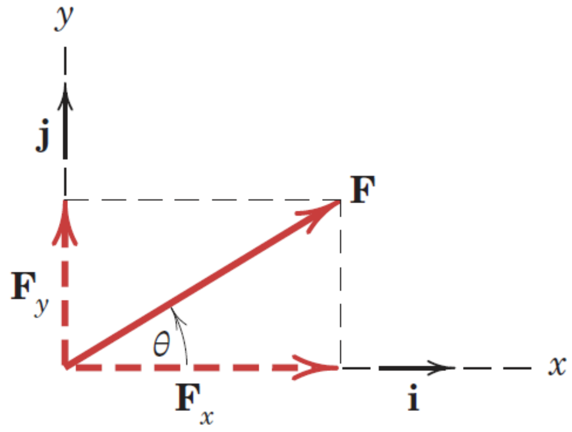
Vector Components: $\mathbf{F}_1$ and $\mathbf{F}_2$

Vector Projections: $\mathbf{F}_a$ and $\mathbf{F}_b$

- The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular* projections onto the same axes.
- The figure shows the perpendicular projections $\mathbf{F}_a$ and $\mathbf{F}_b$ of **R** on to a and b axes. We can understand that the components of a vector are not necessarily equal to the projections onto same axes.

# 2/3 Rectangular Components

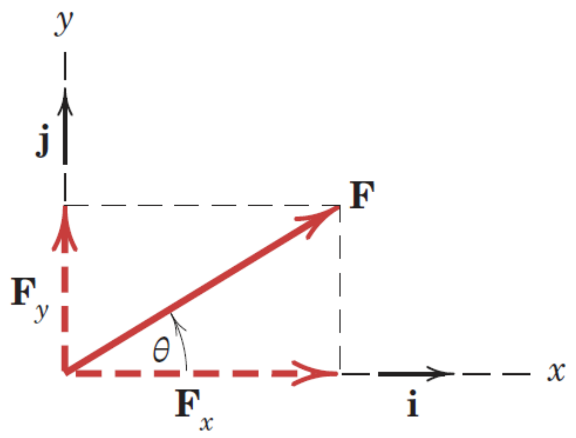
- Vector Components: $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$
- Scalar Components: $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$
- Other Useful Relationships




$$F_x = F \cos \theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F \sin \theta \qquad \theta = \tan^{-1} \frac{F_y}{F_x}$$

# 2/3 Conventions for Describing Vector Components




- Vector Magnitude, *F*
  - Lightface, Italic Font
  - Always Positive
- Scalar Component, $F_x$
  - Lightface, Italic Font
  - Positive or Negative
- Force Vector Depiction
  - Solid, Red Arrow
- Component Vector Depiction
  - Dashed, Red Arrow

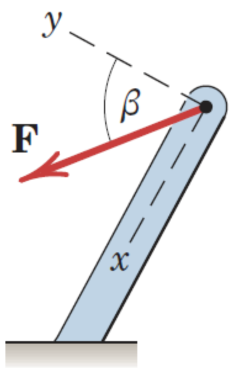
# 2/3 Determining the Components of a Force (1 of 2)

- It is often the case that…
  - Dimensions or axes are not given in horizontal and vertical directions.
  - Angles are not measured counterclockwise from the $x$-axis.
  - Coordinates do not originate from the line of action of a force.

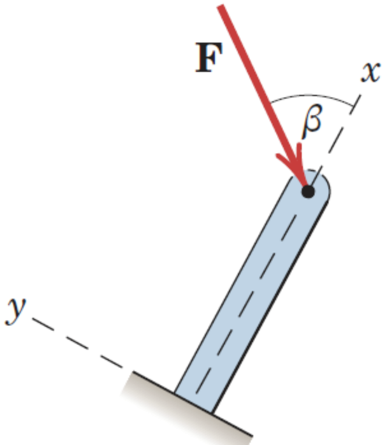
- We still need to be able to find the components of a force vector!

# 2/3 Determining the Components of a Force (2 of 2)

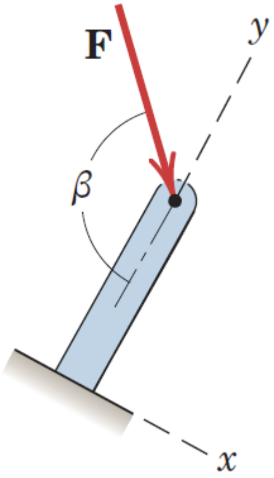
- Some examples



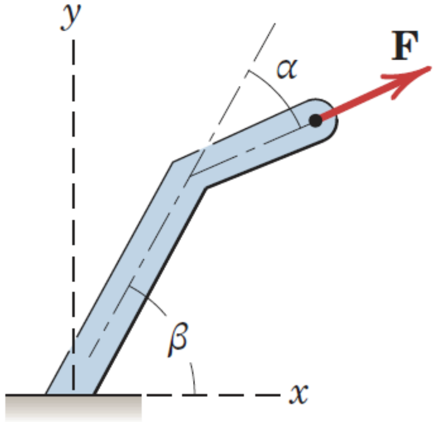

$F_x = F \sin \beta$
$F_y = F \cos \beta$




$F_x = -F \cos \beta$
$F_y = -F \sin \beta$




$F_x = F \sin(\pi - \beta)$
$F_y = -F \cos(\pi - \beta)$




$F_x = F \cos(\beta - \alpha)$
$F_y = F \sin(\beta - \alpha)$

# 2/3 Finding Resultants using Components

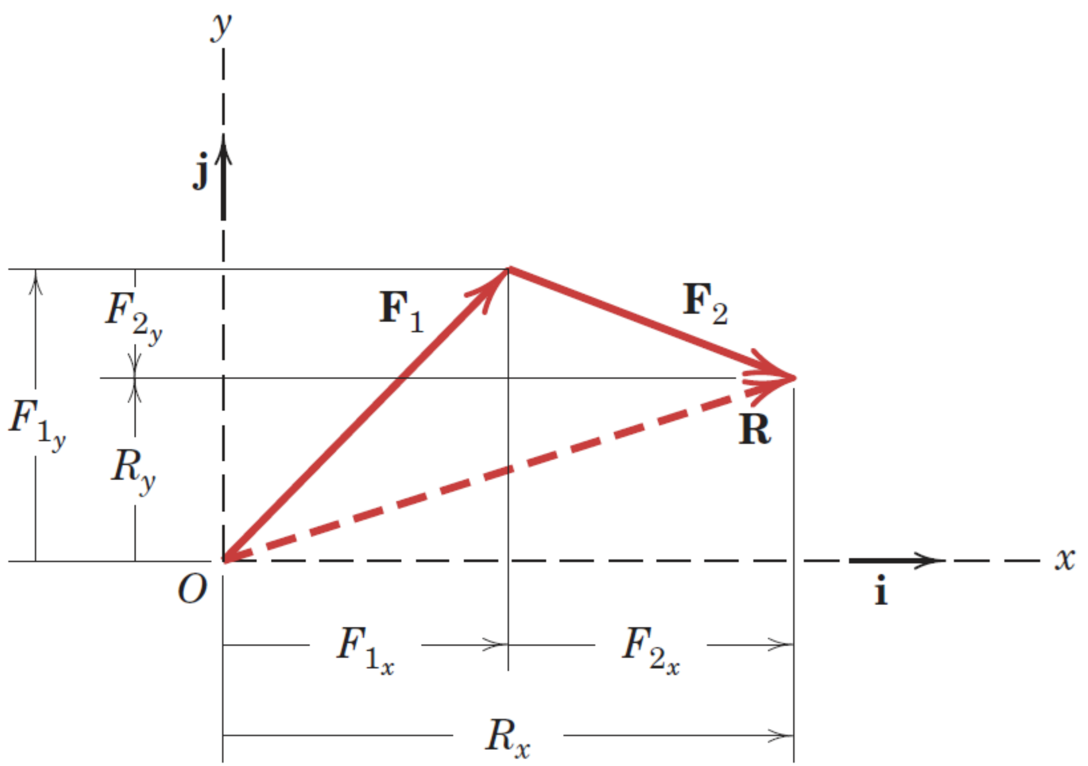
- Rectangular components are convenient for finding the resultant of two forces which are concurrent.

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

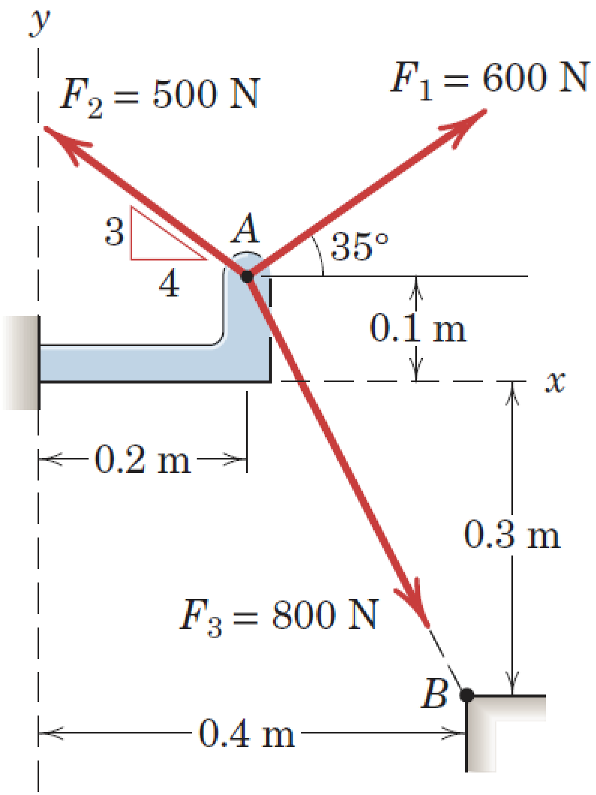
$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$

$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$

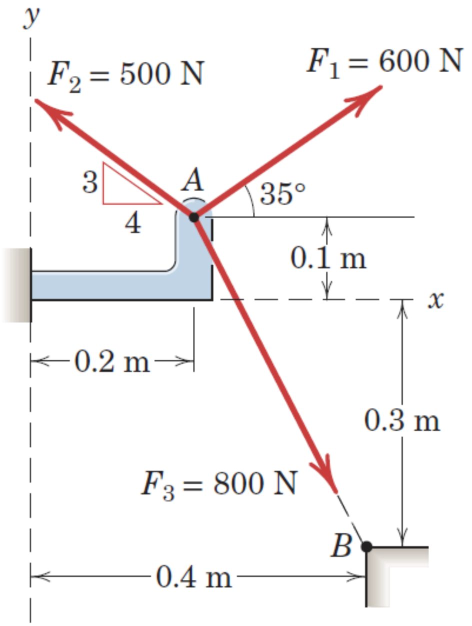


# Article 2/3 – Sample Problem 2/1 (1 of 3)

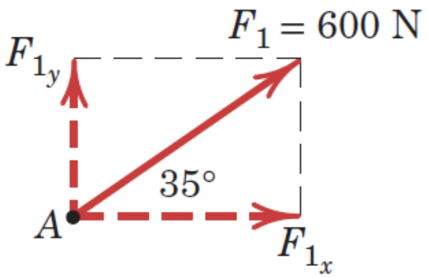
The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point *A* of the bracket, are specified in three different ways. Determine the *x* and *y* scalar components of each of the three forces.



# Article 2/3 – Sample Problem 2/1 (2 of 3)



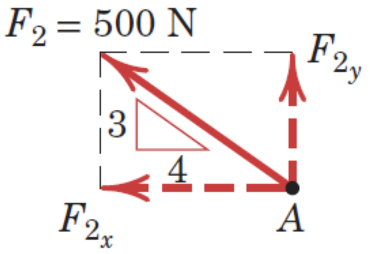

- Scalar Components of $\mathbf{F}_1$




$F_{1_x} = 600 \cos 35° = 491$ N *Ans.*

$F_{1_y} = 600 \sin 35° = 344$ N *Ans.*

- Scalar Components of $\mathbf{F}_2$

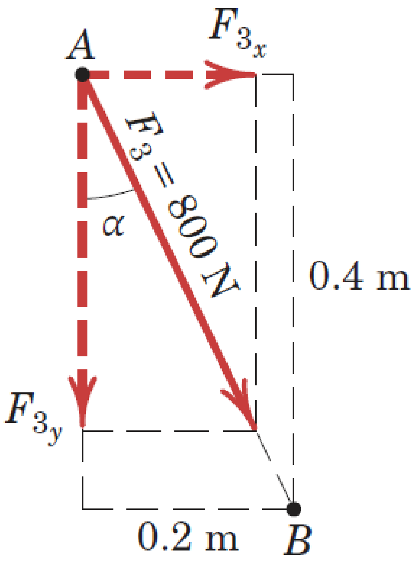



$F_{2_x} = -500(\frac{4}{5}) = -400$ N *Ans.*

$F_{2_y} = 500(\frac{3}{5}) = 300$ N *Ans.*

# Article 2/3 – Sample Problem 2/1 (3 of 3)

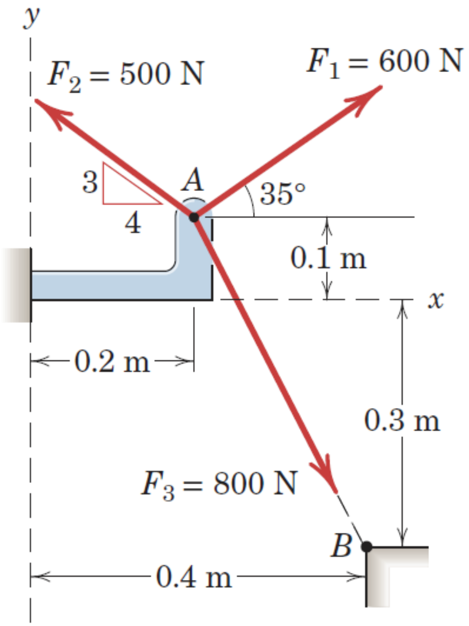
- Scalar Components of $\mathbf{F}_3$




$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

$$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6° = 358 \text{ N} \quad ① \qquad \textit{Ans.}$$

$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6° = -716 \text{ N} \qquad \textit{Ans.}$$




- Alternative Calculation

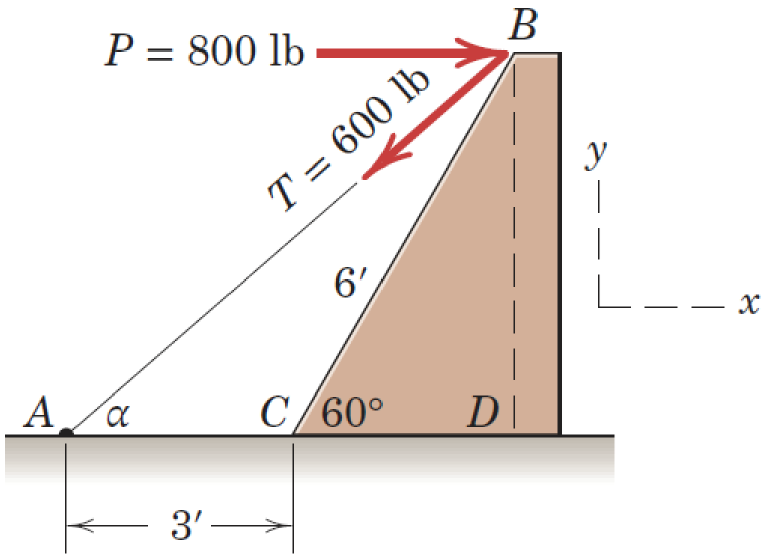
$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{AB} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

$$= 800\,[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j} \text{ N}$$

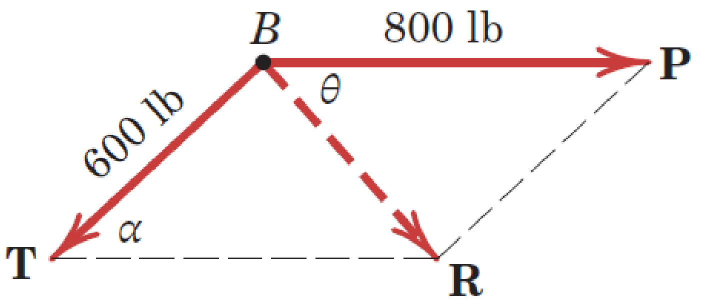
# Article 2/3 – Sample Problem 2/2 (1 of 4)

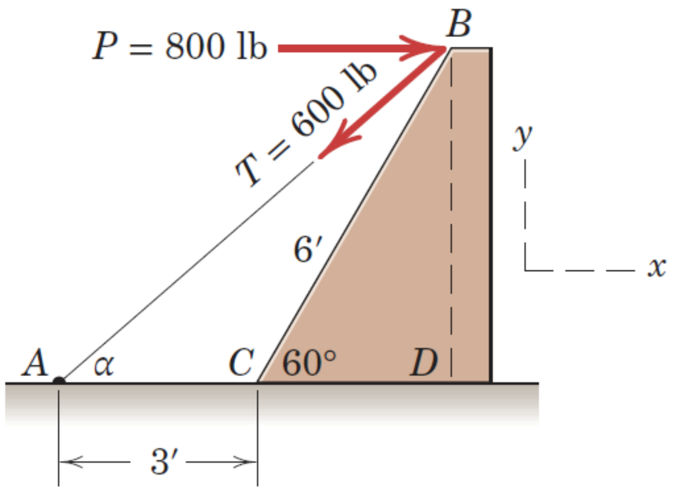
Combine the two forces **P** and **T**, which act on the fixed structure at *B*, into a single equivalent force **R**.



# Article 2/3 – Sample Problem 2/2 (2 of 4)

- Graphical Solution (Scaled Drawing)



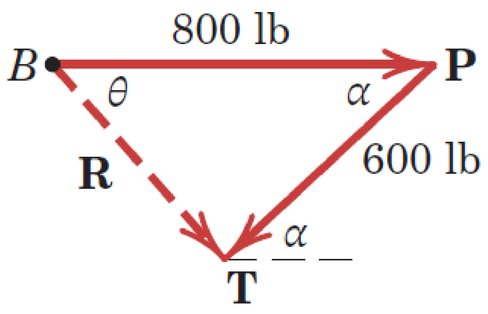

$$\tan \alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6 \sin 60^\circ}{3 + 6 \cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$

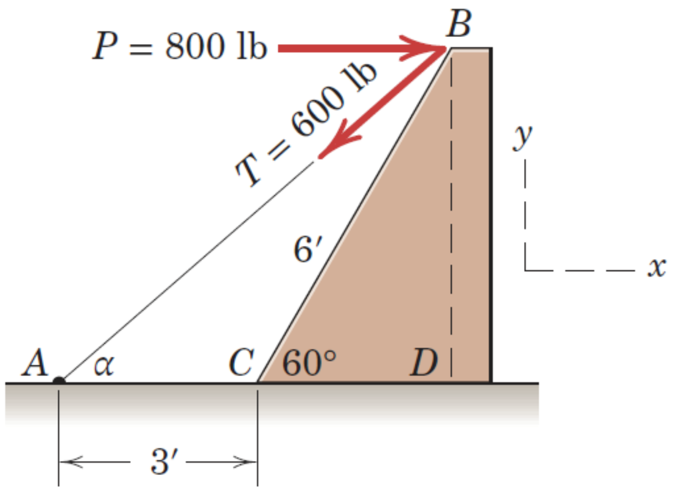
Measurement of the length $R$ and direction $\theta$ of the resultant force **R** yields the approximate results

$$R = 525 \text{ lb} \qquad \theta = 49^\circ \qquad \textit{Ans.}$$

# Article 2/3 – Sample Problem 2/2 (3 of 4)

- Geometric Solution



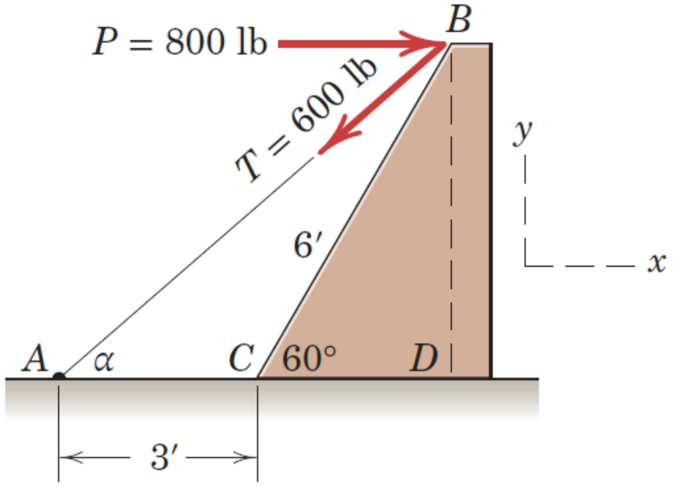

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9° = 274{,}300$$

$$R = 524 \text{ lb} \qquad \textit{Ans.}$$

From the law of sines, we may determine the angle $\theta$ which orients **R**. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9°} \qquad \sin\theta = 0.750 \qquad \theta = 48.6° \qquad \textit{Ans.}$$

# Article 2/3 – Sample Problem 2/2 (4 of 4)




- Algebraic Solution

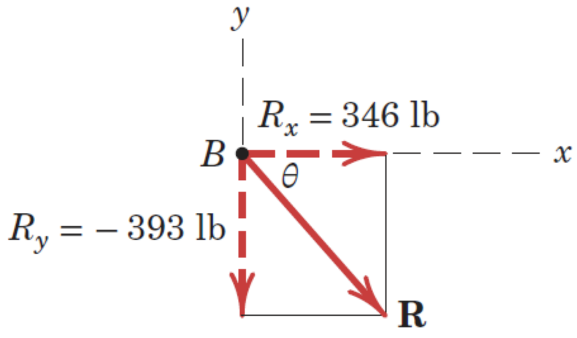
$$R_x = \Sigma F_x = 800 - 600 \cos 40.9° = 346 \text{ lb}$$

$$R_y = \Sigma F_y = -600 \sin 40.9° = -393 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ lb} \qquad \textit{Ans.}$$

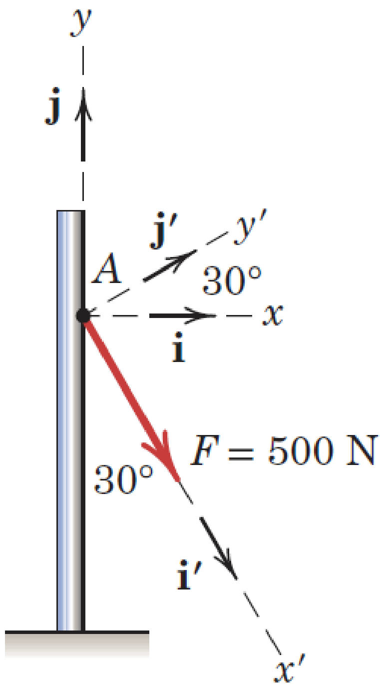
$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6° \qquad \textit{Ans.}$$

- Vector Representation



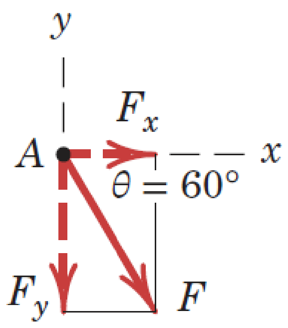
# Article 2/3 – Sample Problem 2/3 (1 of 3)

The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the *x*′- and *y*′-axes. (3) Determine the scalar components of **F** along the *x*- and *y*′-axes.



# Article 2/3 – Sample Problem 2/3 (2 of 3)

- Part 1 Solution

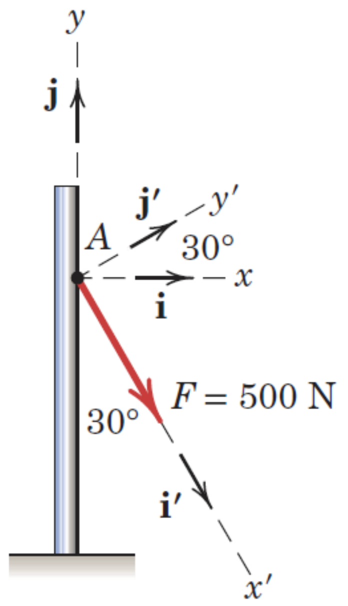



$$\mathbf{F} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$

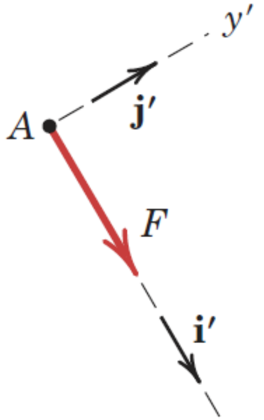
$$= (500\cos 60°)\mathbf{i} - (500\sin 60°)\mathbf{j}$$

$$= (250\mathbf{i} - 433\mathbf{j})\text{ N} \qquad \textit{Ans.}$$

The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.




- Part 2 Solution

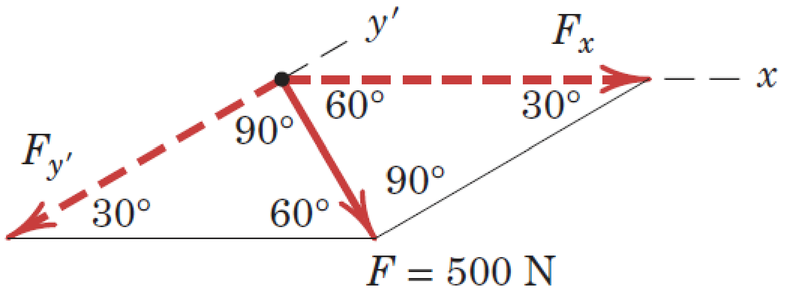


(b)

$$F_{x'} = 500\text{ N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$

# Article 2/3 – Sample Problem 2/3 (3 of 3)

- Part 3 Solution

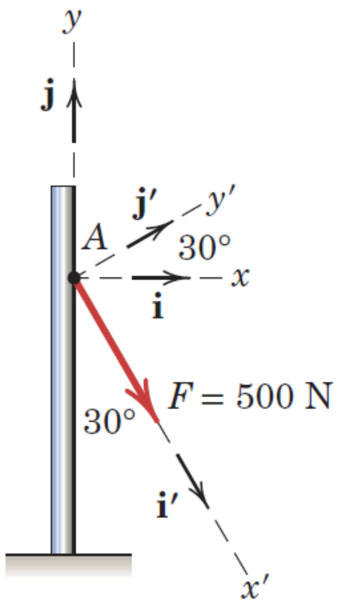



$$\frac{|F_x|}{\sin 90°} = \frac{500}{\sin 30°} \qquad |F_x| = 1000 \text{ N} \quad ①$$

$$\frac{|F_{y'}|}{\sin 60°} = \frac{500}{\sin 30°} \qquad |F_{y'}| = 866 \text{ N}$$

The required scalar components are then

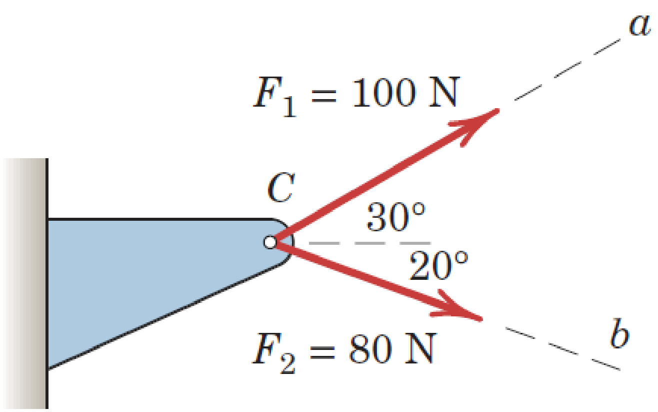
$$F_x = 1000 \text{ N} \qquad F_{y'} = -866 \text{ N} \qquad \textit{Ans.}$$




① Obtain $F_x$ and $F_{y'}$ graphically and compare your results with the calculated values.

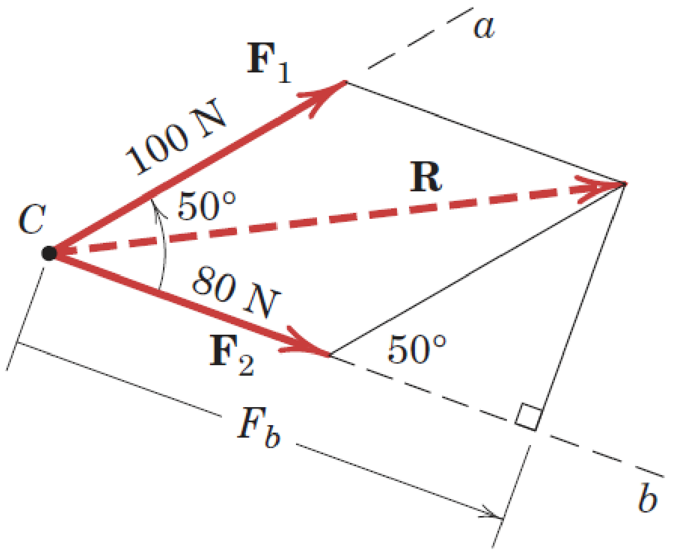
# Article 2/3 – Sample Problem 2/4 (1 of 2)

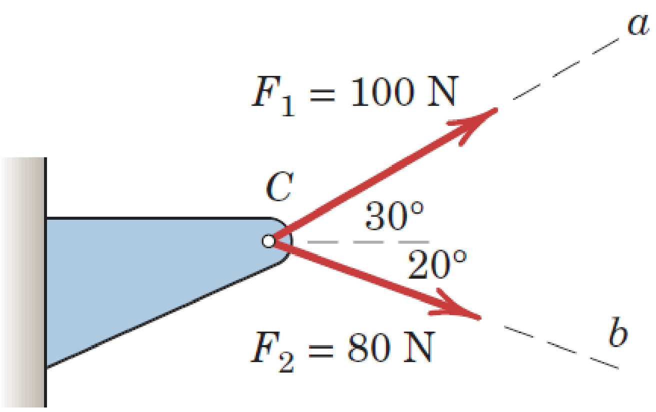
Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection $F_b$ of their resultant $\mathbf{R}$ onto the $b$-axis.



# Article 2/3 – Sample Problem 2/4 (1 of 2)

- Solution




$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4\text{ N}$$

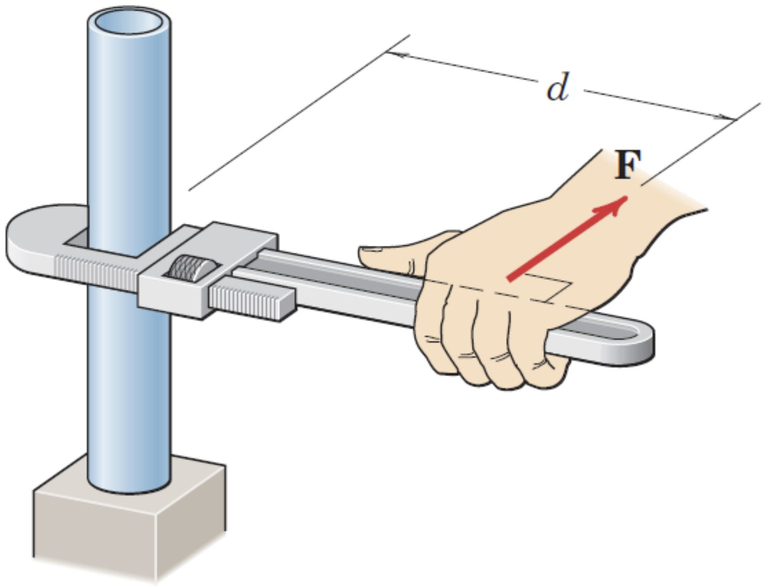
The figure also shows the orthogonal projection $\mathbf{F}_b$ of $\mathbf{R}$ onto the $b$-axis. Its length is

$$F_b = 80 + 100\cos 50^\circ = 144.3\text{ N} \qquad \textit{Ans.}$$

# Article 2/4 Moment

- A moment is the tendency of a force to rotate a body about an axis.

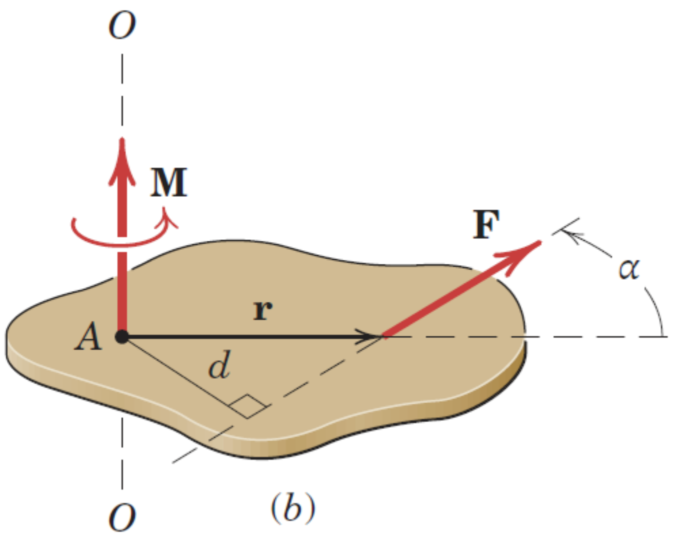
- Things to Note
  - Direction and Orientation of the Force
  - Axis of Rotation
  - Direction of Rotation
  - Effective Length, *d*




Pipe Wrench

# Article 2/4 – Moment about a Point (1 of 3)

- Scalar Development



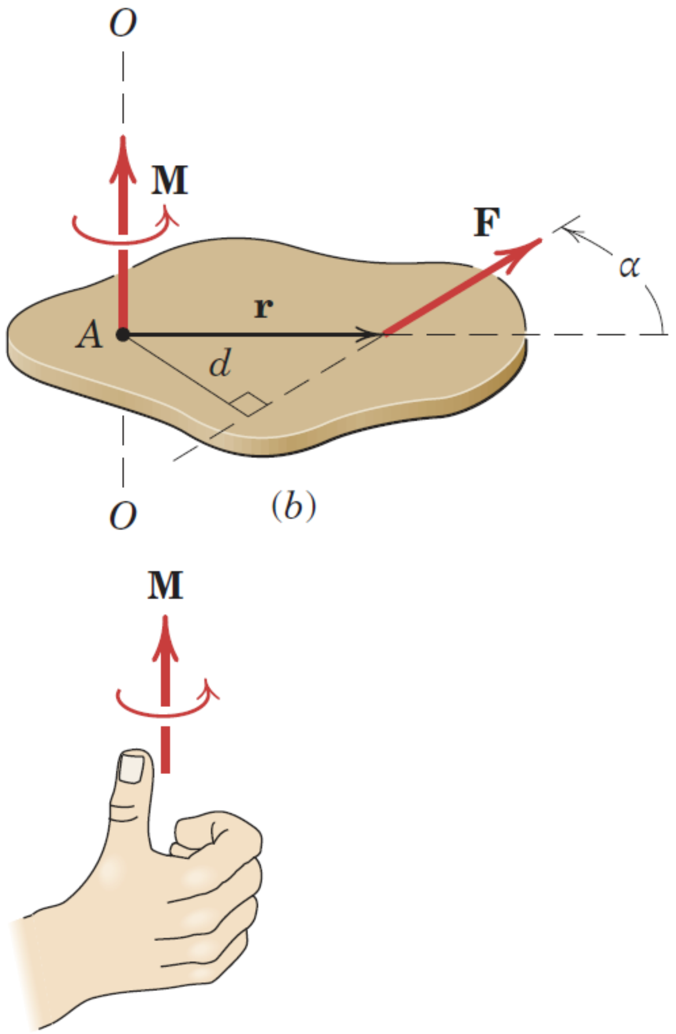
(b)

- Moment Arm, *d* is the perpendicular distance from the axis to the line of action of the force
    - Moment Vector, **M** is perpendicular to plane of body
    - Axis of Rotation, *O*-*O*
    - Moment Magnitude, $M = Fd$
    - Direction of Rotation
    - Units

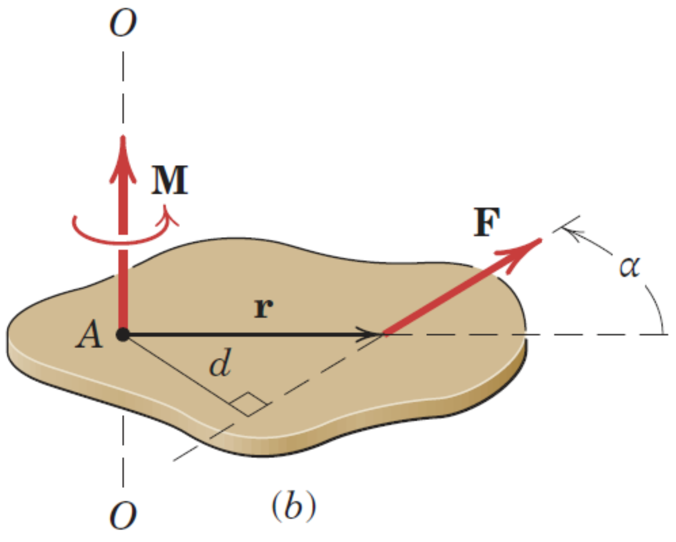
# Article 2/4 – Moment about a Point (2 of 3)

- The Right-Hand Rule

1. Position your right hand such that your fingers point in the same direction as the force.
2. Orient your hand such that the point you are computing the moment about is on the same side as your palm. From the figure at right, your hand is positioned such that the moment arm *d* intersects the middle of your palm.
3. Close your fingers to make a fist and extend your thumb straight up. From the figure at right, imagine closing your fist around line *O*-*O*, and your thumb would point in the direction of the moment vector. Curling your fingers about this line would represent the rotation of the moment about the axis.

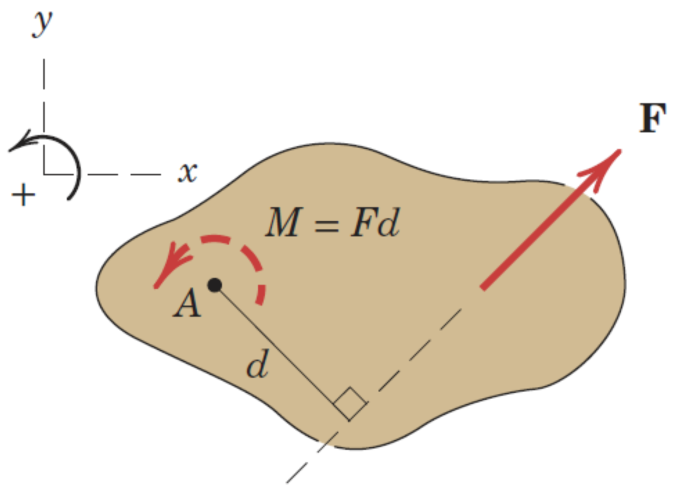


# Article 2/4 – Moment about a Point (3 of 3)




(b)

- Sign Conventions
  - Counterclockwise, CCW positive
  - Clockwise, CW negative
  - User-Defined
- Two-Dimensional Representation

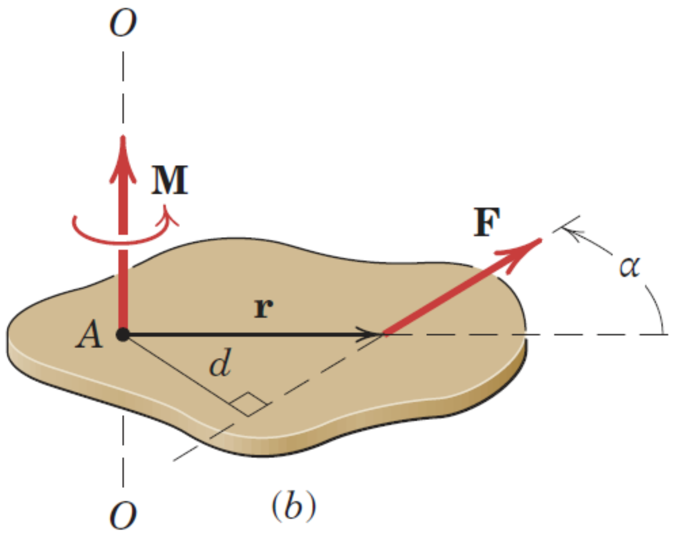


# Article 2/4 – The Cross Product

- Vector Expression for Moments

  - Position Vector, **r**

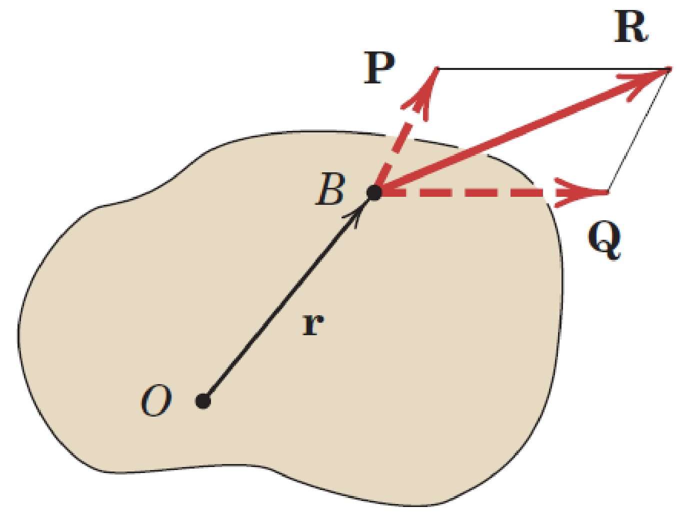
  - Moment Vector, **M = r × F**




- In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations.

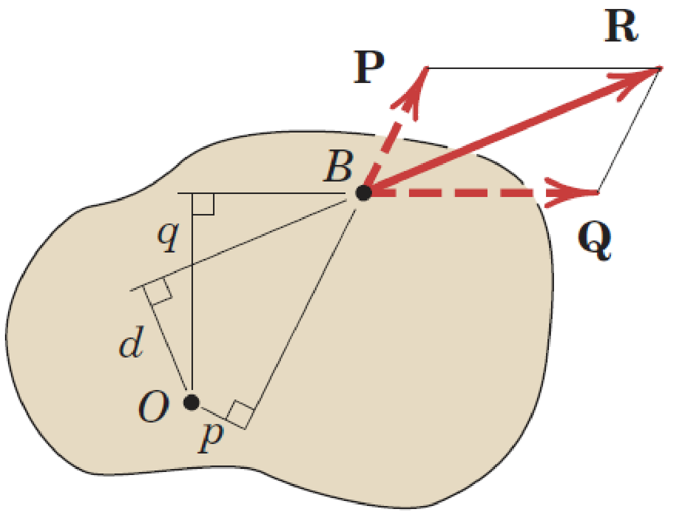
# Article 2/4 – Varignon's Theorem

- The theorem states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.




$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q}) = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

**In Vectoral Form**

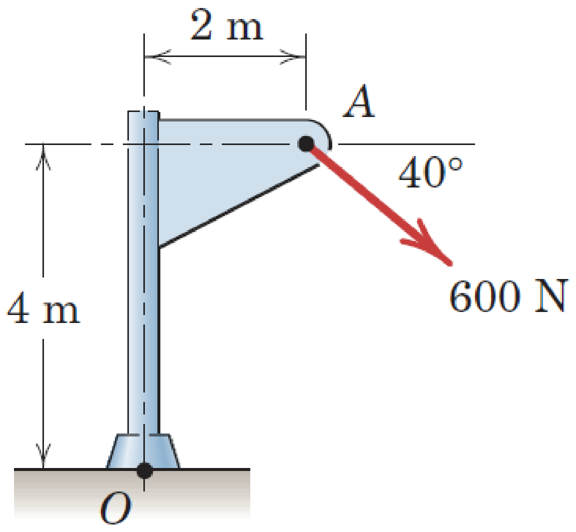



$M_O = Rd = -pP + qQ$ (Assumes CW +)

**In Scalar Form**

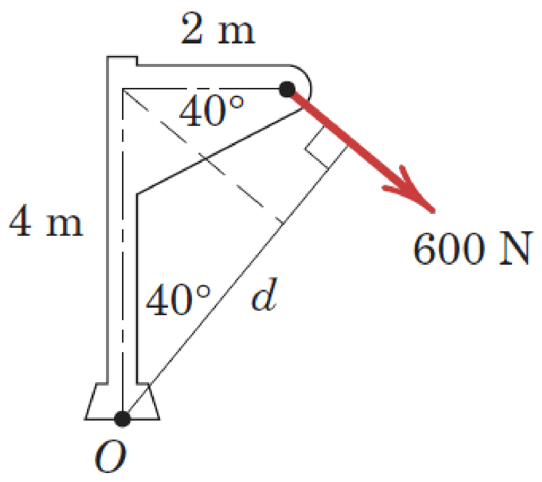
# Article 2/4 – Sample Problem 2/5 (1 of 5)

Calculate the magnitude of the moment about the base point *O* of the 600-N force in five different ways.



# Article 2/4 – Sample Problem 2/5 (2 of 5)

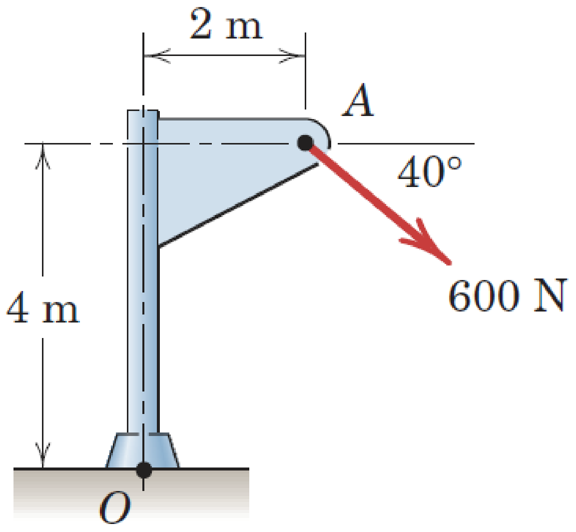
- Method 1: Use the Moment Arm (CW is +)




$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

By $M = Fd$ the moment is clockwise and has the magnitude ①

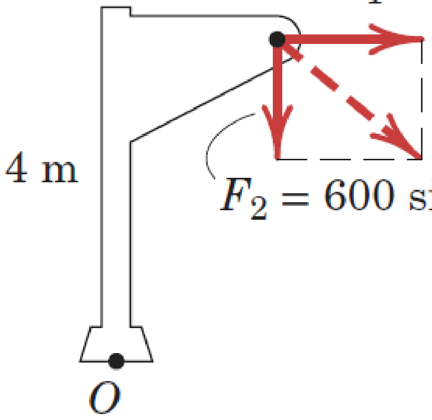
$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

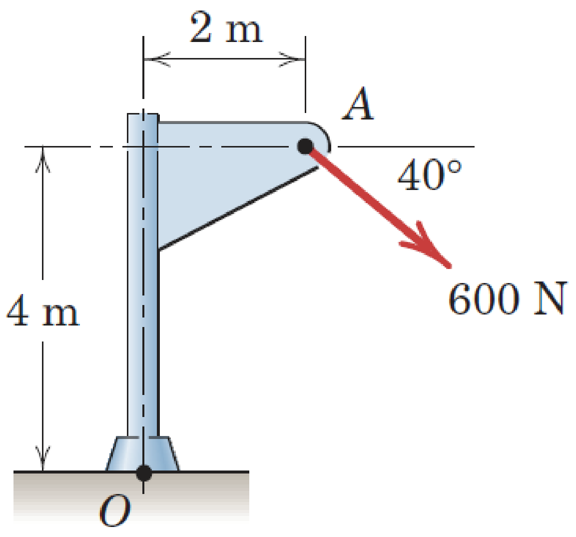



① The required geometry here and in similar problems should not cause difficulty if the sketch is carefully drawn.

# Article 2/4 – Sample Problem 2/5 (3 of 5)

- Method 2: Use Components at *A* (CW is +)




$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

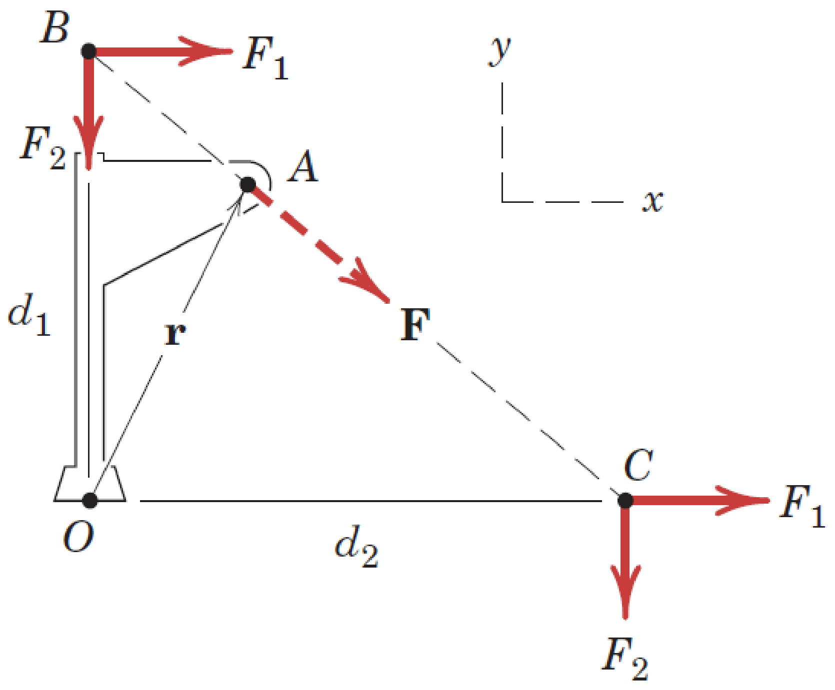
By Varignon's theorem, the moment becomes

$$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \quad ② \qquad \textit{Ans.}$$

② This procedure is frequently the shortest approach.

# Article 2/4 – Sample Problem 2/5 (4 of 5)

- Methods 3 and 4: Alternative Moment Arms

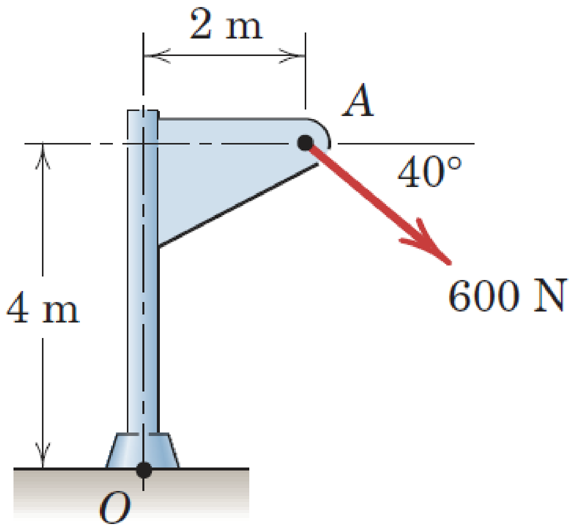



$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$

$$M_O = 460(5.68) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

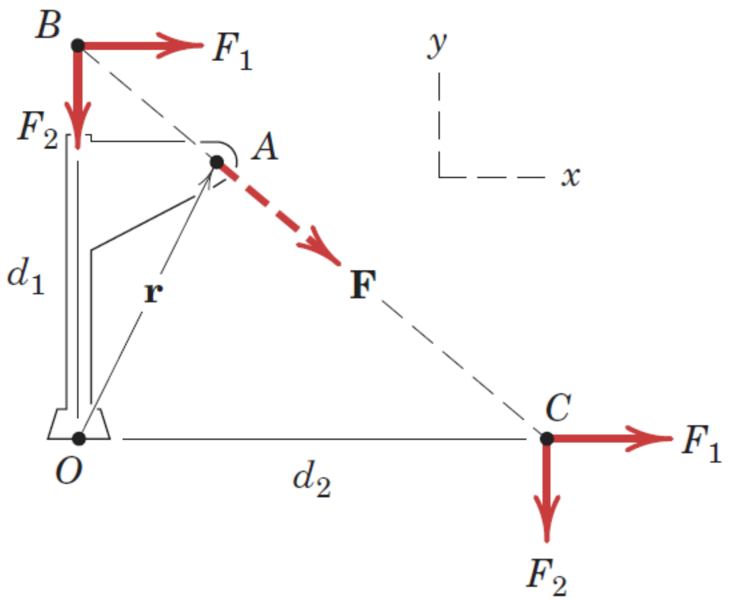
$$M_O = 386(6.77) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

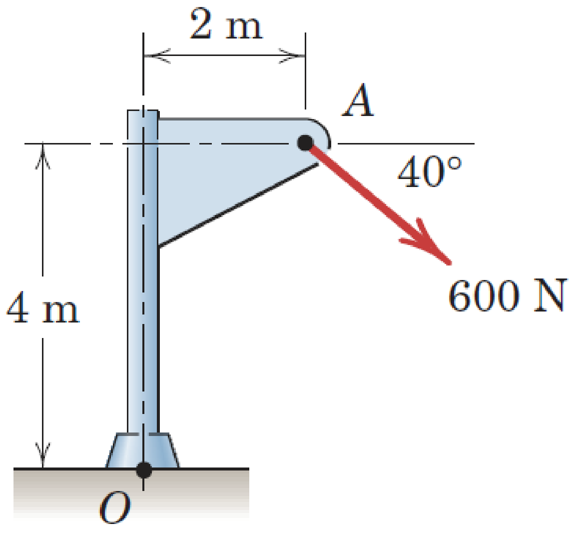



③ The fact that points $B$ and $C$ are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.

# Article 2/4 – Sample Problem 2/5 (5 of 5)

- Method 5: Vector Approach




$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°) \quad ④$$

$$= -2610\mathbf{k} \text{ N}\cdot\text{m}$$

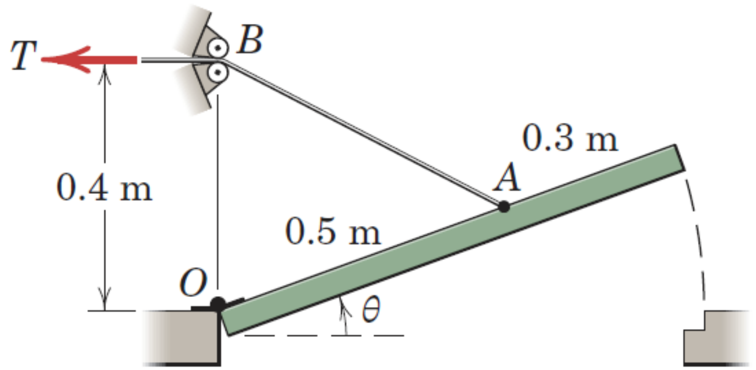
④ Alternative choices for the position vector $\mathbf{r}$ are $\mathbf{r} = d_1\mathbf{j} = 5.68\mathbf{j}$ m and $\mathbf{r} = d_2\mathbf{i} = 6.77\mathbf{i}$ m.

The minus sign indicates that the vector is in the negative $z$-direction. The magnitude of the vector expression is

$$M_O = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

# Article 2/4 – Sample Problem 2/6 (1 of 4)

The trap door *OA* is raised by the cable *AB*, which passes over the small frictionless guide pulleys at *B*. The tension everywhere in the cable is *T*, and this tension applied at *A* causes a moment $M_O$ about the hinge at *O*. Plot the quantity $M_O/T$ as a function of the door elevation angle $\theta$ over the range $0 \leq \theta \leq 90°$ and note minimum and maximum values. What is the physical significance of this ratio?



# Article 2/4 – Sample Problem 2/6 (2 of 4)

$$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{\mathbf{r}_{OB} - \mathbf{r}_{OA}}{r_{AB}} \quad ①$$

Using the $x$-$y$ coordinates of our figure, we can write

$$\mathbf{r}_{OB} = 0.4\mathbf{j} \text{ m} \qquad \mathbf{r}_{OA} = 0.5(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \text{ m} \quad ②$$

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} = 0.4\mathbf{j} - (0.5)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

$$= -0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j} \text{ m}$$

So

$$r_{AB} = \sqrt{(0.5\cos\theta)^2 + (0.4 - 0.5\sin\theta)^2}$$

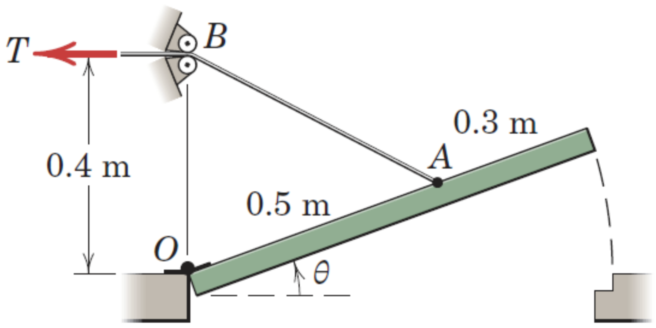
$$= \sqrt{0.41 - 0.4\sin\theta} \text{ m}$$

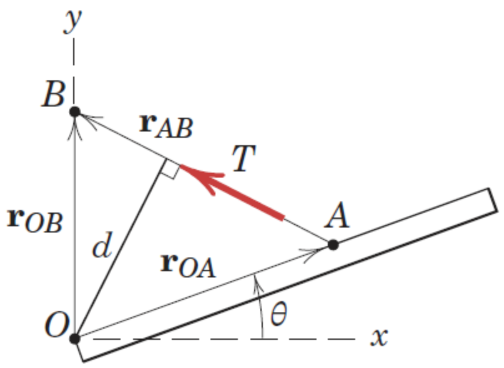
The desired unit vector is

$$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}$$

Our tension vector can now be written as

$$\mathbf{T} = T\mathbf{n}_{AB} = T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]$$




① Recall that any unit vector can be written as a vector divided by its magnitude. In this case the vector in the numerator is a position vector.

② Recall that any vector may be written as a magnitude times an "aiming" unit vector.

# Article 2/4 – Sample Problem 2/6 (3 of 4)

- Moment Vector

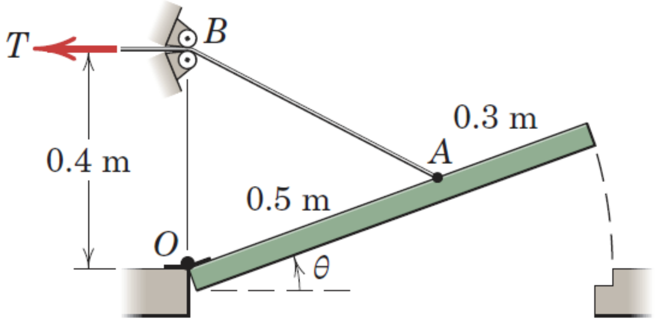
The moment of $\mathbf{T}$ about point $O$, as a vector, is $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T}$, where $\mathbf{r}_{OB} = 0.4\mathbf{j}$ m, or ③

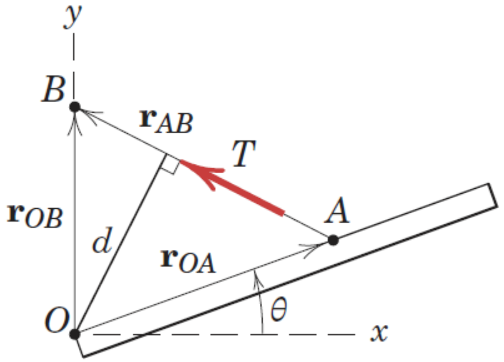
$$\mathbf{M}_O = 0.4\mathbf{j} \times T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]$$

$$= \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}\mathbf{k}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}$$



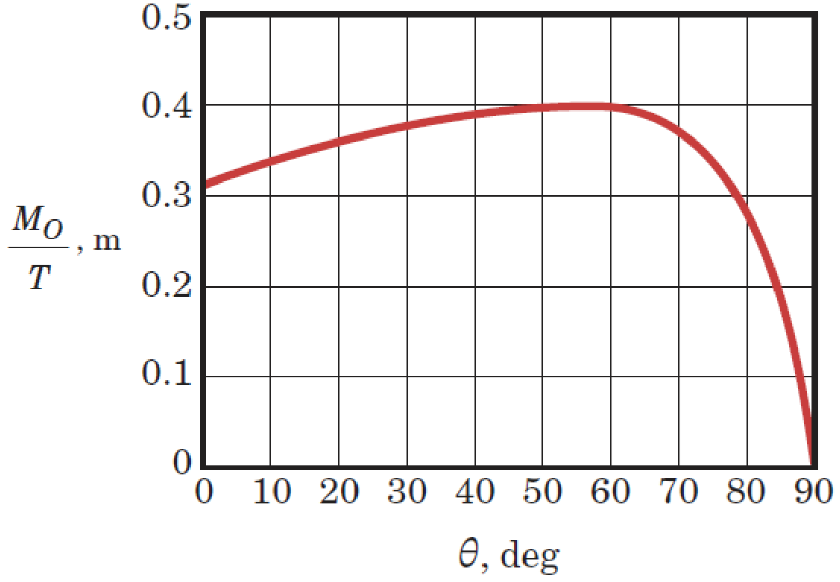

③ In the expression $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, the position vector $\mathbf{r}$ runs *from* the moment center *to* any point on the line of action of $\mathbf{F}$. Here, $\mathbf{r}_{OB}$ is more convenient than $\mathbf{r}_{OA}$.

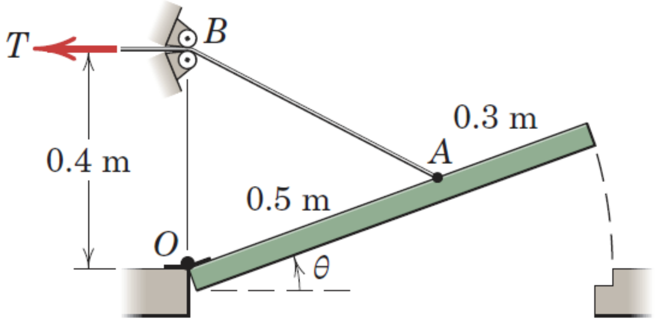
# Article 2/4 – Sample Problem 2/6 (4 of 4)

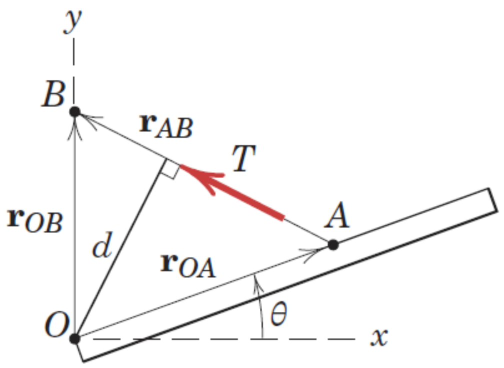
- Desired Expression and Plot

$$\frac{M_O}{T} = \frac{0.2 \cos \theta}{\sqrt{0.41 - 0.4 \sin \theta}} \qquad \textit{Ans.}$$

which is plotted in the accompanying graph. The expression $M_O/T$ is the moment arm $d$ (in meters) which runs from $O$ to the line of action of **T**. It has a maximum value of 0.4 m at $\theta = 53.1°$ (at which point **T** is horizontal) and a minimum value of 0 at $\theta = 90°$ (at which point **T** is vertical). The expression is valid even if $T$ varies.



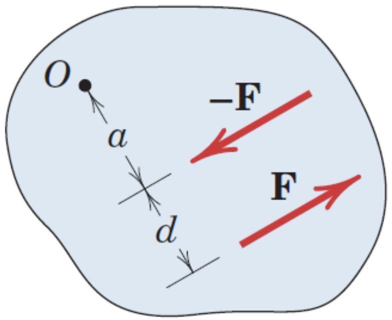
# Article 2/5 Couple

- Definition

  The moment produced by two equal, opposite, and noncollinear forces is called a **couple**.

- Illustration and Derivation (Scalars)

  $M_O = F(a + d) - Fa = Fd$

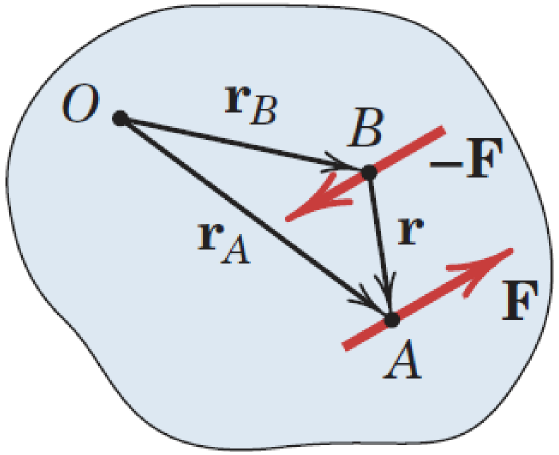



- Note especially that the magnitude of the couple is independent of the distance a which locates the forces with respect to the moment center O.
- It follows that the moment of a couple has the same value for all moment centers.

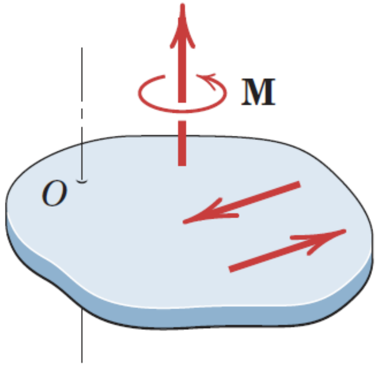
# Article 2/5 – Vector Algebra Method

- Illustration and Derivation (Vectors)

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$

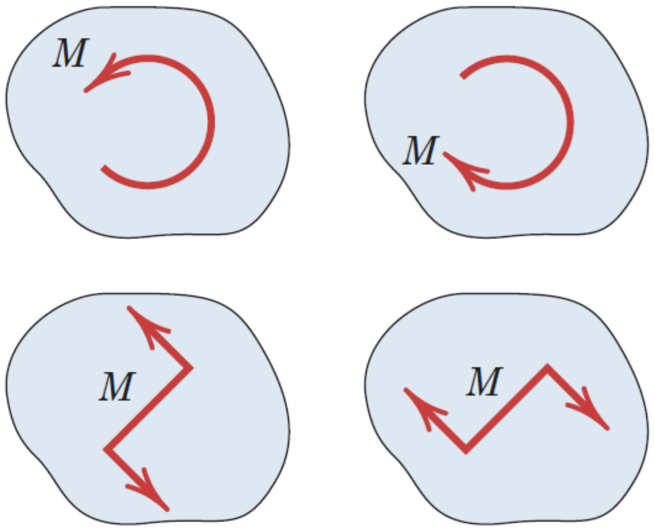



- The Couple is a Free Vector



# Article 2/5 – Alternative Representations of Couples

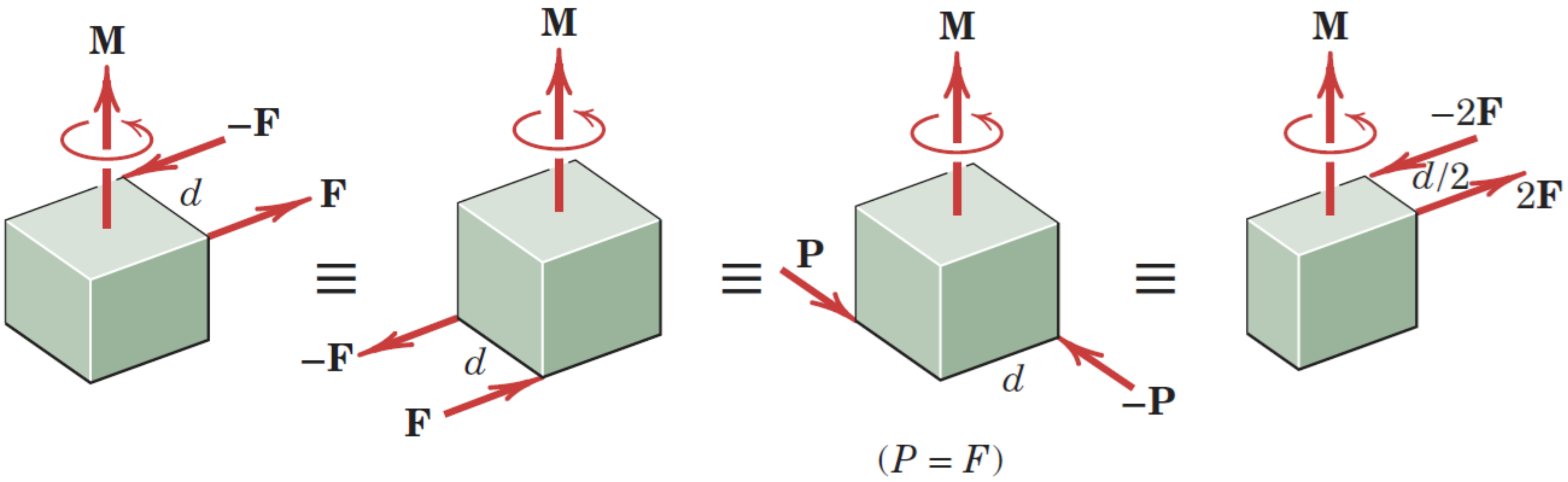
- Two-Dimensional Representations




Counterclockwise couple    Clockwise couple

- Because the couple vector **M** is always perpendicular to the plane of the forces which constitute the couple, in two-dimensional analysis we can represent the sense of a couple vector as clockwise or counterclockwise by one of the conventions shown in Figure.
- Later, when we deal with couple vectors in three-dimensional problems, we will use the notation to represent them.

# Article 2/5 – Equivalent Couples

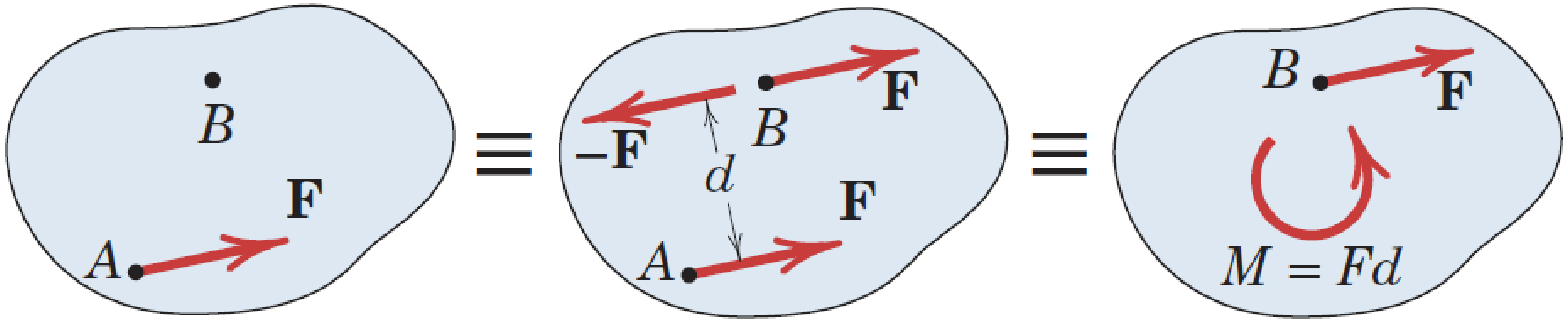


# Article 2/5 – Force-Couple Systems (1 of 2)

- Principle of the Force-Couple System

  Any force which acts at a particular location on a body can be replaced by an equivalent force which acts at a different location and a couple.

- Illustration of the Process



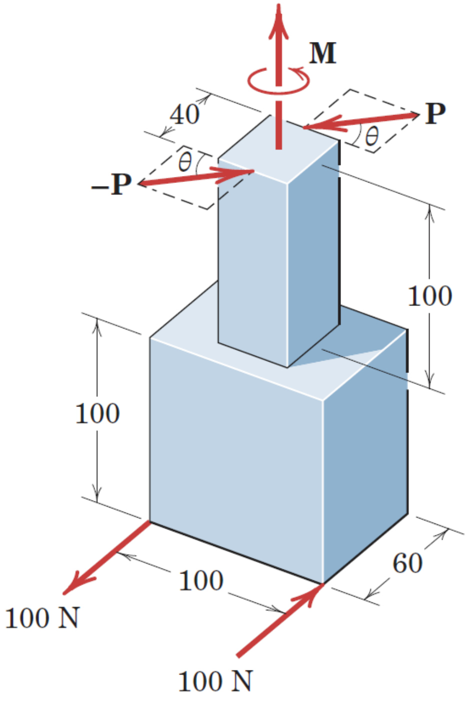
# Article 2/5 – Force-Couple Systems (2 of 2)

- Steps to Create a Force-Couple System
  1. Write the force as a vector.
  2. Compute the moment or couple which the force creates about the point.
  3. Redraw the force acting at the new location.
  4. Sketch the couple acting at the new location.

- Important Reminder
  - The force-couple system has the same effect on the body which the original force had.  It is simply a different way to visualize the effect of the force acting at a new location.

# Article 2/5 – Sample Problem 2/7 (1 of 2)

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and −**P**, each of which has a magnitude of 400 N. Determine the proper angle $\theta$.




Dimensions in millimeters

# Article 2/5 – Sample Problem 2/7 (2 of 2)

- Solution

$[M = Fd]$ $\qquad M = 100(0.1) = 10 \text{ N}\cdot\text{m}$

The forces **P** and –**P** produce a counterclockwise couple

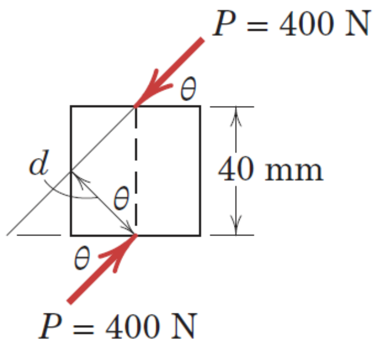
$$M = 400(0.040) \cos \theta$$

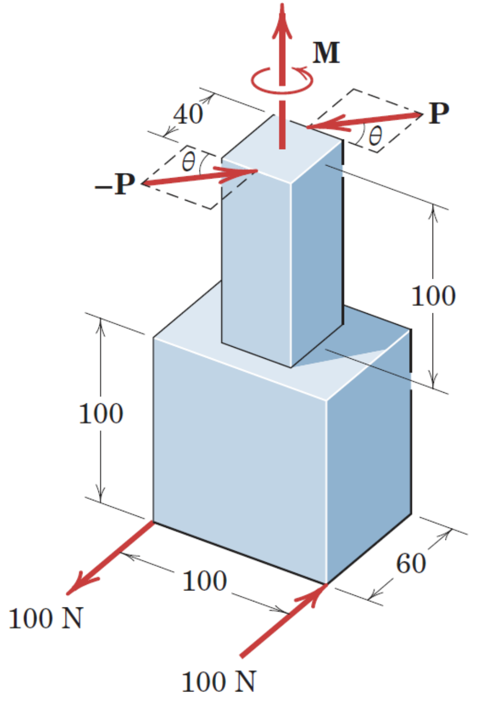
Equating the two expressions gives ①

$$10 = (400)(0.040) \cos \theta$$

$$\theta = \cos^{-1} \frac{10}{16} = 51.3° \qquad \textit{Ans.}$$

① Since the two equal couples are parallel free vectors, the only dimensions which are relevant are those which give the perpendicular distances between the forces of the couples.

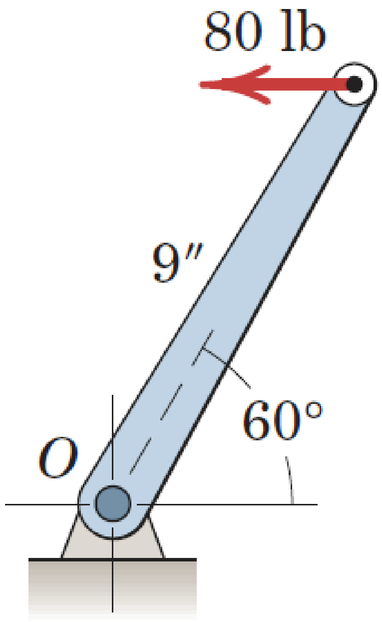



Dimensions in millimeters

# Article 2/5 – Sample Problem 2/8 (1 of 2)

Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at *O* and a couple.

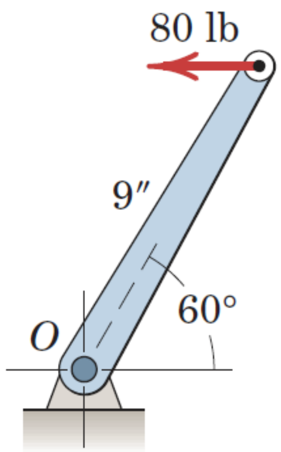


# Article 2/5 – Sample Problem 2/8 (2 of 2)

- Solution

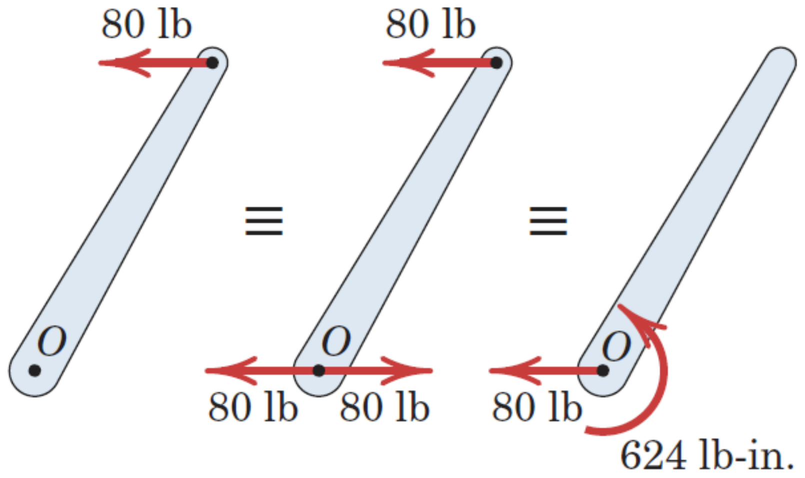
[$M = Fd$] $\quad M = 80(9 \sin 60°) = 624$ lb-in. *Ans.*

Thus, the original force is equivalent to the 80-lb force at $O$ and the 624-lb-in. couple as shown in the third of the three equivalent figures. ①




**HELPFUL HINT**

① The reverse of this problem is often encountered, namely, the replacement of a force and a couple by a single force. Proceeding in reverse is the same as replacing the couple by two forces, one of which is equal and opposite to the 80-lb force at $O$. The moment arm to the second force would be $M/F = 624/80 = 7.79$ in., which is $9 \sin 60°$, thus determining the line of action of the single resultant force of 80 lb.



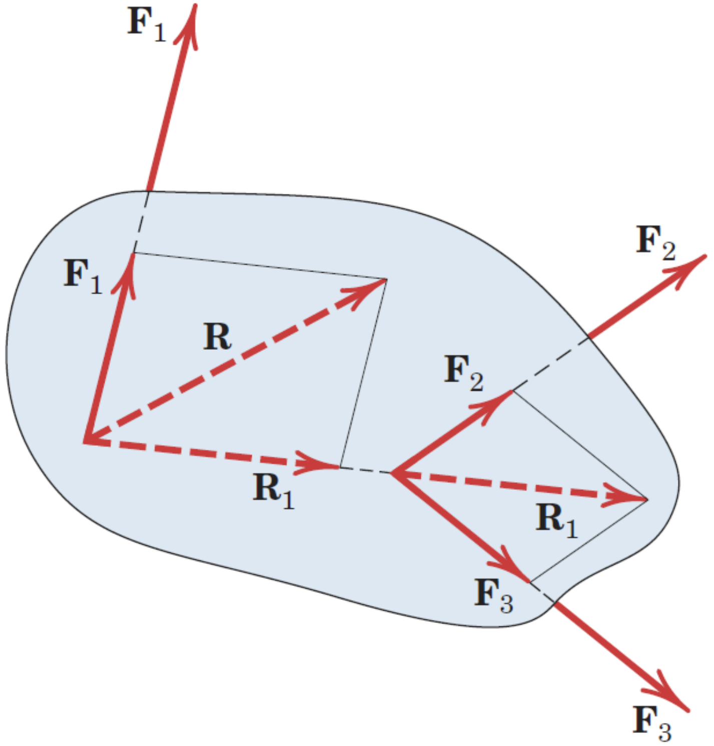
# Article 2/6 Resultants

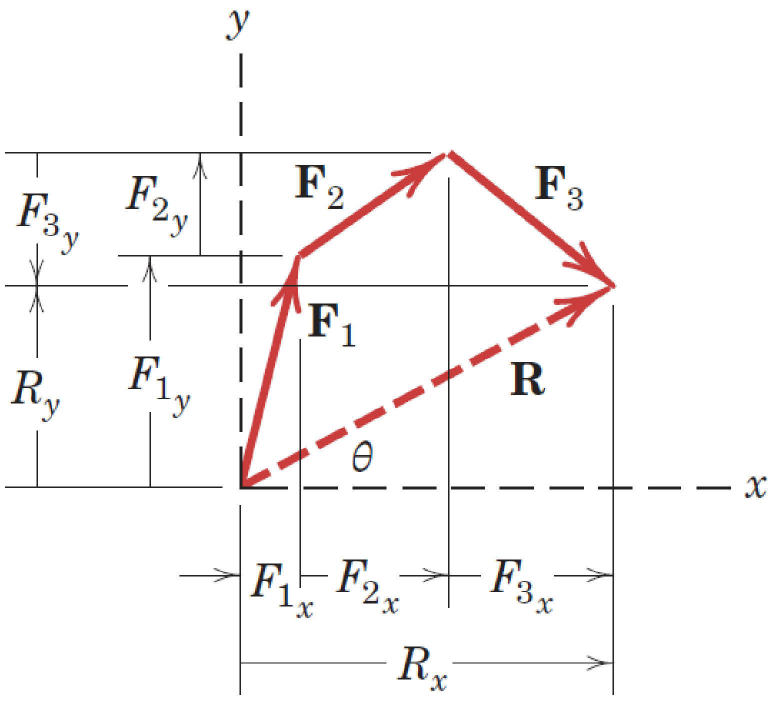
- The *resultant* of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

- Equilibrium Condition: *Equilibrium* of a body is the condition in which the resultant of all forces acting on the body is zero.

- Nonequilibrium Condition:When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of body.

# Article 2/6 – Planar Force System




$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

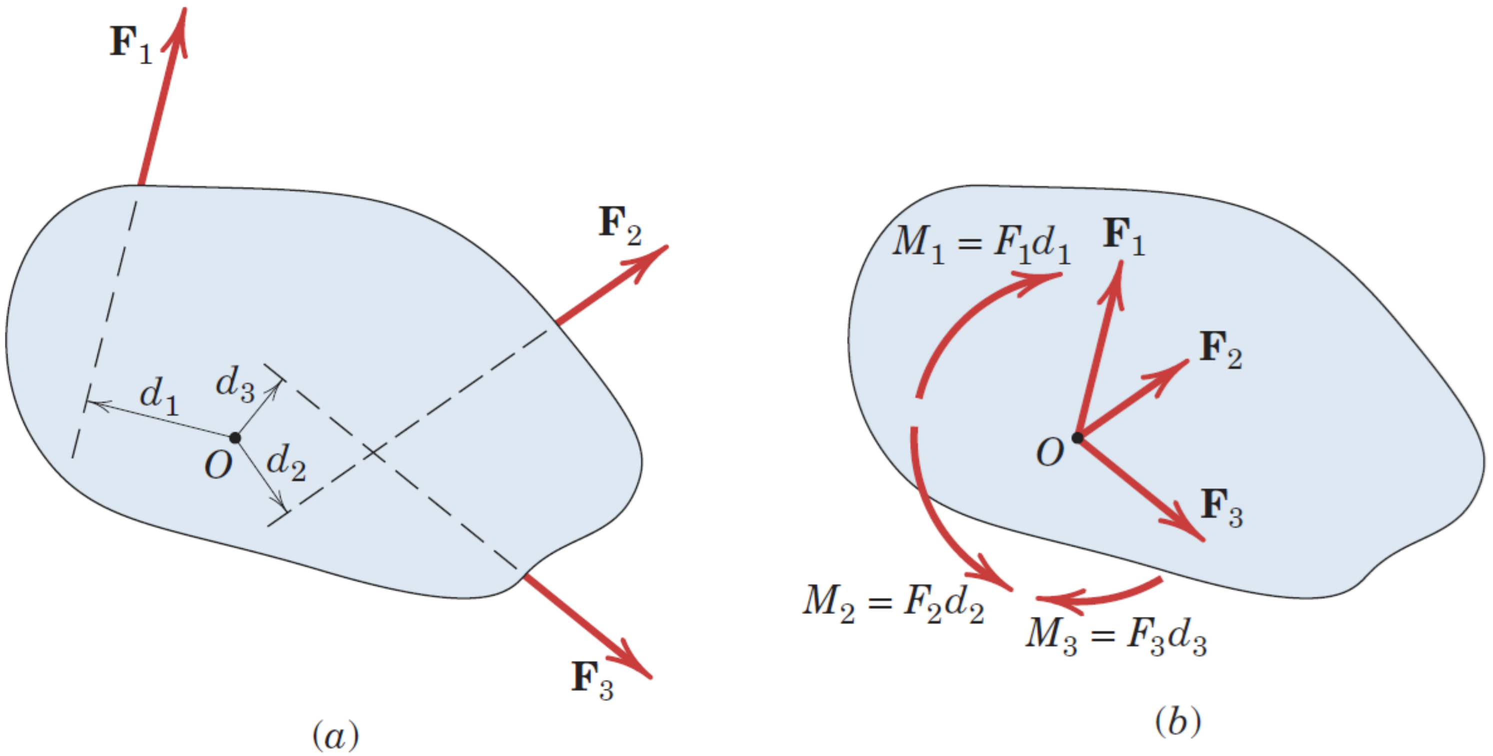
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

# Article 2/6 – Algebraic Method (1 of 3)

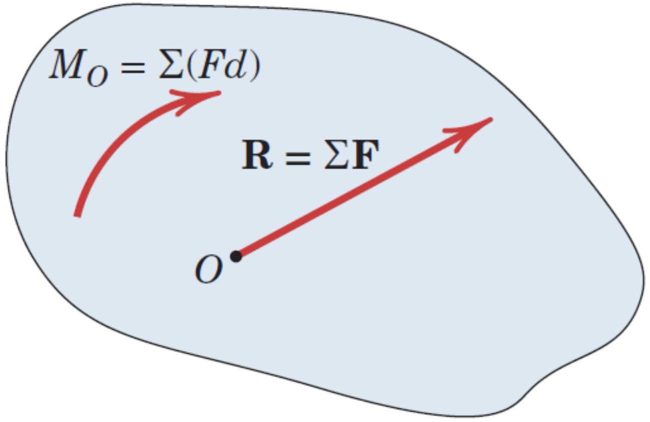
- **Finding the Resultant and Line of Action**

  1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs. (*a*) and (*b*) below where $M_1$, $M_2$, and $M_3$ are the couples resulting from the transfer of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ from their respective original lines of action to lines of action through point *O*.

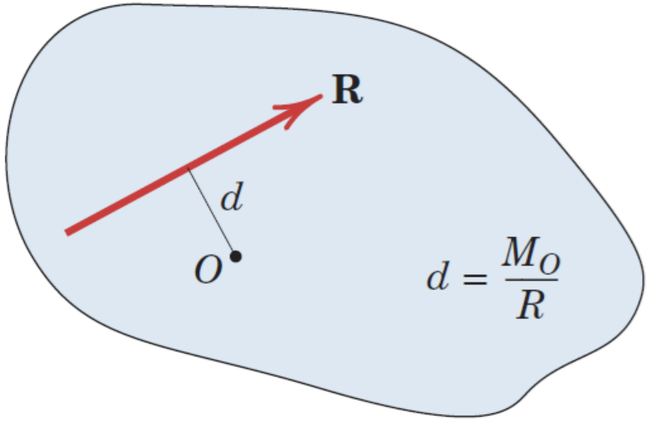


# Article 2/6 – Algebraic Method (2 of 3)

- **Finding the Resultant and Line of Action (cont.)**
  2. Add all forces at *O* to form the resultant force **R**, and add all couples to form the resultant couple $M_O$. We now have the single force–couple system, as shown below in Fig. (*c*).
  3. Find the line of action of **R** by requiring **R** to have a moment of $M_O$ about point *O*. Note that the force system in Fig. (*d*) is equivalent to the initial force system from Fig. (a) and that $\Sigma(Fd)$ in Fig. (*a*) is equal to *Rd* in Fig. (*d*).



(*c*)



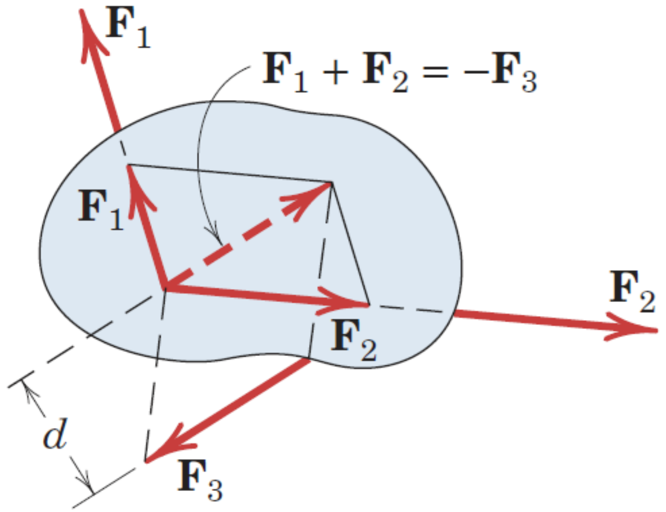
(*d*)

# Article 2/6 – Algebraic Method (3 of 3)

$$\mathbf{R} = \Sigma \mathbf{F}$$

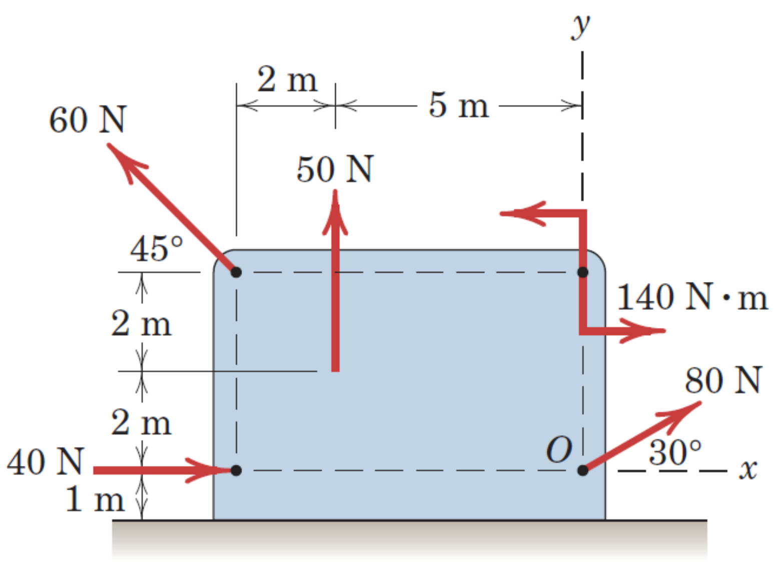
$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$



# Article 2/6 – Sample Problem 2/9 (1 of 4)

Determine the resultant of the four forces and one couple which act on the plate shown.



# Article 2/6 – Sample Problem 2/9 (2 of 4)

- **Equivalent Force-Couple System**

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

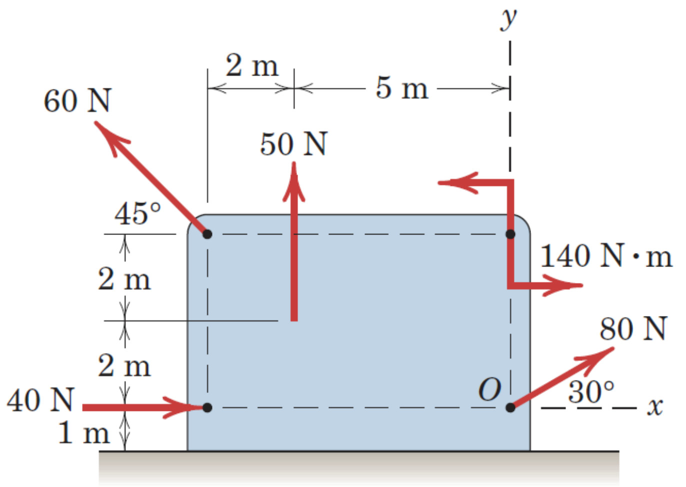
$[R = \sqrt{R_x{}^2 + R_y{}^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

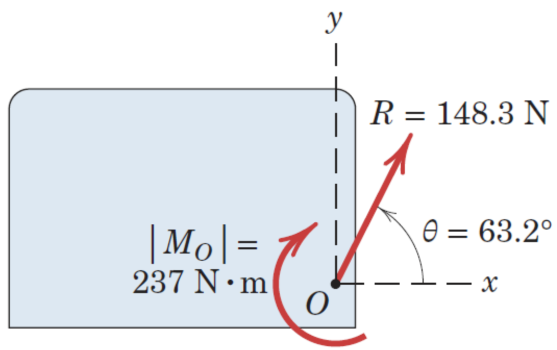
$\left[\theta = \tan^{-1} \dfrac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \dfrac{132.4}{66.9} = 63.2°$ *Ans.*

$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$ ①

$$= -237 \text{ N·m}$$

① We note that the choice of point $O$ as a moment center eliminates any moments due to the two forces which pass through $O$. Had the clockwise sign convention been adopted, $M_O$ would have been +237 N·m, with the plus sign indicating a sense which agrees with the sign convention. Either sign convention, of course, leads to the conclusion of a clockwise moment $M_O$.



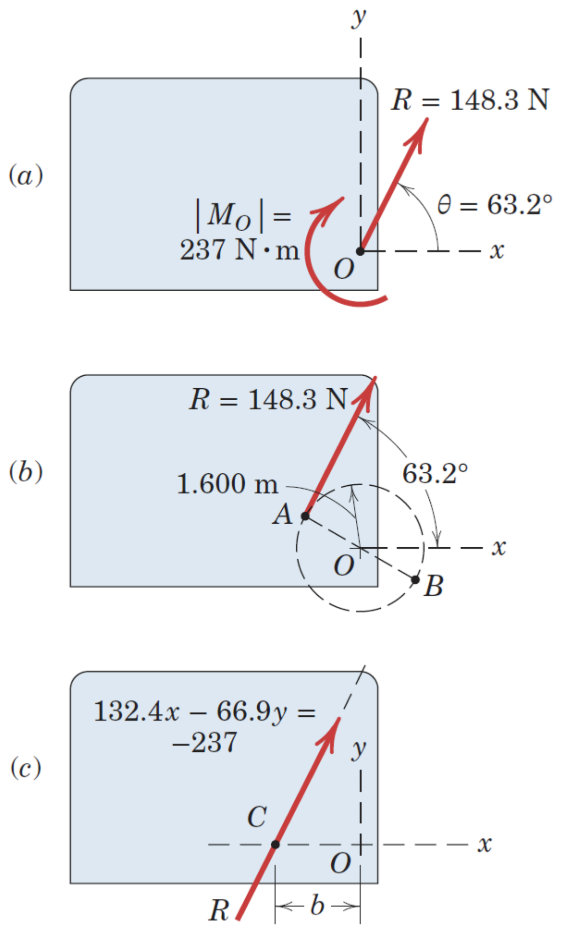
# Article 2/6 – Sample Problem 2/9 (3 of 4)

- Line of Action for the Resultant

$$[Rd = |M_O|] \qquad 148.3d = 237 \qquad d = 1.600 \text{ m} \qquad \textit{Ans.}$$

- Alternative Solution (Point *C* on *x*-axis)

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$



# Article 2/6 – Sample Problem 2/9 (4 of 4)

- Vector Approach for the Line of Action

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is a position vector running from point $O$ to any point on the line of action of $\mathbf{R}$. Substituting the vector expressions for $\mathbf{r}$, $\mathbf{R}$, and $\mathbf{M}_O$ and carrying out the cross product result in

$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

Thus, the desired line of action, Fig. $c$, is given by

$$132.4x - 66.9y = -237$$

By setting $y = 0$, we obtain $x = -1.792$ m, which agrees with our earlier calculation of the distance $b$. ②

② Note that the vector approach yields sign information automatically, whereas the scalar approach is more physically oriented. You should master both methods.