Relation :-

In order to express a relation from the set A to the set B, we always need a statement which connects the elements of A with the element of B.

Ex :- Suppose $A = \{1, 3, 5, 9\}$, $B = \{0, 2, 4, 8\}$. Now suppose a relation from the set A to the set B is expressed by the statement "is less than".

Now taking the first and 2nd co-ordinates of the elements of the sets A and B resp., the ordered pair satisfying the statement "is less than" are as follows;

$(1, 2)\ (1, 4)\ (1, 8)\ (3, 4)\ (3, 8)\ (5, 8)$.

The set R of these ordered pairs given by.

$$R = \{(1, 2)\ (1, 4)\ (1, 8)\ (3, 4)\ (3, 8)\ (5, 8)\}$$

R expresses a relation from the set A to the set B.

Clearly the set R is a subset of $A \times B$.

$$\therefore R \subseteq A \times B$$

Clearly any Relation from A to B is given by;

$$R = \{(x, y) : x R y, \text{ where } x \in A \text{ and } y \in B\}.$$

Ex :- If relation R is greater than; $A = \{1, 3, 5, 9\}$, $B = \{0, 2, 4, 8\}$

$$R = \{(1, 0)\ (3, 0)\ (3, 2)\ (5, 0)\ (5, 4)\ (9, 0)\ (9, 2)\ (9, 4)\ (9, 8)\}$$

(Range) → $\{0, 2, 4, 8\}$

(Domain) → $\{1, 3, 5, 9\}$

* Domain and Range of a Relation :-

Suppose R is a relation from A to B ie $R \subseteq A \times B$.
The Domain of R is the set of all first element of the ordered pairs which belong to R. and
Range of R is the set of all Second element of these ordered pairs.

$$\text{Domain} = \{x : x \in A \text{ and } (x, y) \in R \text{ for some } y \in B\}$$

$$\text{Range} = \{y : y \in B \text{ and } (x, y) \in R \text{ for some } x \in B\}$$

Ex :- Let $A = \{1, 2, 9\}$ and $B = \{1, 3, 7\}$

- Case 1st :- If relation R is "equal to"

$$R = \{(1, 1)\}$$

$$\text{Domain}(R) = \{1\}$$
$$\text{Range}(R) = \{1\}$$

- Case 2nd :- If relation R is "less than"

$$R = \{(1, 3)(1, 7)(2, 3)(2, 7)\}$$

$$\text{Domain}(R) = \{1, 2\}$$
$$\text{Range}(R) = \{3, 7\}$$

- Case 3rd :- If relation R is "greater than"

$$R = \{(2, 1)(9, 1)(9, 3)(9, 7)\}$$

$$\text{Domain}(R) = \{2, 9\}$$
$$\text{Range}(R) = \{1, 3, 7\}$$

*

* Inverse Relation :-

If R is a relation from the set A to B, then the inverse of R is relation from set B to A and is denoted by R^{-1}. Symbolically,

$$R^{-1} = \{(y, x) : (x, y) \in R, x \in A \text{ and } y \in B\}.$$

Thus to find R^{-1}, we write in reverse order all ordered pairs belonging to R.

$\therefore$ Range of R^{-1} = Domain of R.

Domain of R^{-1} = Range of R.

Ex :- 2
Ex :- 3
Ex :- 4
Ex :- 5

$\Big\}$ from Book [Pg → 44, 45].

Properties of Relation :-

(1) Reflexive Relation :
Irreflexive Relation.

(2) Symmetric Relation.
Asymmetric Relation.

(3) Antisymmetric

(4) Transitive Relation.

(5) Identity Relation.

(1) Reflexive Relation :-

If R is a relation in the set A, then R is called Reflexive Relation, if every element of A is R-related to itself. i.e

$$(a, a) \in R, \forall a \in A$$

$$a R a$$

$$R \subseteq A \times A$$

Ex:- If $A = \{1, 2, 3\}$ and $R = \{(1,1)(2,2)(3,3)(1,2)\}$ then relation R is Reflexive ;

$\because (1, 1) \in R, (2, 2) \in R, (3, 3) \in R$ i.e $(a, a) \in R$ $\forall a \in A$.

Ex:- If $A = \{a, b\}$ and $R = \{(a, a), (b, b), (a, b)\}$ then relation R is Reflexive;

$\because (a, a) \in R, (b, b) \in R$, i.e $(a, a) \in R \ \forall a \in A$.

- **Irreflexive Relation :-**

A relation R on a set A is called irreflexive, if $(a, a) \notin R$ for every element $a \in A$.

Ex:- If $A = \{1, 2, 3\}$ and $R = \{(1,1)(2,2)(1,3)\}$ then R is not reflexive $\because (3, 3) \notin R$ while $3 \in A$.

(2) Symmetric Relation :-

If R is a relation in the set A, then R is called Symmetric Relation; if R-related to b then b is also R-related to a. i.e;

i.e $\quad (a, b) \in R \Rightarrow (b, a) \in R$

i.e $\quad a R b \Rightarrow b R a$

where, $a, b \in A$

NOTE :- Clearly a relation R will be Symmetric if $R = R^{-1}$

Ex:- If $A = \{2, 4, 5, 6\}$ and

$R_1 = \{(2,4)(4,2)(4,5)(5,4)(6,6)\}$,

$R_2 = \{(2,4)(2,6)(6,2)(5,4)(4,5)\}$.

Sol :- (a) $R_1 = \{(2,4)(4,2)(4,5)(5,4)(6,6)\}$

Relation R_1 is Symmetric

$\because$

$(2,4) \in R_1 \Rightarrow (4,2) \in R_1$;

$(4,5) \in R_1 \Rightarrow (5,4) \in R_1$;

$(6,6) \in R_1 \Rightarrow (6,6) \in R_1$;

i.e, $(a,b) \in R_1 \Rightarrow (b,a) \in R_1$. is true.

(b) R_2 is not Symmetric $\because$

$(2,4) \in R_2 \Rightarrow (4,2) \notin R_2$.

- Asymmetric :-

A relation R on a set A is called Asymmetric. If $a = b$ whenever $(a,b) \in R \Rightarrow (b,a) \notin R \;\forall\; a, b \in A$.

Ex:- R_2 in above is Asymmetric.

③ Anti-Symmetric Relation :-

If R is ~~Symmetric~~ Relation in the Set A, then R is called Anti-Symmetric Relation.

Condition (I) $(a,b) \in R$ & $(b,a) \in R \Rightarrow a = b$

$a R b$ and $b R a \Rightarrow a = b$

$a, b \in A$.

Condition (II)

$(a,b) \in R$ and $a \neq b$, then $(b,a) \notin R$ [must]

which means,

$(b,a) \notin$ must not hold.

Ex:- $A = \{1, 2, 3\}$

* $R_1 = \{(1,2)\ (2,2)\ (2,1)\}$

$(1,2) \in R$, $(2,1) \in R$ but $1 \neq 2$

So, it is not Anti-Symmetric.

* $R_2 = \{(1,1)\ (2,2)\ (1,3)\}$

$(1,1) \in R$, $(2,2) \in R$ i.e $1 = 1$, $2 = 2$

and $(1,3) \in R$ and [$(3,1) \notin R$ then it is antisymmetric]

Hence R_2 is Anti-Symmetric.

* $R_3 = \{(1,1)(2,2),(3,3)\}$

$(1,1) \in R$, $(2,2) \in R$, $(3,3) \in R$ i.e $1=1$, $2=2, 3=3$

Hence R_3 is Anti Symmetric.

* $R_4 = \{(1,1)\ (2,2)\ (1,2)\ (3,3)\ (1,3)\}$

$(1,1),(2,2)(3,3) \in R$ i.e $1=1$, $2=2$, $3=3$

and

$(1,2) \in R$ and $(2,1) \notin R$

$(1,3) \in R$ and $(3,1) \notin R$

Hence, R_4 is Anti-Symmetric.

* $R_5 = (\leq) \Rightarrow \{(1,2)\ (2,3)\ (3,3)\}$

$1 \leq 2 \in R$, $2 \leq 3 \in R$

Hence, R_5 is Anti symmetric

If $R_5 = \{(1,2)\ (2,3)\ (3,3)(2,1)\}$

$1 \leq 2$, $2 \leq 1$ which is not true

then,

R_5 is Anti-Symmetric.

Ex:-1 In the set of Natural No., the relation "a divide b" is Anti-Symmetric, ∵ "a divide b" and "b divide a" is possible only when, $a = b$ i.e

$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$.

ex:- $a = 2$, $b = 4$

$(2, 4) \in R$ $(4, 2) \notin R$ ∴ $2 \neq 4$

(4) Transitive Relation :-

If R is relation in the set A, then R is called "Transitive Relation if a is R-related to b and b is R-related to c, then a is also R-related to c.

i.e $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

aRb and $bRc \Rightarrow aRc$ $\forall a, b, c \in A$.

Ex:- If $A = \{1, 3, 5\}$ and $R = \{(1, 3)(1, 5)(3, 5)\}$

Ans The relation R is transitive,

∵ $(1, 3) \in R$ and $(3, 5) \in R$ then $(1, 5) \in R$

Hence, it is transitive.

Ex:- If $A = \{a, b, c, d\}$ $R = \{(a, b) (a, c) (b, d) (c, d) (a, d) (b, c)\}$

The Relation R is transitive;

∵ $(a, b) \in R$, $(b, c) \in R$, $(c, d) \in R$ then $(a, d) \in R$

Hence, it is transitive.

∵ $(a, c) \in R$, $(c, d) \in R$, $(a, d) \in R$

∵ $(b, c) \in R$, $(c, d) \in R$ then $(b, d) \in R$

(5) Identity Relation :-

If R is a relation in the A, then R is called Identity Relation if

a is R-related to b then $a = b$

i.e $(a, b) \in R \Rightarrow a = b$

$a R b \Rightarrow a = b \ \forall\ a, b \in A.$

If R is an Identity Relation in the Set A, then it is denoted by I_A.

Ex :- If $A = \{2, 4, 6\}$ then

(a) $I_A = \{(2,2)\ (4,4)\ (6,6)\}$ is an identity Relation in set A.

(b) $R_1 = \{(2,2)\ (4,4)\}$ is not an identity relation $\because (6,6) \notin R_1$.

(c) $R_2 = \{(2,2)\ (4,4)\ (6,6)\ (2,6)\}$ is not identity relation $\because (2,6) \in R_2$ but R_2 is Reflexive Relation.

NOTE : Every identity Relation is Reflexive but its converse is not true. If in a Set A identity relation is I_A and Reflexive Relation is R, then

$$I_A \subset R.$$

*

Equivalence Relation :-

If R is relation in the Set A, then R is called Equivalence Relation if;

(1) R is Reflexive; $(a, a) \in R \ \forall\ a \in A$.

(2) R is Symmetric; $(a, b) \in R \Rightarrow (b, a) \in R$ where $a, b \in A$.

(3) R is transitive; $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall\ a, b, c \in A$.

Questions Based on Relation :

Pg - 44

Ex:-2 If a relation $R = \{(x, y) : x, y \in N \text{ and } x + y = 8\}$, then find the Domain and Range of R.

Sol:- Here, $(x, y) \in N = \{1, 2, 3, 4, 5, \ldots\}$.

The values of x and y satisfying $x + y = 8$ are given :-

$x =$	1	2	3	4	5	6	7	$x + y = 8$
$y =$	7	6	5	4	3	2	1	$x + y = 8$

$\therefore R = \{(1, 7)\ (2, 6)\ (3, 5)\ (4, 4)\ (5, 3)\ (6, 2)\ (7, 1)\}$

Domain; $R = \{\text{value of } x\} = \{1, 2, 3, 4, 5, 6, 7\}$

Range; $R = \{\text{value of } y\} = \{7, 6, 5, 4, 3, 2, 1\}$ - Ans

*

Ex:-3 If Relation $R = \{(x, y) : y = |x - 2|\}$ has its domain $\{-1, 0, 1, 2, 3\}$, then find its Range.

Ex:-4 If Domain of Relation $R = \{(x, y) : y = x - 1\}$ is $\{x : 0 \leq x \leq 4,\ x \in I\}$ then find its Range.

Ex:-5 A set of first ten natural No. 1 to 10. A relation R is defined as follows:

$$x R y \iff x + 2y = 10, \text{ where } x, y \in A,$$

the evalute the following:

i) Domain of R.

ii) Range of R.

iii) R^{-1}.

Ex:8:- Let $R = \{(1,2)(2,3)\}$ be a relation in the set of Natural No. Insert Minimum No. of ordered pair in R so that this Extended Relation become Reflexive, Symmetric and transitive.

Sol:- The given relation $R = \{(1,2)\ (2,3)\}$ will be

Reflexive if $(a,a) \in R\ \forall a$.

Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$

Transitive if $(a,b) \in R,\ (b,c) \in R \Rightarrow (a,c) \in R$

Hence,

Extended $R = \{(1,2)(2,3)(1,3)(2,1)(3,2)(3,1)(1,1)(2,2)(3,3)\}$

Hence, ordered pair are insted

$\{(1,3)\ (2,1)\ (3,2)\ (3,1)\ (1,1)\ (2,2)\ (3,3)\}$

Ex.9:- If $A = \{1,2,3,4\}$, then examine the following Relation for transitivity:

$R_1 = \{(1,3)\ (3,4)\ (1,4)\ (4,3)\}$

$R_2 = \{(1,2)\ (2,2)\ (2,1)\ (3,1)\ (4,3)\}$

Ex.25:- If $A = \{1,2,3,4\}$ then analyse the following relation reflexive, Symmetric and transitive;

(i) $R_1 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (2,3)\}$

(ii) $R_2 = \{(1,2)\ (2,1)\}$

(iii) $R_3 = \{(1,2)\ (2,3)\ (1,3)\}$

(iv) $R_4 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (2,1)\}$

(v) $R_5 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (2,3)\ (1,3)\}$

(vi) $R_6 = \{(1,2)\ (2,1)\ (2,3)\ (3,2)\ (1,3)\ (3,1)\ (1,1)\ (2,2)\ (3,3)\}$

Ex:- 14

(a) Show that the Relation $R = \{(a,b) : a-b = \text{even integer and } a,b \in I\}$ in the set I of Integers is an Equivalence relation.

Sol:- we know that R will be an Equivalence relation if it is reflexive, Symmetric and transitive.

(i) R is reflexive :- Let $a \in I$ then $a - a = 0$, which is an Even I.

$\therefore (a,a) \in R, \ \forall a \in I.$

(ii) R is Symmetric :-

If $(a-b)$ is an Even Integer than $b-a$ is also an Even I.

$\therefore (a,b) \in R \Rightarrow (b,a) \in R$ is true.

Hence, R is Symmetric Relation.

(iii) Transitive R :- Let $a, b, c \in I$.

If $(a-b)$ and $(b-c)$ are even integers, then

$a - c = (a - b) + (b - c)$ = even integer

$\therefore (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ is true.

Hence R is transitive Relation.

$\therefore$ In the set I, the relation R is reflexive, Symmetric, and Transitive,

$\therefore$ R is an Equivalence Relation.

*

Partial Ordering Relation :-

A Relation R on a set A is called Partial Ordering relation, if

(i) R is reflexive : i.e ; $a R a$, $\forall a \in A$

(ii) R is Anti-Symmetric : i.e; $a R b$ and $b R a$ then $a = b$ $\forall a, b \in A$

(iii) R is transitive : i.e, $a R b$ and $b R c \Rightarrow a R c$ for each $a, b, c \in A$.

Ex:- Show the the Relation $R = \{(1,1)\ (2,2)\ (1,2)\ (3,3)\ (2,3)\ (1,3)\}$ of set $A = \{1, 2, 3\}$ is partial ordering Relation.

Sol:- R is reflexive i.e

$(1,1) \in R$, $(2,2) \in R$ $(3,3) \in R$ $\forall$ $1, 2, 3 \in A$

R is Antisymmetric i.e

$(1,1) \in R$, $(2,2) \in R$ $(3,3) \in R$

$(1,2) \in R$ and $(2,1) \notin R$

$(2,3) \in R$ and $(3,2) \notin R$

$(1,3) \in R$ and $(3,1) \notin R$

R is transitive ; i.e

$(1,2) \in R$, $(2,3) \in R$ and $(1,3) \in R$

Hence, R satisfy all 3 properties,

Hence, it is Partial Ordering Relation.

*

Composition Relation of 2 Relations :-

Let A, B, C be 3 sets.
Suppose R is a relation from A to B and S is a relation from B to C.
The composition relation of the 2 relation R and S is a relation from the set A to the set C. and is denoted by SOR. and is defined as,

$$S \circ R = \{(a,c) : \exists \text{ an element } b \in B \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

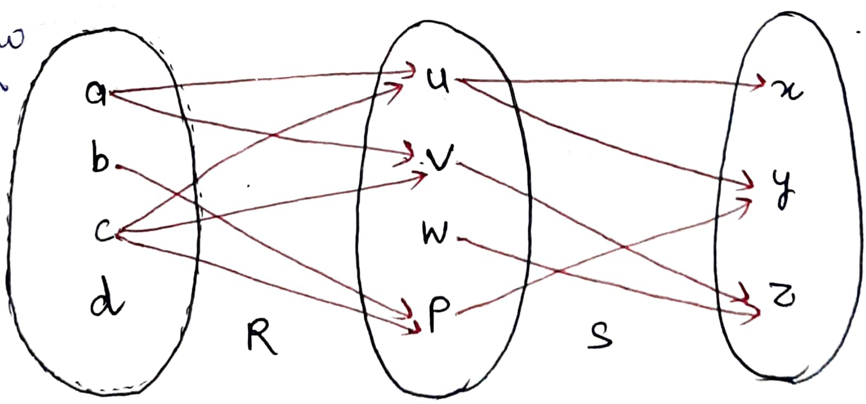
Hence,

$$(a,b) \in R,\ (b,c) \in S \Rightarrow (a,c) \in S \circ R.$$

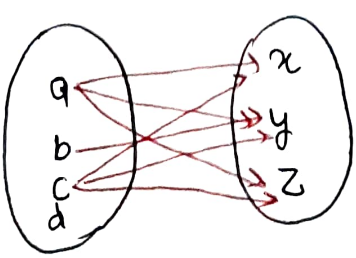
Ex:- Let $A = \{a, b, c, d\}$, $B = \{u, v, w, p\}$, $C = \{x, y, z\}$
and Let $R = \{(a,u), (a,v), (b,p), (c,u) (c,v) (c,p)\}$ and
$S = \{(u,x), (u,y), (v,z), (w,z), (p,y)\}$.
Find SoR and draw the arrow diagram.

Sol:- we have,
$S \circ R = \{(a,x), (a,y), (a,z) (b,y) (c,x) (c,z) (c,y)\}$

Arrow diagram for A, B, C



Arrow diagram for SoR.



Ex:- $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, $C = \{p, q, r\}$ and

Let $R = \{(1,a)\ (2,d)\ (3,b)\ (3,d)\}$ and $S = \{(b,p)\ (b,r)\ (c,q)\ (d,r)\}$

then draw arrow diagram and find SoR also.

*Some - More Question's on Relation :-

Ex:- 23 Give an example of relation which is reflexive but neither symmetric nor transitive.

Sol:- Let $A = \{a, b, c\}$ be a set. Consider a relation R in the set A defined by;

$$R = \{(a,a), (a,b)\ (b,c), (b,b)\ (c,c)\}$$

(i) ∵ every element of A is R-related to itself i.e,

$(a,a) \in R$, $(b,b) \in R$, $(c,c) \in R$

∴ R is reflexive Relation.

(ii) $(a,b) \in R$ but $(b,a) \notin R$

$(b,c) \in R$ but $(c,b) \notin R$

Hence, R is not Symmetric relation.

(iii) we have, $(a,b) \in R$, $(b,c) \in R$ but $(a,c) \notin R$

Hence, R is not Transitive Relation.

*

Pictorial Representation of Relation :-

Ex:- Let A and B are 2 finite set.
Let R be a relation from A to B, then we can picture R in following ways.

WAY I^{st} :- we form a rectangular array whose column is labeld by elements of A and whose row is labeled by elements of B. we put 1 or 0 in each position of the array according as $a \in A$ is or not related to $b \in B$. The array so formed is known as The matrix of the Relation.

WAY II^{nd} :- we write down the elements of the set A and B in 2 different disks. Now we draw an arrow from the element $a \in A$ to the element $b \in B$ whenever a is related to b. The picture so obtained is called The arrow diagram of Relation.

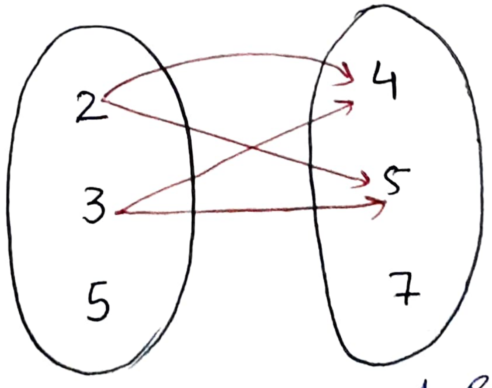
Ex:- Draw i) Matrix of Relation R and, ii) Arrow diagram of Relation R, where $A = \{2, 3, 5\}$, $B = \{4, 5, 7\}$

$$R = \{(2,4)\ (2,5)\ (3,4)\ (3,5)\}$$

Sol:-

	4	5	7
2	1	1	0
3	1	1	0
5	0	0	0

(Matrix of Relation)



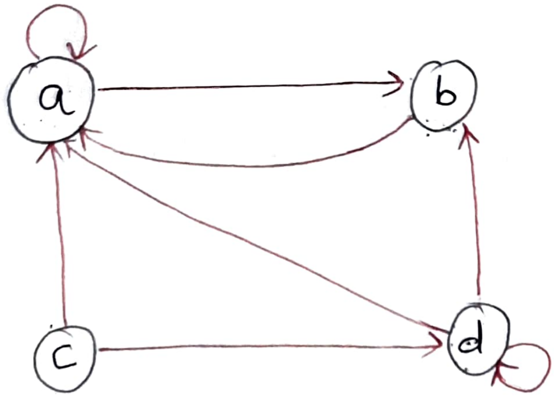
Arrow diagram of Relation.

Way (III^rd) :- Graph Representation of Relation :-

A direct graph consist of Nodes or vertices connected by Directed Edges or arcs. [Explained with Example]

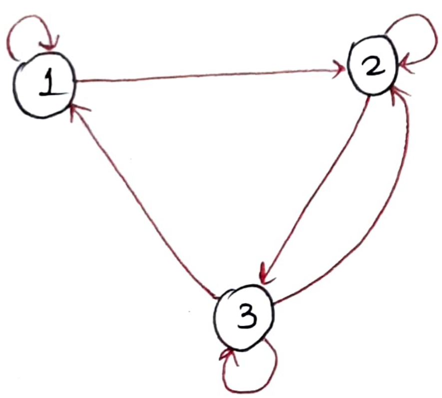
Ex:- Let $A = \{a, b, c, d\}$ and let R be a relation on the set A defined by
$R = \{(a,a)\ (a,b)\ (b,a)\ (c,a)\ (c,d)\ (d,a)\ (d,b)\ (d,d)\}$

Ans :-

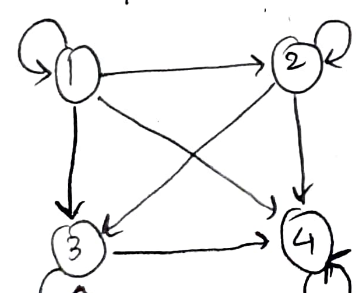


Ex:- Draw the direct graph that represent the relation
$R = \{(1,1)\ (2,2)\ (1,2)\ (2,3)\ (3,2)\ (3,1)\ (3,3)\}$ on $A = \{1,2,3\}$.

Graph :-



Ex:- Determine wheather the Relation for the directed graph shown in figure, are Reflexive, Symmetric, Anti-Symmetric and/or transitive ?



Reflexive : (1,1) (2,2) (3,3) (4,4) (✓)

Symmetric : (1,2) but (2,1) dosent exist (x)

Anti-Symmetric : (1,1) (2,2) (3,3) (4,4) (1,2) (2,4) (✓)

Transitive : $(1,2) \to (2,4) \to (4,1)$ exist (✓)

Function :-

A function is Special type of Relation.

Let A and B be two non-~~negative~~ Empty Sets. If every element of the first set A is related to an Unique Element of B by some define rule f, then f is called a function/Mapping from A to B and is denoted by

$$f : A \rightarrow B \quad \text{and} \quad A \xrightarrow{f} B$$

OR

A subset f of $A \times B$ is called a function or Mapping from A to B if to every element x of A there is associated an unique element y to B such that $(x, y) \in f$ i.e, $f \subseteq (A \times B)$

※ Condition for mapping from the set A to the set B :-

f will be a mapping from A to B if following condition are satisfied :

(i) Every element of the first set A should be definitely f-related to some element of the second set B,

(ii) two or more element of B should not be associated to any one element of A but two or more element of A may be associated to any one element of B.

(A) Image and Preimage :-

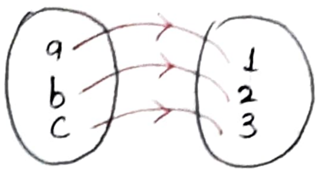
Let $f : A \rightarrow B$ be a mapping. Let one element x of A is associated to element y of B by the mapping f, then y is called the f-image of x and is denoted by $f(x)$.

Hence, f-image of $x = y = f(x)$.

x is called Pre-Image of y.

Types of Function :-

(1) One - One function [Injective]



$f : A \to B$ is called one - one, if different element in A have diff. image in B.

$f(x_1) = f(x_2)$
$\Rightarrow x_1 = x_2$ OR

$f(x_1) \neq f(x_2)$
$x_1 \neq x_2$

$\forall \; x_1, x_2 \in A$.

STEPS :- 1) Let $x, y \in A$
2) Put $f(x) = f(y)$ and solve it.
3) If $x = y \Rightarrow f$ is 1-1 otherwise NOT.

Ex :- One person has one passport, passport is associated with single person. collage ID also.

Ex :- Show that the functⁿ $f : R \to R$ given by $f(x) = 2x$, is 1-1.

Sol :- Let $a, b \in R$ such that $f(a) = f(b)$ then

$f(x) = 2x$
$2a = 2b$
$a = b$

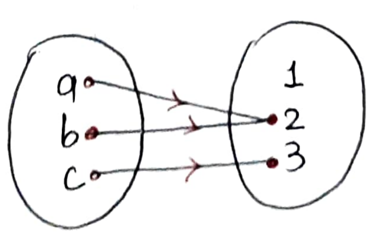
Hence, $f : R \to R$ is 1-1

(2) Many - One function

$f : A \to B$ is called many - one, if 2 or more than 2 image different element in A have the same f-image in B i.e,

$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$
$\forall \; x_1, x_2 \in A$

the fuction which is not 1-1 is Many one.



Ex :- In family children have same parents.
Sibling → Same parents.

Ex :- Show that the functⁿ $f : Z \to Z$ given by $f(x) = (x^2 + x)$ is many -1

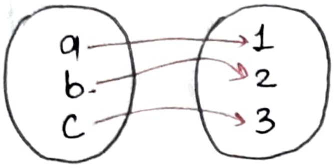
Sol :- Let $x, y \in Z$

$f(x) = f(y)$
$x^2 + x = y^2 + y$
$(x^2 - y^2) + (x - y) = 0$
$(x - y)[(x + y) + 1] = 0$
$x - y = 0 \quad , \quad x + y + 1 = 0$
$x = y \quad , \quad x = -1 - y$
$x = y, (-1, -y)$

x has 2 pre-image.

Hence, f is many one not One - One

(3) Onto Mapping
[Surjective]



$f: A \to B$ is called Onto if, Every Element of B is the image of some element of A.

for each $b \in B$, $\exists\, a \in A$ such that

$$f(a) = b$$

Range = Co-domain = Set B.

STEPS:-

1) Let $y \in B$ such that, $f(x) = y$
2) Solve: $f(x) = y$ and find x.
3) If $x = g(y) \in A$ then f is Onto otherwise not.

Ex:- Student $\to$ Roll No.

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Ex:- If $f: R \to R$, $f(x) = x^3 + 1$

Then show it is onto or not?

Sol:- Let $y \in R$

$$f(x) = y$$
$$x^3 + 1 = y$$
$$x^3 = y - 1$$
$$x = (y-1)^{1/3}$$

x is real No. i.e $x \in R$

f is onto

Ex:-2 $f: R \to R$

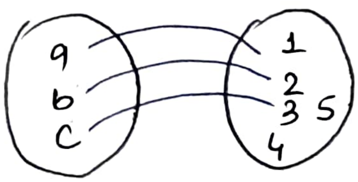
$y \in R$: $y = 2x - 3$

$= 2x = y + 3$

$$x = \frac{y+3}{2} \in R$$

Range $f = R$

(4) Into function



$f: A \to B$.

If there is an element in B having no pre-Image. then, f is Into.

If f is not onto then f is Into.

Range $\neq$ Codomain

Ex:- $f: R \to R$, $f(x) = x^2 + 1$

then, show f into funct^n.

Sol:- $f: R \to R$

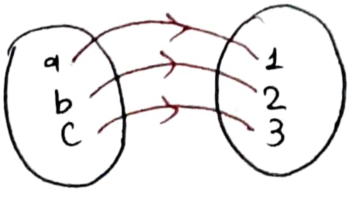
$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

Range $= \{1, 0, -1\}$

Range $\neq$ Codomain

{Co-domain is all Real No but we are getting only 3 elements}

(5) One-One Onto mapping :-
[Bijective]



A mapping $f: A \to B$ is called one-one into Map. if f is One-One and onto both in B

$$f: A \xrightarrow[\text{1-1}]{\text{Onto}} B$$

Questions Based on Mapping | Function :-

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Ex 4 :- Show that the mapping $f : N \to N$ defined by $f(x) = x^2$ is one-one into, where N is a set of Natural No.

Sol :- Let $x_1, x_2 \in N$,

then $f(x_1) = f(x_2)$

$$x_1^2 = x_2^2$$

$$x_1 = x_2$$

$\therefore$ f is 1-1

Again, using the funct^n $f(x) = x^2$, the f-image of x is its square i.e x^2.

Ex :- f-image of 2 is 4, 3 is 9, 4 is 16, etc.

$\therefore$ The natural No. 2, 3, 5, 6, 7, 8.... etc are not f-image of any natural No.

$\therefore$ f is into Mapping.

Hence, the given mapping is 1-1 into.

Ex:-5 Show that the mapping $f : R \to R$, $f(x) = \frac{1}{x}$, $x \neq 0$ and $x \in R$ is 1-1 onto, where R is the set of non-zero real No.

Sol :- Let $x_1, x_2 \in R$ be any 2 non-zero real No, then

$$f(x_1) = f(x_2)$$

$$\frac{1}{x_1} = \frac{1}{x_2}$$

$$x_1 = x_2$$

$\therefore$ f is 1-1. mapping.

Let $y \neq 0$ be any real No. Such that $f(x) = y$.

$$y = \frac{1}{x} \implies x = \frac{1}{y}$$

$\because$ $y \neq 0$,

$\therefore$ $\frac{1}{y}$ is also a non-zero real No.

Hence, for every $y \neq 0$ $\exists$ its pre-Image $\frac{1}{y} \neq 0$ in R shown below,

$$f(x) = \frac{1}{x}$$

$$f\left(\frac{1}{y}\right) = \frac{1}{\frac{1}{y}} = y$$

It shows that every $0 \neq y \in R$ has its pre-image in domain R.

$\therefore$ f is onto.

Hence, the given Mapping is 1-1 onto.

*

Easy Example:- Show that the function $f: R \to R$ defined by $f(x) = 3x^3 + 5$ $\forall$ $x \in R$ is a Bijection.

Sol:- Let $x, y \in R$, Then

$$f(x) = f(y)$$

$$3x^3 + 5 = 3y^3 + 5$$

$$x = y$$

$\therefore$ f is 1-1

Now, Let $y \in R$

Then, $f(x) = y \Rightarrow 3x^3 + 5 = y \Rightarrow x = \left(\frac{y-5}{3}\right)^{1/3} \in R$ Such that

$$f(x) = f\left(\frac{y-5}{3}\right)^{1/3}$$

$$f(x) = \cancel{3}\left(\frac{y-5}{\cancel{3}}\right)^{1/\cancel{3} \times \cancel{3}} + 5$$

$$f(x) = y$$

f is onto, then

f is Bijection means 1-1 and onto Both.

*