Unit :- I

SET - THEORY

Difinition of Sets, Venn Diagram, Complements, Cartesian Product, Power ~~Setsles~~, Counting Principle, Cardinality and Countability {countable and Uncountable Sets}, Pigeonhole Principle, Relation: Definition, Type of Relation, domain and Range of Relation, Pictorial Representation of Relation, Properties of Relation, partial ordering Relation, Fuction : Definition and Type of funct^n.

NOTES!!

* Definition of Sets :-

A well defined collection or class of objects is called SET. Every object included in the set is called an Element of the set.

By well defined collection of object we mean that; If A is a set and "a" is any element then either "a" is an element of A. Denoted by $a \in A$ and if it dosen't belong to A then it is denoted by $a \notin A$.

Usually Capital letter of English Alphabet are used to ~~donate~~ denote set and Small letter are used to denote elements of set.

Example :- (1) $A = \{a, b, c, d\}$ is a set consisting of four elements a, b, c, d.

Some Standard Notations :-

N : The set of natural No. $= \{1, 2, 3, 4, \ldots\ldots\}$

W : The set of whole No. $= \{0, 1, 2, 3, 4 \ldots\ldots\}$

Z or I : The set of all Integer $= \{\ldots\ldots -2, -1, 0, 1, 2, \ldots\ldots\}$

Q : The set of Rational No. $= \left\{\frac{p}{q} : p \in I, q \in I, q \neq 0\right\}$

R : The set of all Real Numbers
- $R^+ \to$ Set of +ve Real No.
- $R^- \to$ Set of -ve Real No.

C : The set of all Complex No. $= \{x + iy : x \in y \in R\}$.

Sets can be described or represented in following two ways :-

1) Tabular form / Roster form :
In this form, we actually list the elements of set and then the elements are separated by commas and enclosed in Braces { }.

Ex :- (1) The Set of odd Natural No. less than 14.
$= \{1, 3, 5, 7, 9, 11, 13\}$

(2) The set of all factor of 24 usually denoted by D_{24} :-
$D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

2) Set Builder form or Property Form :-
In this form, the property of properties Satisfied by all elements of the set is listed.

Ex :- (1) The set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ in Set builder form is described as;
$A = \{x : x \in N \text{ and } x < 9\}$.

(2) The set $A = \{a, e, i, o, u\}$ in set builder form is written as;
$A = \{x : x \text{ is a Vowel in English Alphabets}\}$

(3) The Set $A = \{3, 9, 27, 81\}$ in i.e;
$A = \{x : x = 3^n, n \in N \text{ and } n < 5\}$

Types of SETS :-

1. Null Set | Empty Set | Void Set :-

The set which contains No elements at all is called the Empty Set. Denoted { } or ϕ.

Ex :- $\{x : x \neq x\} = \phi$

$\{x : x^2 = 2, x \text{ is rational No.}\}$

$\{x : x^2 + 4 = 0, x \text{ is real No.}\}$

2. Singleton Set :-

The set which contains only one elements is called the Singleton Set.

Ex :- $\{x : x + 5 = 8\} \Rightarrow \{3\}$.

$\{x : 6 < x < 8, \text{ and } x \in N\} \Rightarrow \{7\}$

3. Finite Set :-

The set which contains finite (or limited) no. of element is called a Finite Set.

Ex :- $\{x : 1 < x < 7 \text{ and } x \in N\}$

$\Rightarrow \{2, 3, 4, 5, 6\}$

$\{x : x \text{ is vowel}\} \Rightarrow \{a, e, i, o, u\}$

4. Infinite Set :-

A set which is not finite is called Infinite Set.

Ex :- N, I, Q, R, etc.

(5) SUB-SETS :-

Let A and B are 2 sets. If every element of A is also an element of B then A is called a SUBSET of B and it is denoted by the symbol $A \subseteq B$. and is read as " A is a subset of B".

Symbolically; $A \subseteq B \Rightarrow \{x \in A \rightarrow x \in B\}$.

OR

- A set is called a subset of the set B if;

$\forall x \in A \Rightarrow x \in B$. it is expressed as $A \subseteq B$

Ex :- $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{1, 3, 5, 6\}$

$B \subseteq A$

(6) Proper Subset :-

Let A and B any 2 sets. then A is called Proper Subset of B if;

1) A is Subset of B i.e $A \subseteq B$ and
2) their Exist at least one element in B which does not belong to A, denoted by $A \subset B$.

Ex :- (1) $A = \{a, b\}$,

$B = \{a, b, c, d\}$

$A \subset B$ OR $B \supset A$

(2) $A = \{x : x$ is multiples of 3 b/w 1 to 20$\}$
$B = \{x : x$ is multiples of 3 b/w 3 to 15$\}$

$A = \{3, 6, 9, 12, 15, 18\}$, $B = \{6, 9, 12\}$

$B \subset A$ OR $A \supset B$

7. Improper Subset :-

A set "A" is called an Improper subset of B if and only if ;

$$A = B$$

OR

A subset which contains all the elements of the original set is called Improper Subset and it is denoted by $\subseteq$.

NOTE :- (1) Every set is an improper subset of itself.
(2) Empty set is a proper subset of every set $A \neq \phi$.

8. EQUAL SET :-

Two sets A and B are called Equal if ;

(1) Every Element of A is also an element of B i.e $A \subseteq B$.
(2) Every Element of B is also an element of A i.e $B \subseteq A$

then, Set (A) = Set (B).

* Working Rule :- In order to prove $A = B$, we shall prove ;

$$A \subseteq B \quad \text{and} \quad B \subseteq A.$$

9. Power SET :-

The family of all subsets of set A is called the power set of "A". and denoted by P(A).

$$P(A) = \{ \xi : \xi \subseteq A \}$$

Ex:- $A = \{a, b\}$

$$P(A) = \{ \phi, \{a\}, \{b\}, \{a, b\} \}.$$

If a finite set A has n elements, then the power set of A has 2^n elements., i.e, $\boxed{P(A) = 2^n}$

where, n = no. of elements in SET.

$\boxed{\text{Total No. of Subsets} = 2^n}$

* $A = \{1, 2, 3\}$

$P(A) = \{2^n\}$ $(n = 3) \Rightarrow 2^3 = 8$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}\}$.

(10) Universal Set :-

The set containing all objects or element of all other sets. In other words. It is a set consisting of all the elements present in other given set.

Ex:- $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{6, 7, 8\}$

So, Universal set will have all the elements of set A, B, C

$\therefore U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Operations on Set :-

(1)

Union	$A \cup B$	$\{x \mid x \in A \text{ or } x \in B\}$
Intersection	$A \cap B$	$\{x \mid x \in A \text{ and } x \in B\}$
Difference	$A - B$	$\{x \mid x \in A \text{ and } x \notin B\}$
	$B - A$	$\{x \mid x \in B \text{ and } x \notin A\}$

Ex:- $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 6\}$ Find $A \cup B$, $A \cap B$, $B - A$, $A - B$?

Sol:- $A \cup B = \{1, 2, 3, 4, 5, 6\}$ [all]

$A \cap B = \{3, 4\}$ [common]

$A - B = \{1, 2, 5\}$ [difference]

$B - A = \{6\}$

(2) Difference of 2 Sets:-

Let A and B are 2 sets. The difference of A and B (in that order) is the set of elements which belong to A but not belong to B. The difference of B from A is denoted by $A - B$ OR $B \sim A$.

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

$$x \in (A - B) \Rightarrow \{x \in A \text{ and } x \notin B\}$$

Similarly;

$$B - A = \{x : x \in B \text{ and } x \notin A\}.$$

NOTE:- (1) $A - B \subseteq A$

(2) $B - A \subseteq B$

(3) $(A - B) \cap (B - A) = \phi$.

(3) Symmetric difference of 2 Sets:-

If A and B are any 2 sets then their Symmetric difference is denoted by $A \Delta B$ or $A \oplus B$ and is read as "A symmetric difference B".

$$A \Delta B = (A - B) \cup (B - A)$$

$$A \Delta B = (A \cup B) - (A \cap B)$$

from definatⁿ, it is clear that; The set $A \Delta B$ does not contain the elements common to A and B.

In other words, It is a set A and B, denoted by $A \oplus B$ consist of those elements which belong to A or B but not to both i.e,

$$A \oplus B = (A \cup B) - (A \cap B)$$

$$A \oplus B = (A - B) \cup (B - A)$$

(4) Disjoint Set :-

Two set A and B are called Mutually Disjoint if they have No element in Common or in other words their Intersection is Empty set.

Such that ;

$$\boxed{A \cap B = \phi}$$

Ex :- $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

$$\boxed{A \cap B = \phi}$$

(5) Complement of Set :-

Let A be any Set and U be any Universal Set. The Set $U - A$ is called the Complement set of A. and denoted by A' or A^c or $C(A)$ or $\bar{A}$.

$$A' \text{ or } \bar{A} = \{x : x \in U \text{ and } x \notin A\}$$

Ex :- If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$. find A' :

Sol :- $A' = U - A$

$A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}$

$A' = \{2, 4, 6, 8, 10\}$.

Algebric Properties of SET Operations :-

① Commutative Law :-
- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

② Associative Law :-
- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$

③ Idempotent Law :-
- $A \cup A = A$
- $A \cap A = A$

④ Distributive Law :-
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

⑤ De-Morgan's Law :-
- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

⑥ Identity Law :-
- $A \cup \phi = A$
- $A \cap U = A$

⑦ Domination Law :-
- $A \cup U = U$
- $A \cap \phi = \phi$

⑧ Complimentation Law :-
- $(A')' = A$

⑨ Absorbtion Law :-
- $A \cup (A \cap B) = A$
- $A \cap (A \cup B) = A$

⑩ Complement Law :-
- $A \cup A' = U$
- $A \cap A' = \phi$

Questions Based on Union / Intersection / Difference :-

(1) Let $A = \{x : x$ is natural No. and a factor of $18\}$
$B = \{x : x$ is natural No. and less than $6\}$
find $A \cup B$, $A \cap B$, $A - B$, $B - A$.

(2) Let $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7\}$
verify, $(A \cup B) \cap C = A \cup (B \cup C)$
$(A \cap B) \cap C = A \cap (B \cap C)$

(3) If $P = \{$ multiples of 3 between 1 to 20 $\}$ and
$Q = \{$ even Natural No. upto 15 $\}$. Find Intersection of 2 given product SET and Difference Also.

(4) Given 3 Sets P, Q, R, Such that,
$P = \{x : x$ is a natural No. between 10 to 16 $\}$
$Q = \{y : y$ is a even No. between 8 to 20 $\}$
$R = \{7, 9, 11, 14, 18, 20\}$
Find ; $P - Q$, $Q - R$, $R - P$, $Q - P$

(5) Let, $A = \{a, b, d, e\}$, $B = \{b, c, e, f\}$ and $C = \{d, e, f, g\}$
then verify : $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(6) If, $A = \{1, 3, 7, 9, 10\}$, $B = \{2, 5, 7, 8, 9, 10\}$, $C = \{0, 1, 3, 10\}$
$D = \{2, 4, 6, 8, 10\}$, $E = \{0\}$
find ; $A \cup B$, $E \cup D$, $C \cup B$, $A \cap E$, $C \cap E$, $B \cap D$
$(A \cup B) \cup (A \cap D)$, $(A \cup E) \cap (C \cup D)$.

(Solved in class.)

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Ex:- 4 Show that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Sol:- Let x be any arbitary element of $A \cap (B \cup C)$;

$x \in A \cap (B \cup C) \Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ & $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ & $x \in B)$ or $(x \in A$ & $x \in C)$

$\Rightarrow [x \in (A \cap B)]$ or $[x \in (A \cap C)]$

$\Rightarrow x \in [(A \cap B) \cup (A \cap C)]$

$\therefore \quad A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad — \quad (1)$

Again, Let $x \in (A \cap B) \cup (A \cap C) \Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow [x \in A$ and $x \in B]$ or $[x \in A$ & $x \in C]$

$\Rightarrow (x \in A)$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A)$ and $(x \in B \cup C)$

$= x \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \quad — \quad (11)$

from Eq^n (1) and (11) we get.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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Ex:- 9 If X and Y are 2 sets, then prove $(X - (X - Y)) = X \cap Y$.

Sol:- By def^n $X - (X - Y)$ is the set of those elements which belong to X and does not belong to $(X - Y)$.

* Now, the element which is not in $(X - Y)$ will not be in X but it will be in Y.

i.e $\quad a \notin (X - Y) \Rightarrow a \notin X$ and $a \in Y$.

$X - (X - Y) = \{x / x \in X$ and $x \notin (X - Y)\}$

$X - (X - Y) = \{x / x \in X$ and $x \in Y\}$

$X - (X - Y) = \{x / x \in X \cap Y\}$

$X - (X - Y) = X \cap Y$

Hence Proved

Pg - 11 (12)

Ex:- 10 If A and B are 2 sets, then prove,

$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$.

Sol:- Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow (x \in A$ & $x \notin B)$ or $(x \notin A$ & $x \in B)$ or $(x \in A$ & $x \in B)$

$\Rightarrow (x \in (A - B))$ or $(x \in (B - A))$ or $(x \in A \cap B)$

$\Rightarrow x \in [(A - B) \cup (B - A) \cup (A \cap B)]$

$(A \cup B) \subseteq [(A - B) \cup (B - A) \cup (A \cap B)]$

Converse,

$x \in [(A - B) \cup (B - A) \cup (A \cap B)]$

$\Rightarrow x \in (A - B)$ or $x \in (B - A)$ or $x \in (A \cap B)$

$\Rightarrow (x \in A$ & $x \notin B)$ or $(x \in B$ & $x \notin A)$ or $(x \in A$ or $x \in B)$

$\Rightarrow (x \in A$ or $x \in B)$ or $(x \in A$ and $B)$

$\Rightarrow x \in (A \cup B)$

$(A + B) \cup (B - A) \cup (A \cap B) \subseteq (A \cup B)$

Hence, $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Pg - 12
Ex:- 12 H.W

Ex:- 16 Pg - 13

Ex:- 20 Pg - 14

Ex: 12 :- If A, B, C are 3 sets, prove $(A - C) \cap (B - C) = (A \cap B - C)$.

Ex: 16 :- If A, B, C are 3 sets, prove :- $A - (B \cap C) = (A - B) \cup (A - C)$.

$A - (B \cup C) = (A - B) \cap (A - C)$.

Ex:-20 :- Prove: "$B - A' = B \cap A$."

* Ex:-14 :- If U is Universal Set and its 2 subsets are A and B, then prove the following De-Morgan's Laws:

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$.

Sol:- (a) Let x is any arbitary element of $(A \cup B)'$ then,

$x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$(A \cup B)' \subseteq A' \cap B'$ — (1)

Converse,

$x \in A' \cap B' \Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ ~~and~~ or $x \notin B$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in (A \cup B)'$

$A' \cap B' \subseteq (A \cup B)'$

Hence, $(A \cup B)' = A' \cap B'$

(b) Let x is any arbitary element of $(A \cap B)'$ then,

$x \in (A \cap B)' \Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A' \cup B'$

$(A \cap B)' \subseteq A' \cup B'$

Converse, $x \in A' \cup B' \Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin (A \cap B)$

$= x \in (A \cap B)'$

$(A' \cap B)' \subseteq (A \cap B)'$

Hence

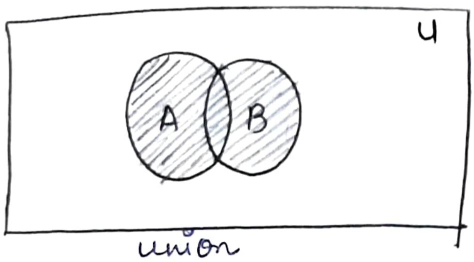
$(A \cap B)' = A' \cup B'$.

*

VENN DIAGRAM :-

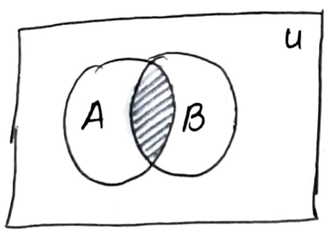
Venn Diagram is a pictorial representation of set that uses circle to show the relation among things or finite group.

① $A \cup B$



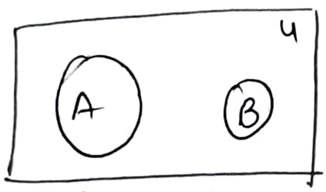
union

② $A \cap B$



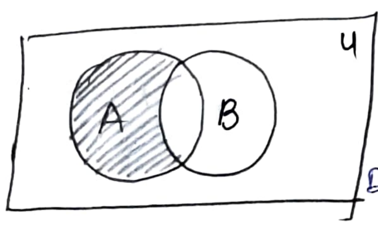
Intersection

③ $A \cap B = \phi$



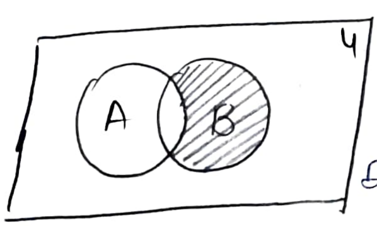
Disjoint sets.

④ $A - B$



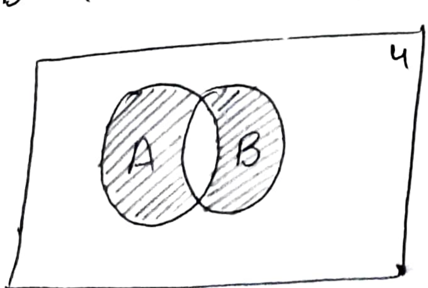
Difference

⑤ $B - A$



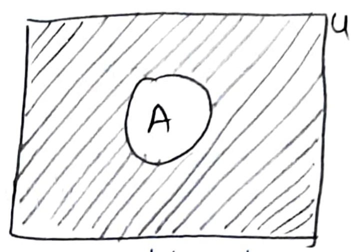
Difference

⑥ $A \Delta B = (A-B) \cup (B-A)$



Symmetric Difference

⑦ A'

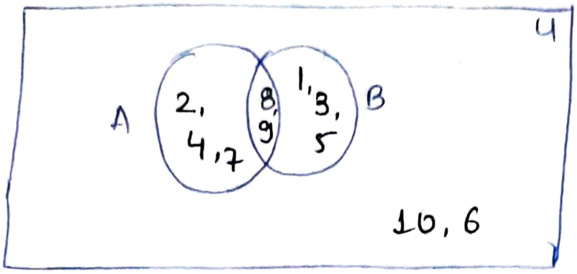


complement

Ex:- $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 7, 8, 9\}$

$B = \{1, 3, 5, 8, 9\}$



$A \cup B = \{1, 2, 3, 4, 5, 7, 8, 9\}$

$A \cap B = \{8, 9\}$

$A - B = \{2, 4, 7\}$

$B - A = \{1, 3, 5\}$

~~$(A \cup B)' = \{1, 3, 5, 6, 8, 9,$~~

$A' = \{1, 3, 5, 6, 8, 9, 10\}$

$B' = \{2, 4, 6, 7, 8, 9, 10\}$

$(A \cup B)' = \{10, 6\}$

$(A' \cap B') = \{8, 9, 6, 10\}$

Ex:- $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4\}$

$B = \{8, 9, 10\}$

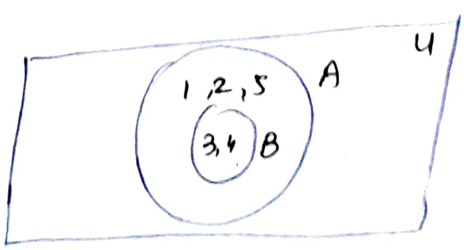
then show by Venn Diagram $A \cap B = \phi$?

$\because$ A and B are disjoint set.

Ex:- $A = \{1, 2, 3, 4, 5\}$ $B = \{3, 4\}$

$A - B = ?$

$B - A = ?$



$A - B = \{1, 2, 5\}$

$B - A = \phi$.

Cartisian Product of 2 Sets :-

Let A and B are 2 sets. The cartisian product of the set A and B is the set of all those ordered pair whose first co-ordinate is an element of A and 2^{nd} co-ordinate is an element of B and it is denoted by $A \times B$.

Symbolically ; $A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

Similarly $B \times A = \{(b,a) : b \in B \text{ and } a \in A\}$

Ex:- $A = \{1,2\}$, $B = \{3,4\}$ find $A \times B$ & $B \times A$?

* $A \times B = \{1,2\} \times \{3,4\}$

$A \times B = \{(1,3)\ (1,4)\ (2,3)\ (2,4)\}$

* $B \times A = \{3,4\} \times \{1,2\}$

$B \times A = \{(3,1)\ (3,2)\ (4,1)\ (4,2)\}$

we can clearly see that,

$A \times B \neq B \times A$

Pg - 33
Ex :- 1
Ex :- 2. H.W

Ex:- 1 If $A = \{1,2\}$, $B = \{2,3\}$, then find $A \times B$ and $B \times A$?
Is $A \times B = B \times A$?

Ex:- 2 If $A = \{1,2\}$, $B = \{2,3\}$ and $C = \{3,5\}$ then find
$(A \times B) \cap (A \times C)$

Pg-33

Ex:-5 Prove that; $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Sol:- Let (x,y) be any arbitary Element of $A \times (B \cup C)$

$(x,y) \in A \times (B \cup C) \Rightarrow x \in A$ and $y \in B \cup C$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow [x \in A$ and $y \in B]$ or $[x \in A$ or $y \in C]$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\Rightarrow (x,y) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ — (1)

Converse,

Let $(x,y) \in (A \times B) \cup (A \times C)$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\Rightarrow [x \in A$ and $y \in B]$ or $[x \in A$ and $y \in C]$

$\Rightarrow (x \in A)$ and $[y \in B$ ~~and~~ or $y \in C]$

$\Rightarrow (x \in A)$ and $(y \in B \cup C)$

$\Rightarrow (x,y) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ — (11)

from Eq^n (1) and (11) we get

$(A \times B) \cup (A \times C) = A \times (B \cup C)$

*

Ex:-6 Prove that; $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol:- Let (x,y) be any arbitary Element of $A \times (B \cap C)$

$(x,y) \in A \times (B \cap C) \Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow [x \in A$ and $y \in B]$ and $[x \in A$ and $y \in C]$

$\Rightarrow (x,y) \in (A \times B)$ and $(x,y) \in (A \times C)$

$\Rightarrow (x,y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ — (1)

Converse,

$(x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow (x,y) \in (A \times B)$ and $(x,y) \in (A \times C)$

$\Rightarrow [x \in A$ and $y \in B]$ and $[x \in A$ and $y \in C]$

$\Rightarrow x \in A$ and $[y \in B$ and $y \in C]$

$\Rightarrow x \in A$ and $[y \in B \times C]$

$\Rightarrow (x,y) \in A \cap (B \times C)$

$\therefore (A \times B) \cap (A \times C) \subseteq A \cap (B \times C)$ — (11)

from Eq^n (1) and (11) we get;

$$A \cap (B \times C) = (A \times B) \cap (A \times C)$$

*

Pg-35

Ex:-10 Show that : $(P \cap Q) \times (R \cap S) = (P \times R) \cap (Q \times S)$ for some arbitary set. P, Q, R and S.

Sol:- Let (x,y) be any arbitary element of $(P \times R) \cap (Q \times S)$

$(x,y) \in (P \times R) \cap (Q \times S)$

$\Rightarrow (x,y) \in (P \times R)$ and $(x,y) \in (Q \times S)$

$\Rightarrow [x \in P$ and $y \in R]$ and $[x \in Q$ and $y \in S]$

$\Rightarrow [x \in P$ and $x \in Q]$ and $[y \in R$ and $y \in S]$

$\Rightarrow x \in (P \cap Q)$ and $y \in (R \cap S)$

$\Rightarrow (x,y) \in (P \cap Q) \times (R \cap S)$

$\therefore (P \times R) \cap (Q \times S) \subseteq (P \cap Q) \times (R \cap S)$ — (1)

Converse,

Let $(x,y) \in (P \cap Q) \times (R \cap S)$

$\Rightarrow x \in (P \cap Q)$ and $y \in (R \cap S)$

$\Rightarrow [x \in P$ and $x \in Q]$ and $[y \in R$ and $y \in S]$

$\Rightarrow [x \in P \text{ and } y \in R] \text{ and } [x \in Q \text{ and } y \in S]$

$\Rightarrow [(x, y) \in (P \times R)] \text{ and } [(x, y) \in (Q \times S)]$

$\Rightarrow (x, y) \in (P \times R) \cap (Q \times S)$

$\therefore (P \cap Q) \times (R \cap S) \subseteq (P \times R) \cap (Q \times S)$ — (ii)

Hence, $(P \cap Q) \times (R \cap S) = (P \times R) \cap (Q \times S)$

*

Pg-34

Ex:- Prove that $(A-B) \times C = (A \times C) - (B \times C)$.

Sol:- Let (x, y) be any arbitrary element of $(A-B) \times C$

$(x, y) \in (A-B) \times C \Rightarrow x \in (A-B) \text{ and } y \in C$

$\Rightarrow [x \in A \text{ and } x \notin B] \text{ and } y \in C$

$\Rightarrow [x \in A \text{ and } y \in C] \text{ and } [x \notin B \text{ and } y \in C]$

$\Rightarrow (x, y) \in (A \times C) \text{ and } (x, y) \notin (B \times C)$

$\Rightarrow (x, y) \in (A \times C) - (B \times C)$

$\therefore (A-B) \times C \subseteq (A \times C) - (B \times C)$ — (i)

Converse,

$(x, y) \in (A \times C) - (B \times C) \Rightarrow (x, y) \in (A \times C) \text{ and } (x, y) \notin (B \times C)$

$\Rightarrow [x \in A \text{ and } y \in C] \text{ and } [x \notin B \text{ and } y \in C]$

$\Rightarrow [x \in A \text{ and } x \notin B] \text{ and } (y \in C)$

$\Rightarrow x \in (A-B) \text{ and } y \in C$

$\Rightarrow (x, y) \in (A-B) \times C$

$\therefore (A \times C) - (B \times C) \subseteq (A-B) \times C$ — (ii)

Hence,

$(A-B) \times C = (A \times C) - (B \times C)$

*

Counting Principle :-

"To Count" or Counting can be defined as " The act of determining the ~~quali~~ quantity or the total No. of object in a SET or a group.

The fundamental Counting Principle states that if there are p ways to do one thing and q ways to do another things then there are $p \times q$ ways or $p+q$ ways to do both things.

Ex :- You have 3 Shirts and 4 pants; that means

$3 \times 4 = 12$

12 different outfits.

There are Some Basic Counting Principle :- Addition, multiplication, Cardinality, Inclusion-Exclusion Principle.

(1) Addition Principle :-

Rule :- A task can be performed either in n_1 or n_2 ways where the 2 task cannot be performed at same time. then, $n_1 + n_2$ ways to perform task.

Ex :- Suppose there are 15 NIT's and 20 IIT's in India and you qualified for the JEE. So there are : $(15+20) = 35$ ways you can select your college.

* Consider you go to a restaurant where you find 8 paneer dishes and 5 vegetable dishes. Now you want to order a dish among these. So, you can choose a dish in $(8+5) = 13$ dishes.

(2) Multiplication Principle :-

Rule :- There are n_1 ways to perform first task and n_2 ways to perform second task ; after the first task is complete ; then there are $n_1 \times n_2$ ways to perform task.

Ex :- * Suppose there are 20 IIT's each with 7 Branches. The the qualified student can select his Branch in

$20 \times 7 = 140$ ways.

* A new company with just 2 Employees, Rohan & Raj rents a floor of a Building with 12 offices, How many ways are there to assign different office to these 2 Employees ?

n_1 task $= 12$

n_2 task $= 11$

$n_1 \times n_2 = 12 \times 11 = 132$ choices.

(3) Cardinality :-

Let A be a finite Set. The No. of distinct elements in the set A is called the order or Cardinality of A and is denoted by $O(A)$, $n(A)$, $|A|$.

It is Basically About the no. of Elements present in SET.

Ex:- * $\{x : x$ is days in a week$\} = \{$Sun, Mon, Tue, Wed, Th, Fri, Sat$\} = 7$

* $\{x : x$ is even No. less than $10\} = \{0, 2, 4, 6, 8\} = 5$

* $A = \{1, 2, 3, 4, 5\} \Rightarrow |A| = 5$.

* $B = \{1, 2, 3 \ldots n\} \Rightarrow |B| = n$.

* $\{x : x$ is month of year$\} = 12$

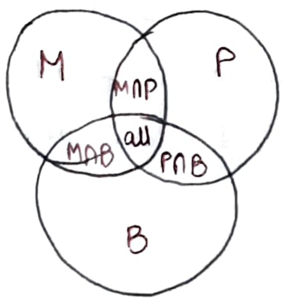
* $\{x : 10 < x < 20\} = 9$.

Ex:- 7 Among 100 Students, 32 Study Mathematics, 20 Study Physics, 45 Study Biology, 15 Study Mathematic and Biology, 7 Study Mathematics and Physics, 10 Study Physics and Biology and 30 do not Study any of 3 Sub.

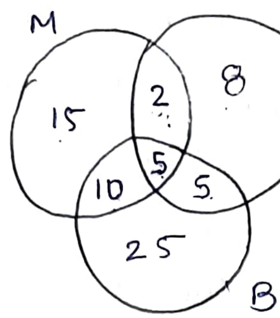
i) Find the no. of Students studying all 3 Subjects.

ii) Find the no. of Students Studying exactly 1 of 3 Subjects.

Sol:- VENN DIAGRAM:-



$\Rightarrow$



(i) Let X denote the set of all students.

Let M, P, B denote the set of students ~~stydiiy~~ Studying Mathematics, Physics, and Biology resp. Then we have.

$$|X| = 100$$

$$|M| = 32, \quad |P| = 20, \quad |B| = 45$$

$$|M \cap B| = 15, \quad |M \cap P| = 7, \quad |P \cap B| = 10$$

$$|(M \cup P \cup B)'| = 30$$

$$\therefore \; |M \cup P \cup B| = |X| - |(M \cup P \cup B)'| = 100 - 30 = 70.$$

By the Principle of Inclusion and Exclusion, we have

$$|M \cup P \cup B| = |M| + |P| + |B| - |M \cap P| - |M \cap B| - |P \cap B| + |M \cap P \cap B|$$

$$|M \cap P \cap B| = |M \cup P \cup B| - |M| - |P| - |B| + |M \cap P| + |M \cap B| + |P \cap B|$$

$$= 70 - 32 - 20 - 45 + 7 + 15 + 10$$

$$|M \cap P \cap B| = 5$$

Hence, the No. of Students Studying all the 3 Subjects is 5.

(iii) No. of students studying exactly one of 3 subjects

$= |M| + |P| + |B|$ {value of $|M|$ $|P|$ and $|B|$ by venn Diagram.}

$= 15 + 8 + 25$

$= 48$

Ex:- Out of 120 students surveyed it was found that 20 students have studied French, 50 students have studied English, 70 students have studied Hindi, 5 have studied English and French, 20 have studied English & Hindi, 10 studies Hindi and French Only 3 students have studied all the 3 language. Find How many students have studied:

i) Only Hindi 2) French Only

3) English but not Hindi 4) Hindi but not french.

and 12 students do not study at all.

Sol:- Let X denote the total No. of students;

F, E, H denote the students who study only French, English, Hindi, resp.

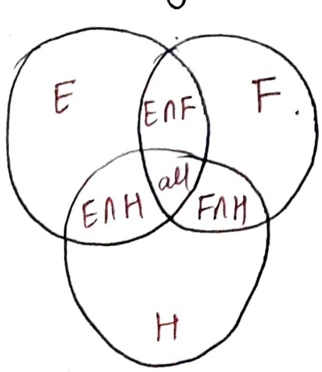
$|X| = 120$

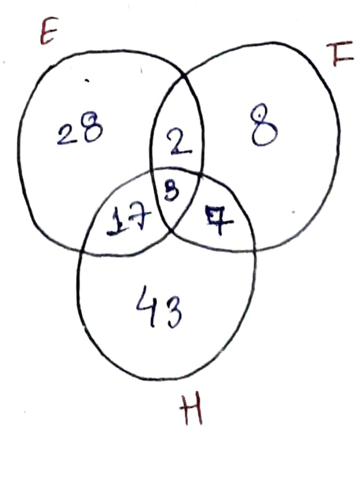
$|F| = 20$, $|E| = 50$, $|H| = 70$

$|E \cap F| = 5$, $|E \cap H| = 20$, $|H \cap F| = 10$

$|F \cap E \cap H| = 3$

Venn Diagram:-





1) Only Hindi =

$$|H| = |H - E - F|$$
$$= |H| - |E \cap H| - |H \cap F| + |H \cap E \cap F|$$
$$= 70 - 20 - 10 + 3$$
$$= 73 - 30$$
$$= \underline{\underline{43}} \text{ students}$$

(2) Only French =

$$|F| = |F - E - H|$$
$$= |F| - |F \cap H| - |F \cap E| + |F \cap E \cap H|$$
$$= 20 - 10 - 5 + 3$$
$$= 23 - 15$$
$$= 8 \text{ students}$$

(3) English but not Hindi = 28 + 2 = 30 Students

(4) Hindi but not french = 43 + 17 = 60 Students

*

Ex:-6] A class has 175 students. The following description showing:
Mathematics student : 100, Physics 70, Chemistry 46, Mathematics and Physics 30, Mathematics and chemistry 28, Physics and Chemistry 23, None of the 3 subjects 22. Find:
1) How many students study all 3 Subjects?
2) How many students study exactly one subject out of 3?

Ex:-9] Out of total 130 students, 60 are wearing hats to class, 51 are wearing scarves and 30 are wearing both Hats & Scarves. Of the 54 students who are wearing sweaters, 26 are wearing Hats, 21 are wearing scarves and 12 are wearing both hats and Scarves. Everyone wearing neither a hat nor a scarf is wearing gloves:

i) How many students are wearing gloves?
ii) How many students not wearing a sweater are wearing Hats but not Scarfs.

Countability :-

* Equivalent Set :-

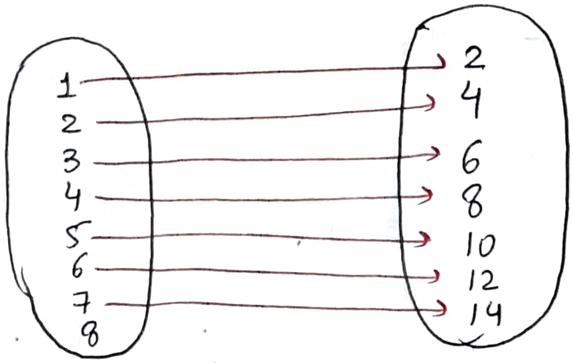
A set "A" is said to be equivalent to set B. if there is one to one correspondence between 2 sets.

Ex :- Let $N = \{1, 2, 3, 4, 5, 6 \ldots\ldots\}$ (Natural No.)

$E = \{2, 4, 6, 8, 10 \ldots\ldots\}$ (Even No.)

A function defined by $f(x) = 2x \ \forall \ x \in N$ which is one - one, onto.

$N \sim E$.



$f(x) = 2x$

$x = 1$, $f(1) = 2$

$x = 2$ $f(2) = 4$

$x = 3$ $f(3) = 6$

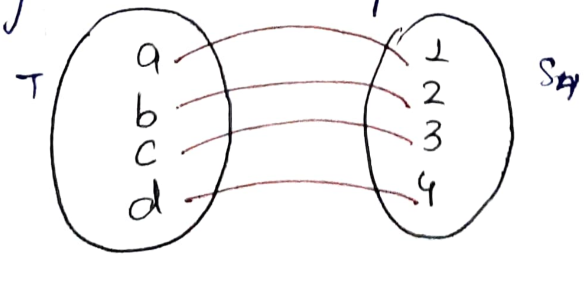
so on

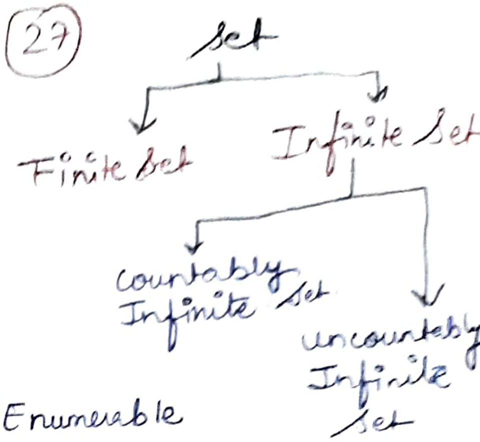
* Finite Set :-

A set having 1-1 correspondance with the empty set and set $S_n = \{1, 2, 3, 4 \ldots\ldots n\}$; $n \in N$ are called Finite SET.

Ex :- Let $T = \{a, b, c, d\}$, $S_n = \{1, 2, 3, 4\}$

T having one - one correspondence with the set $S_4 = \{1, 2, 3, 4\}$.





* Infinite Set :-

A set which is not finite is called Infinite set.

Ex:- N, Z, Q, R, etc

They are of 2 Types :-

1) Countably Infinite / Denumerable / Enumerable
2) Un-Countably Infinite set.

Countably Infinite :-

A set "A" is called countably Infinite if it is Equivalent to set of natural No.
It is also known as a Denumerable | Enumerable.

In other words,
If the set A is Infinite, then A is called "Countable" if its elements can be put in 1-1 correspondence with the set of natural No. otherwise Un-Countable if such a correspondence does not exist.

Ex:- $A = \{0, 1, 2\}$ is Countable.
Z, N are Countable.

RESULT :-

i) Cardinality of Countably Infinite Set is $\aleph_0$ {alpha not}
ii) An Infinite Set is Countable iff its elements can be labelled by Natural No.
iii) Finite product of Countable Set is Countable.
iv) Countable Union of Countable Set is Countable.

{Countably Infinite Set's Cardinality is Denoted by $\aleph_0$}

Uncountable Set :-

A set which is not Countable is known as Uncountable set and A set which we can not find 1 to 1 corrospondence to Natural No.

Ex:- Real No., Complex No, etc.

Que: Show that the set of all Integer is Countable.

Sol:- Let, $Z = \{-\infty, \ldots\ldots -2, -1, 0, 1, 2 \ldots\ldots\ldots \infty\}$

$$Z = \{\infty \ldots\ldots\ldots \pm 2, \pm 1, 0\}$$

Let $f : N \longrightarrow Z$ be a mapping defined as ;

$$f(n) = \begin{cases} \frac{n}{2}, \text{ if } n \text{ is even} & \forall\, n \in N. \\ -\frac{(n-1)}{2}, \text{ if } n \text{ is odd} & \forall\, n \in N. \end{cases}$$

$N = \{1, 2, 3, 4, 5, 6 \ldots\ldots\}$.

Now put the value of Natural No. to $f(n)$ So we get the set of Integer or Not ?.

$(n = 1)$, $f(1) = \frac{-(1-1)}{2} = 0$

$(n = 2)$, $f(2) = \frac{\cancel{2}}{\cancel{2}} = 1$

$(n = 3)$, $f(3) = -\frac{(3-1)}{2} = -1$

$(n = 4)$, $f(4) = \frac{\cancel{4}\,2}{\cancel{2}\,1} = 2$

Hence, Z is 1-1 Corrospending to Natural No.

So, Z is Countably Infinite Set.

*