UNIT 3

NUCLEAR PHYSICS

INTRODUCTION

Nuclear Constituents

From the scattering of α-particles, Rutherford concluded that the atom of any element consists of central core called ***nucleus*** and ***electrons*** moving around it. The entire mass of the atom and positive charge is concentrated inside the nucleus. The mass of the electron (9.108×10^{-31} kg) is insignificantly small. The nucleus is supposed to consist of two particles, the ***proton*** and the ***neutron.*** Their masses are nearly the same as shown below :

$$\text{Mass of the proton} = 1.67261 \times 10^{-27} \text{ kg}$$

$$\text{Mass of the neutron} = 1.67492 \times 10^{-27} \text{ kg}$$

The proton is a positively charged particle while the neutron is a neutral particle, *i.e.*, it carries no charge. The charge on the proton is 1.6×10^{-19} coulomb. Both the proton and neutron together in the nucleus are called ***nucleons.*** The number of protons in the nucleus is called the ***atomic number*** and the sum of protons and neutrons is called as ***mass number.*** The stability of a nucleus depends on the relative number of protons and neutrons.

The conventional symbols for nuclear species follow the following pattern

$$_Z X^A$$

M = P+N
P = Atomic no.

where X = Chemical symbol of the element

Z = Atomic number (*i.e.*, number of protons in the nucleus)

and A = Mass number (*i.e.*, number of protons and neutron in the nucleus)

Atomic no. = Mass no. − Neutron

Therefore, the number of neutrons in the nucleus

$$N = A - Z$$

For example, $_3\text{Li}^7$ has $Z = 3$ and $A = 7$

Therefore, the number of neutrons $N = 7 - 3 = 4$

General Properties of Nucleus

Some of the important properties of atomic nucleus are given below :

(a) Nuclear mass : The mass of the nucleus is the sum of the masses of the neutrons and protons contained in it. This is usually expressed in terms of atomic mass unit (a.m.u.). One a.m.u. $= 1.66 \times 10^{-27}$ kg. For example $_6\text{C}^{12}$ nucleus has a mass of 12 a.m.u. so that its mass number $A = 12$.

It is assumed that the mass of the nucleus should be

$$Z\,m_p + N\,m_n$$

where m_p and m_n are the masses of proton and neutron respectively.

Here N is the number of neutrons.

Experimentally, it has been observed that

real nuclear mass $< Z\,m_p + N\,m_n$

The difference is assumed and real mass is called as ***mass defect.***

(b) Nuclear charge : The charge on the nucleus is due to protons contained in it. The charge on each proton is $+1.6 \times 10^{-19}$ coulomb which is equal in magnitude to the charge of an electron. Taking the charge of a proton as one unit, the total charge on the nucleus is numerically equal to the number of protons. For example, a hydrogen nucleus (*i.e.*, proton) carries a single unit charge.

(c) Nuclear radius : Nuclear diameters can be measured in a number of ways. Rutherford, as a result of his experiments on α-particle scattering by thin metallic foils, concluded that the distance of the closest approach of the α-particle to the nucleus of the scatterer can be regarded as a measure of the size of the nucleus. He found this distance of the order of 10^{-14} m.

It has been observed that volume of the nucleus is directly proportional to the number of nucleons A, *i.e.*,

$$\text{Volume} \propto A$$

or $$\frac{4}{3}\pi r^3 \propto A$$

or $$r^3 \propto \left(\frac{3}{4\pi}\right) A$$

or $$r \propto \left(\frac{3}{4\pi}\right)^{1/3} A^{1/3} = C\left(\frac{3}{4\pi}\right)^{1/3} A^{1/3}$$

or $$\boxed{r = r_0\,A^{1/3}}$$

where $r_0 = C\left(\dfrac{3}{4\pi}\right)^{1/3}$, C = proportionality constant

Here r_0 is a linear constant and has an average value of 14×10^{-15} m. A is mass number of the nucleus. Radius of the nuclei of few elements are given below :

Carbon ($A = 12$)

$$r = 1.4 \times 10^{-15} \times (12)^{1/3} = 3.21 \times 10^{-15}\ \text{m}$$

Copper ($A = 62$)

$$r = 1.4 \times 10^{-15} \times (62)^{1/3} = 5.97 \times 10^{-15}\ \text{m}$$

Uranium ($A = 238$)

$$r = 1.4 \times 10^{-15} \times (238)^{1/3} = 8.68 \times 10^{-15}\ \text{m.}$$

(d) **Nuclear density :** The density of the nucleus can be calculated as follows :

Volume of the nucleus $= \frac{4}{3}\pi r^3$,

where r is the radius of the nucleus.

Taking $r = 1.5 \times 10^{-15} A^{1/3}$ metre, we have

Volume of the nucleus

$$= \frac{4}{3}\pi (1.5 \times 10^{-15})^3 A \text{ metre}^3 = 14.15 \times 10^{-45} A \text{ metre}^3$$

Mass of the nucleus

$$= A \times \text{mass of proton (approximately)} = 1.673 \times 10^{-27} A \text{ kg}$$

$\therefore$ Density of nucleus

$$= \frac{1.673 \times 10^{-27} A}{14.15 \times 10^{-45} A} \text{ kg/metre}^3$$

$$= 1.18 \times 10^{17} \text{ kg/metre}^3$$

(e) **Nuclear quantum states :** From the study of artificial radioactivity and α- and γ-ray spectra it would become clear that every nucleus possesses a set of quantum states in a corresponding number of discrete energy levels. Transititon between different nuclear states are accompanied by the emission of γ-rays.

(f) **Nuclear spin (I) :** Both the proton and neutron, like the electron, have an intrinsic angular momentum, commonly referred to as its spin. The magnitude of spin angular momentum is $\frac{1}{2}\hbar$ just like electron. In addition, the nucleons (protons and neutrons) possess orbital angular momentum due to motion about the centre of the nucleus. The resultant angular momentum of the nucleus is obtained by adding spin and orbital angular momenta of all nucleons within the nucleus. In general, the total angular momentum of the nucleus I is referred to as the nuclear spin. Corresponding to total angular momentum quantum number I, the absolute magnitude of total angular momentum is $\hbar[I(I+1)]^{1/2}$. The value of I depends on the type of interaction between the nucleons.

(g) **Magnetic dipole moment of nuclei (μ) :** We know that a charged particle moving in a closed path produces a magnetic field. The magnetic field, at a large distance, may be regarded as due to a magnetic dipole located at the current loop. The spinning electron has an associated magnetic dipole moment of 1 bohr magneton

i.e.,
$$\mu_e = \frac{e\hbar}{2m_e}.$$

$\mu_e = \frac{e\hbar}{2m_e}$

Here e is the charge and m_e is the mass of the electron.

Proton has a positive elementary charge (e). Due to its spin, it has magnetic dipole moment μ_N. According to Dirac's theory

$$\mu_N = \frac{e(h/2\pi)}{2m_p} = \frac{e\hbar}{2m_p}$$

$= \frac{e(h/2\pi)}{2m_p}$

where m_p is the proton mass. Here μ_N is called as *nuclear magneton*. Since $m_p = 1836\, m_e$, so nuclear magneton is only (1/1836) of Bohr magneton.

Neutron is a neutral particle. So, it is hard to understand that how it can have a magentic moment. It is found that neutron has a magentic moment $\mu_n = -1.9128\, \mu_N$. To explain μ_n, it was assumed that neutron contains equal amount of positive and negative charges. If these charges are not uniformly distributed then a spin magnetic moment may arise. The anomalous values of magnetic moments of proton and neutron can be understood on the basis of meson theory.

(h) Electric quadrupole moment (Q) : In addition to its magnetic moment, a nucleus may have an electric quadrupole moment. This is a consequence of the symmetry of nuclei about the centre of mass. In general, the shape of the nucleus is not spherical but it is an ellipsoid of revolution. Indeed, most nuclei do assume approximately such a shape. ***The deviation from the spherical symmetry is expressed in terms of a quantity known as electric quadrupole moment.***

The electric quadrupole moment is given by

$$Q = \frac{2}{5} Z\, e\,[b^2 - a^2]$$

where Z is atomic number and $Z\,e$ is the total charge on the nucleus.

It is obvious from this expression that

(i) $Q = 0$, for spherical shaped nucleus,

(ii) $Q = -$ ve, for ellipsoid ($a > b$), and

(iii) $Q = +$ ve, for ellipsoid ($b > a$).

M, C, G
F

3.1 NUCLEAR LIQUID DROP MODEL

To account for the prominent aspects of nuclear properties and behaviour various nuclear models have been proposed. First of all, we shall discuss the liquid drop model.

This model was proposed by Bohr. According to this model, the nucleus is similar to a small electrically charged liquid drop, *i.e.*, the nucleus takes a spherical shape for its stability. The nucleons (protons and neutrons) move within this spherical enclosure like molecules in a liquid drop.

The motion of nucleons within nucleus is a measure of nuclear temperature as the molecular motion of molecules in liquid is the measure of its temperature.

The nucleons always remain a constant distant apart and share among them the total energy of the nucleus. The nucleons deep inside the nucleus are attracted from all sides by neighbouring nucleons while those on the surface are attracted from one side only.

The spherical surface which encloses the nucleus may be regarded as analogous to surface tension which holds a water drop to the evaporation of particles from a liquid surface.

The evaporation in a liquid drop takes place when Maxwellian energy distribution within the drop causes a particular molecule to have sufficient energy to overcome the intermolecular attraction and escape. Similar is the case with compound nucleus. The nucleus remains in the excited state until the nucleon or a group of nucleons happen to possess sufficiently large excitation energy escape through the potential barrier.

coulomb force nuclear

Assumptions

(i) The material of the nucleus is incompressible and the density of all the nuclei is the same.

(ii) The forces in the nucleus consist of (a) Coulomb forces between protons and (b) powerful attractive nuclear forces.

Analogies between liquid drop and a nucleus

(i) Both are spherical in nature.

(ii) In both the cases, the density is independent of its volume with the exceptions that the density of the nucleus is independent of the nucleus while density of the liquid depends upon its type.

(iii) The molecules in liquid drop interact over short ranges and so is true for nucleons in nucleus.

(iv) As the surface tension forces act on the surface of a drop similarly a potential barrier acts on the surface of nucleus.

(v) When the temperature of the molecules in a liquid drop is increased, evaporation of molecules takes place. Similarly, when the nucleons in the nucleus are subjected to external energy, a compound nucleus is formed which emits nucleons almost immediately. The process is known as nuclear fission.

(vi) When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops. The process of nuclear fission is similar in which the nucleus breaks up into two smaller nuclei.

Merits : Following are the merits of liquid drop model :

(1) It has been successfully applied in describing nuclear reactions and explaining nuclear fission.

(2) The calculation of atomic masses and binding energies can be done with good accuracy.

However this model fails to explain other properties, in particular, the magic numbers.

3.2 SEMIEMPIRICAL MASS FORMULA

In 1935, Von Weizsacker expressed the atomic mass of a nuclide in terms of the series of binding energy correction terms with the main mass contribution from proton and neutron. The modified expression of the mass is known as semiempirical mass formula. Later on this formula was modified by Bethe and others but the main outlines remain the same. The mass of neutral atom can be expressed as

$$_Z M^A = Z M_p + N M_n - B \quad \ldots(1)$$

Mass of neutron atom = No. of protons (Mass of proton) + No. of neutrons (Mass of neutron) − Binding energy in mass units

where $_Z M^A$ = Atomic mass of the nuclide

Z = Number of protons

M_p = Mass of the proton

N = Number of neutrons

M_n = Mass of the neutrons

B = Binding energy expressed in mass units.

The value of B is calculated empirically as made up of a number of correction terms given by

$$B = E_v + E_s + E_c + E_r + E_p \quad ...(2)$$

where E_v, E_s, E_c, E_r and E_p are volume energy correction, surface energy, Coulomb energy, asymmetry energy and pairing energy respectively.

Now we shall proceed to find out empirically the values of these energies :

Volume energy correction E_v : It has been experimentally verified that the binding energy per nucleon is approximately constant over a wide range of mass number. To a first approximation we may assume that the total binding energy of nucleus is proportional to the total number of nucleons A in it, *i.e.*,

$$E_v \propto A \quad \text{or} \quad E_v = a_v A \quad ...(3)$$

where a_v is an undetermined constant. Here subscript v is used to show that the contribution to the binding energy comes from the entire volume of the nucleus.

Surface energy E_s : Nucleus of an atom has some nucleons on its surface. As a molecule at the surface of a liquid is attracted less strongly in one direction in comparison to a molecule in the interior (a molecule in the interior of a liquid is attracted equally in all directions due to the other surrounding molecules). Similarly, the nucleons at the surface of nucleus are bound less tightly. This gives rise to a *slight reduction* in the total binding energy of the nucleus. So it has a negative contribution. The number of nucleons on the surface are proportional to surface area of the nucleus and therefore to r^2. We know that $r = r_0 A^{1/3}$ where r_0 is a constant. So, the number of nucleons on the surface are proportional to $A^{2/3}$. Hence,

$$\boxed{E_s = -a_s A^{2/3}} \quad r_0 = 1.5 \times 10^{-15} \quad ...(4)$$

where a_s is a constant.

Surface energy is just analogous to the surface tension of a liquid.

Coulomb energy E_c : Coulomb's repulsive forces are produced due to the mutual repulsion of protons which are positively charged particles of the nucleus. Due to these repulsive forces, the binding energy is decreased. Assuming that the nuclear charge Ze is uniformly distributed throughout the nuclear volume, the electrostatic potential energy is given by $3/5 \cdot (Z^2e^2/r)$, [because we know that electrostatic potential energy of a uniform spherical distribution of charge is given by $3/5 \cdot (q^2/r)$, where q is the total charge and r is the radius of the sphere]. The Coulomb repulsion is proportional to electrostatic potential energy, hence Coulomb repulsion of binding energy E_c is written as

$$E_c = -a_c' \cdot \frac{3}{5}\frac{Z^2 e^2}{r}$$

But

$$\boxed{r = r_0 A^{1/3}}$$

$$E_c = -a_c' \cdot \frac{3}{5}\frac{Z^2 e^2}{A^{1/3}} = -a_c \frac{Z^2}{A^{1/3}}.$$

More accurate calculations show that

$$E_c = -a_c \frac{Z(Z-1)}{A^{1/3}}. \quad ...(5)$$

Asymmetry energy E_r : It has been observed that nuclei are most stable when nucleus contains equal number of protons and neutrons *i.e.* ...

$N = Z$. This is called *symmetry effect*. As the value of A increases, the number of neutrons increases and alternately the binding energy decreases. This decrement is known as *asymmetric energy correction*. If the atomic number of a nucleus is Z and its mass number is A, then there will be Z protons and $(A - Z)$ neutrons in it. The excess of neutrons over protons is $(A - Z) - Z = (A - 2Z)$. The decrease in binding energy due to neutron excess may be taken as inversely proportional to A. It has been shown that the contribution of asymmetric effect to the binding energy of the nucleus is given by

$$E_r = -a_r \frac{(A-2Z)^2}{A} \qquad \ldots(6)$$

where a_r is a constant.

Pairing energy : It has been observed that nuclei containing even number of protons and even number of neutrons are most stable. On the other hand, nuclei containing odd number of protons and odd number of neutrons are least stable. Moreover, nuclei containing even number of protons and odd number of neutrons or *vice versa* have intermediate stability. This pairing effect changes the binding energy as shown below :

Number of protons Z	Number of neutrons N	E_δ
even	even	$+\delta A^{-3/4}$
odd	even	0
even	odd	0
odd	odd	$-\delta A^{-3/4}$

where δ is a constant.

Combining all these above correction terms, the semiempirical mass formula is given by

$$_Z M_A = Z M_p + (A-Z) M_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_r \frac{(A-2Z)^2}{A} + E_p \qquad \ldots(7)$$

The formula is not exact expression but is fairly valid for mass number $A > 15$.

3.3 THE SHELL MODEL

According to shell model, the nucleus consists of a series of protons and neutrons placed in certain discrete levels or shells just like the electrons in the discrete shells of an atom. According to Pauli's exclusion principle two protons with opposite spins and two neutrons having opposite spins are accommodated in a particular shell. In this way the first shell accommodates two protons and two neutrons and is more tightly bound than other shells.

The concept of the shell model is based on the following fact :

It has been observed that nuclei containing protons and neutrons number 2, 8, 20, 50, 82, 126, etc. known as magic numbers or shell numbers are exceptionally stable as in case of electronic shell rule of 2, 8, 18, etc. This shows that there are definite energy shells in the nuclei.

The observation is based on the following experimental evidences :

(i) The nuclei for which Z (number of protons) and $A-Z$ (number of neutrons) are $2[{}_2He^4, Z=2$ and $A-Z=2]$ and $8[{}_8O^{16}, Z=8$ and $A-Z=8]$ are more stable than their neighbours.

(ii) The nuclei for which Z or $A-Z$ is magic number are specially stable. For example,

$${}_{28}Ni^{62} \qquad [Z=28, A-Z=34]$$
$${}_{38}Sr^{88} \qquad [Z=38, A-Z=50]$$
$${}_{50}Sn^{120} \qquad [Z=50, A-Z=70]$$
$${}_{58}Ce^{140} \qquad [Z=58, A-Z=82]$$
$${}_{82}Pb^{208} \qquad [Z=82, A-Z=126]$$

The binding energy curve shows breaks or kinks at these nuclei which correspond to sudden increase in their binding energies per nucleon.

(iii) The electric quadrupole moments of magic number nuclei are very low (nearly zero) compared with those other nuclei. This shows that these nuclei have almost spherical charge distribution. This is expected for more stable nuclei.

From our investigations of the nucleus, we can say that nuclear forces are strongly attractive and they extend over a very short range from the centre of the nucleus. Whatever may be the form of potential, the shell model is based on the following two assumptions :

(i) Each nucleon moves freely in a force field described by the potential, which is a function of radial distance from the centre of the system.

(ii) The energy levels or shells are filled according to Pauli exclusion principle.

Consider a nucleon of mass M with angular momentum $\sqrt{[l(l+1)]}\,\hbar$ is moving in a potential $V(r)$. Assuming that $V(r)$ is independent of θ and ϕ, the Schroedinger wave equation can be written as

$$\frac{d^2}{dr^2}(rR)+\frac{2M}{\hbar^2}\left[E-V(r)-\frac{l(l+1)\hbar^2}{2Mr^2}\right](rR)=0 \qquad ...(1)$$

where R is the radial wave function and E is the energy eigen value. Here the same quantum numbers, n, l, j, m_l result in shell model is an atomic model.

The square well potential is represented by

$$V(r)=-V_0 \quad \text{for } r\le R$$
$$=0 \quad \text{for } r>R \qquad ...(2)$$

The harmonic-oscillator potential is given by

$$V(r)=-V_0+\frac{1}{2}kr^2 \qquad ...(3)$$

If we combine the square-well potential and harmonic oscillator potential, the new form of the potential will be

$$V(r)=-V_0\left(1-\frac{r^2}{R}\right) \quad \text{for } r\le R$$
$$=0 \quad r>R \qquad ...(4)$$

Using the above potential in eq. (1) and solving it, we get all the nuclear magic numbers (shell structure) except magic number 28. In this way the shell model of the nucleus in an attempt to account for the existence of magic numbers and certain other nuclear properties in terms of nucleon behaviour in a common force field.

Few explanations based on shell model

(i) **Explanation of comparative stability of even-even, odd-odd nuclei :** It has been observed that even-even nuclei are, in general, more stable than odd-odd nuclei. This is obvious from shell model. The explanation is as follows :

According to Pauli's principle, a single energy sub-level can have a maximum of two nucleons. One has spin up and the other has spin down. In this way, an even-even nucleus only completed sub-level. This shows a greater stability. On the other hand, an odd-odd nucleus contains incomplete sub-levels for both kinds of nucleons. This means lesser stability.

(ii) **Prediction of angular momenta of nuclei :** The shell model is able to predict the total angular momenta of nuclei. Consider the case of even-even nuclei. All protons and neutrons pair off as to cancel out one another's spin and orbital angular momenta. So, the total angular momentum is zero for such nuclei. This has been observed with no exception. Now consider the case of odd-odd nuclei. They have an extra proton and an extra neutron. Each has an half-integral spin. Therefore, they give rise an integral total angular momentum. Similar is the case with even-odd and odd-even nuclei. This has been experimentally confirmed.

Merits : This model has been successful to account for the magic numbers. The model has also been able to explain the observed angular momenta, magnetic moments and electric quadrupole moments of nuclei.

Demerits : The shell model fails to explain and account for large nuclear quadrupole moments and spheroidal shapes of many nuclei.

Both the liquid-drop and shell models account, in their different ways, a number of observed facts and properties of nuclei. But neither is entirely satisfactory. Recently, attempts have been made to combine the best features of each of these models and some success has been achieved. The resulting model is known as *"Collective model"*.

INTRODUCTION TO PARTICLE ACCELERATORS

A particle accelerator is a device built to accelerate (i.e., to increase the kinetic energy) the charged particles so as to give them desired energy. The maximum energy that can be acquired by a particle depends upon the type of machine. In present days by accelerators techniques, energies of hundreds of billion electron volts (B.e.V.) have been reached. In principle, there is no upper limit to such high energies of particles but it is the cost of construction and maintenance of the machine that limits the highest energy.

The accelerated particles can be used as a powerful projectile to bombard various targets. Now-a-days, accelerators have become basic tools in rapidly growing branch of nuclear science known as high energy particle physics.

Nuclear reactions carried out by high energy particles led to the discovery of new particles like antiprotons, antineutrons, hyperons, K-mesons, etc. High energy particles also provide valuable information about the complex problem of nuclear structure and nuclear forces.

Operation

The operation of an accelerator consists of the following steps :

The particles are first produced in the source, and then injected by a suitable accelerating voltage into the machine. Then the particles are collimated into a beam and are constrained to follow a certain trajectory in a vacuum under the action of an electric field. Now, they acquire the desired energy. After this, the accelerated particles are made to hit the selected target placed inside the machine. The resultant interactions are now studied.

Types of Accelerators

Accelerators may be divided into the following three classes :

(a) Linear, (b) static and (c) circular

(a) **Linear accelerators or Lineac :** Linear accelerators can accelerate electrons and protons upto maximum energies of 40 GeV and 50 MeV respectively.

(b) **Static accelerators of Statitron :** This type of accelerator demands the production of high voltage so that an atomic particle can be given energy upto 12 electron volts. The example of this type of accelerator is Van de Graaff generator.

(c) **Circular accelerators :** In circular accelerators, the particles are accelerated while moving a spiral or circular path and the energy is given to the particles in series of successive additive state. They are further classified as under :

(i) Lawrence cyclotron or fixed frequency cyclotron : It can be used for accelerating particles like protons, deutrons and α-particles upto energies 40 MeV.

(ii) Synchrocyclotron or frequency modulated cyclotron : This is a modified form of Lawrence cyclotron which gives comparatively much higher energies to the particles.

(iii) Betatron : It is used for accelerating electrons upto energies 300 MeV.

(iv) Electron cyclotron : This is modified form of betatron and gives electron energies upto 6 GeV.

(v) Proton cyclotron : This is proton accelerator which gives proton energies upto 10 GeV.

(vi) Alternating Gradient Sychrotron (AGS) : It is a proton accelerator which gives proton energies upto 70 GeV.

(vii) Omnitron : It is an accelerator which gives strong beams of accelerated particles of all masses from hydrogen to uranium.

3.4 LINEAR PARTICLE ACCELERATORS

As the name indicates, in a linear accelerator, the charged particles move in a straight line. The linear accelerator is shown in fig. (1). It consists of a number of hollow cylindrical tubes $T_1, T_2, T_3, \ldots$, etc., arranged co-axially called as ***drift tubes***. These tubes are of increasing lengths and are arranged in a glass vacuum chamber. The alternate tubes are connected together to form two common junctions. These junctions are connected to high frequency power supply, *i.e.*, radio frequency (R.F.) oscillator. Thus, when tube T_1, T_3 and T_5 are positive, the oth

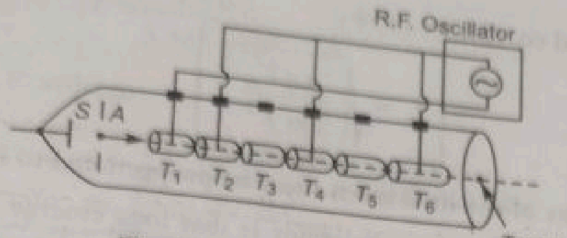


Fig. (1) Linear accelerators.

According to the applied frequency, the reversal of the potential takes place periodically. The positive ions are produced by the source S which travel along the axis of the tube. Since, a potential difference is maintained between two neighbouring cylinders, the ions are accelerated in the gap between two cylinders. Here, it should be remembered that the ions travel with constant velocity in field free space within cylinders themselves, of course, with increasing velocity as successive cylinders.

Let, a positive ion leaves A when tube T_1 is negative with respect to A. The ion will be accelerated and on entering in tube T_1 it travels with uniform velocity which it has acquired during acceleration between the gap A and tube T_1. The length of tube T_1 is selected in such a way with relation to the frequency of oscillator than when the ion leaves tube T_1, tube T_2 becomes negative. (This is only possible ***when the time taken by the ion travel tube T_1 is exactly equal to half the time period of R.F. oscillator***). The ion will again face an accelerating field, *i.e.*, it is further accelerated between the gap of tube T_1 and tube T_2. Now the ion travels through T_2 at constant but increased velocity. Thus, if the lengths of the tubes and frequency of R.F. oscillator are adjusted such that every time the ion faces accelerated field, it will acquire an additional energy every time as it passes through the gap between two tubes. Finally, the ion is emitted from the final drift tube towards the target with extremely high velocity.

If n be the number of tubes, V_m be the maximum voltage of the oscillator, then the energy acquired by ion of charge q is given by

$$\boxed{n\,q\,V_m = \frac{1}{2} m v_n^{\;2}} \qquad \ldots(1)$$

where m is the mass of the ion and v_m its velocity while entering the n^{th} tube.

The time interval for an ion to pass through any tube

$$= t = \text{half period} = \frac{T}{2} = \frac{1}{2f} \qquad \ldots(2)$$

where f is the frequency of the oscillator.

Let, l_n be the length of n^{th} tube and v_n the velocity of the ion when travelling through it, then

$$t = \frac{l_n}{v_n} = \frac{1}{2f}$$

$$\therefore \quad l_n = \frac{v_n}{2f} \qquad \ldots(3)$$

From eq. (1) and eq. (3), we have

$$\boxed{l_n = \sqrt{\left(\frac{n\,q\,V_m}{2\,m\,f^2}\right)}} \qquad \ldots(4)$$

Thus, the lengths of the drift tubes must be proportional to $1 : \sqrt{2} : \sqrt{3} : \sqrt{4}$, etc.

The chief advantage of the accelerator is that ions emerge as a well collimated beam. It does not require intense magnetic fields like circular accelerators. We know that charged particles (particularly electrons) moving with high speeds in circular paths (as in circular accelerators) lose energy by radiation. Such a difficulty does not arise in linear accelerators.

The linear accelerator for energizing protons is similar but a linear accelerator for electrons presents a considerable problem because the mass of the electron is very small. At 1 MeV, the electron has a velocity 0.94 c where c is the velocity of light. The transit times through the tubes are so short that a frequency of about 100000 mc/s would be needed. As the velocity of electron approaches that of light, its mass increases. For this region linear accelerators, for electrons are of cavity resonator or wave guide resonator type.

NUMERICAL EXAMPLES

Example 1. *In a linear accelerator, proton accelerated thrice by a potential of* 40 kV *leaves a tube and enters an accelerating space of length* 30 cm *before entering the next tube. Calculate the frequency of the R.F. voltage and the length of the tube entered by the proton.*

Solution. Let, v_1 be the entering velocity and v_2 be the leaving velocity. Then

$$\frac{1}{2}\, v_1^{\,2} = 3 \times e \times 40000$$

or

$$v_1 = \sqrt{2 \times 3 \times 40000 \times (e/m)}$$

$$= \sqrt{2 \times 3 \times 40000 \times (9.578 \times 10^7)}$$

$$= \mathbf{4.794 \times 10^6\ m/sec}$$

4.794
×1.732
9588

Similarly, $$v_2 = \mathbf{5.536 \times 10^6\ m/sec}$$

$$\text{Mean velocity} = \frac{(4.794 \times 10^6) + (5.536 \times 10^6)}{2}$$

$$= \mathbf{5.165 \times 10^6\ m/sec}$$

Time taken to travel a distance 0.3 m

$$= \frac{T}{2} \text{ of R.F. voltage} \quad \text{or} \quad \frac{T}{2} = \frac{0.3}{5.165 \times 10^6}$$

$$T = \frac{2 \times 0.3}{5.165 \times 10^6}$$

Frequency of R.F. voltage

$$f = \frac{1}{T} = \frac{5.16 \times 10^6}{2 \times 0.3} = 8.608 \times 10^6 \text{ Hz}$$

$$= \mathbf{8.608\ MHz}$$

The proton travel through the next tube for half a period with a velocity of 5.536×10^6 m/sec.

∴ Length of the tube entered by the protons

$$L = 5.536 \times 10^6 \times \frac{1}{2 \times 8.608 \times 10^6}$$

$$= \mathbf{0.3216\ m}$$

3.5 CYCLOTRON

Construction

Cyclotron was devised by Lawrence in 1932. The cyclotron is shown in fig. (2). It consists of two hollow flat semicircular metal boxes D_1 and D_2 called the dees on

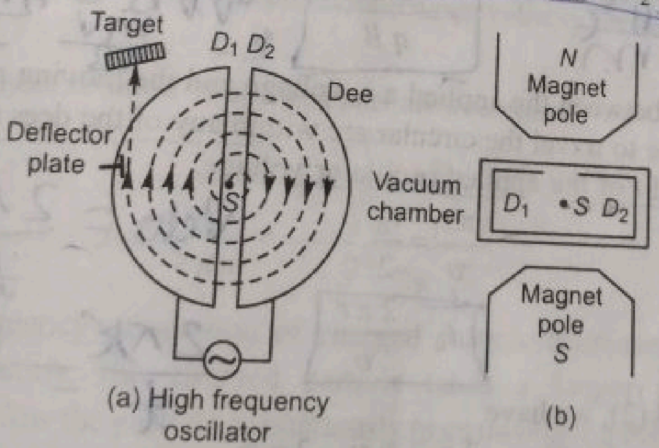


Fig. (2)

account of their shape like the letter D. The two dees are separated by a narrow parallel gap. A high frequency oscillator, which provides an alternating potential of the order of 10^4 to 10^5 volts at a frequency of 10^7 cycles, is connected between the two dees. The oscillator establishes an alternating electric field in the air gap, *i.e.*, the electric field is be directed towards D_1 and then towards D_2. Thus, D_1 and D_2 become alternately positive and negative at the same rate as the frequency of the oscillator. A source S is placed at the centre of the dees which supplies the positive ions to be accelerated. These dees are mounted inside a vacuum chamber. The chamber is mounted horizontally between the pole pieces NS of a huge electromagnet capable of producing a vertical field of about 1.6 Web/metre^2.

Working

The positive ions emitted from the source will be accelerated in the gap towards the dee which is negative at the time. Let it be D_2. Since, there is no electric field inside the dees, the positive ions move with constant velocity along circles of constant radius under the influence of magnetic field which is perpendicular to the dees. If by the time the ions emerge from D_2, the polarity of the applied potential is reversed, (*i.e.*, the dee D_1 now becomes negative), the positive ions will again face the negative dee and thus will be again accelerated by the field in the gap. Since, their velocity is increased, they will now move through D_1 along circular arc of greater radius as shown in the figure. Here, the time of passage to complete the semi circle in the dee remains the same as in D_2. If the time of travel in D_1 is equal to half the time period of the oscillator voltage, the positive ions after coming from D_1 will find the reversed field and hence, they are accelerated again in the gap D_1 D_2. In this way, the positive ions move faster and faster moving in ever-expanding circles until they reach the outer edge of the dees where they are deflected by deflector plate and strike the target. Here, it should be remembered that ***the time required for the positive ions to make one complete turn within dees is the same for all speeds and is equal to the time period of the oscillator.***

Theory

When a particle of mass m and charge q moves with a velocity v in the magnetic field of flux density B, then the radius r of the circular path is given by

$$v = \frac{qBr}{m} \qquad \boxed{r = \frac{mv}{qB}} \qquad qvB = \frac{mv^2}{r} \qquad r = \frac{mv}{qB} \qquad \text{...(1)}$$

For resonance between the applied a.c. voltage and the moving particle, the time taken by the particle to travel the circular arc within any of the dees must be equal of half the time period t of the applied oscillator voltage.

$$\therefore \qquad \frac{\pi r}{v} = \frac{t_0}{2} \qquad \text{Velo} = \frac{2\pi r}{t}$$

or

$$\boxed{t_0 = \frac{2\pi r}{v}} \qquad \frac{2\pi r}{t} = \frac{qBr}{m} \qquad \text{...(2)}$$

For eqs.(1) and (2), we have

$$t_0 = \frac{2\pi}{v} \times \frac{mv}{qB} = \frac{2\pi m}{qB} \qquad \text{...(3)}$$

Now, the orbital frequency, or the cyclotron frequency f_0 is given by

$$\boxed{f_0 = \frac{1}{t_0} = \frac{qB}{2\pi m}} \qquad \text{...(4)}$$

The value of q/m being fixed for an ion hence, the value of f_0 is adjusted corresponding to B or *vice-versa*.

Energy of a particle accelerated by a cyclotron

The ion will have maximum energy when it will travel at the boundary of dee. If the outside radius of dee is R then according to eq. (1), the maximum velocity v_m of the ion may be written as

$$\boxed{v_m = \frac{R\,q\,B}{m}}$$

$\frac{mv^2}{r} = qvB$

$r = \frac{mv}{qB}$

$V = \frac{qBr}{m}$

and so the maximum kinetic energy of the ion will be given by ...(4)

$$E_m = \frac{1}{2} m\, v_m^{\ 2} = \frac{R^2\, q^2\, B^2}{2\,m} \qquad ...(5)$$

$E = \frac{1}{2} m V^2$

We know that 1 MeV $= 1.6 \times 10^{-13}$ joules

$E = \frac{1}{2} m \frac{q^2B^2r^2}{m^2}$

$$E_m = \frac{R^2\, q^2\, B^2}{2\,m \times 1.6 \times 10^{-13}} = \frac{3.12 \times 10^{12}\, R^2\, q^2\, B^2}{m}\ \text{MeV}$$

$\boxed{E_m = \frac{r^2 q^2 B^2}{2m}}$

Limitations of cyclotron

The maximum available particle energy is limited due to the following factors :

(i) due to the limited power and frequency of the oscillator,

(ii) due to the maximum strength of the magnetic field which can be produced, and

(iii) the energy of charged particles emerging from cyclotron, is limited due to variation of mass with velocity, *i.e.*,

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$m_0 = \frac{m_0}{\sqrt{\left(1 - \frac{V^2}{c^2}\right)}}$

where m_0 is the rest mass, m the mass in motion when velocity is v and c, the velocity of light.

The frequency of rotation of charged particle becomes

$$f_0 = \frac{q\,B}{2\,\pi\, m} = \frac{q\,B\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}{2\,\pi\, m_0}$$

Thus, the frequency of rotation of charged particle decreases with increase of velocity. Consequently, the charged particle takes a longer time to complete semicircular path. Now the particle continuously goes on lagging behind the alternating potential differences till a stage is reached when it is no longer be accelerated further.

The frequency of the charged particle can be kept constant by making $B\sqrt{(1 - v^2/c^2)}$ constant. To achieve this, the magnetic field B is increased as the velocity of particle increases. Alternatively the frequency of applied alternating potential difference may be varied so that it is always equal to the frequency of rotation of charged particle.

Numerical Examples

Example 1 *In a cyclotron, the frequency applied to the dees is* 8.6 *mega cycles/sec. Calculate the magnetic field of induction required to accelerate the protons. (mass of*

$\boxed{freq = \frac{qB}{2\pi m}}$

Solution We know that

$$f_0 = \frac{q\,B}{2\,\pi\,m} \quad \text{or} \quad B = \frac{2\,\pi\,m\,f_0}{q}$$

Substituting the given values, we have

$$B = \frac{2 \times 3.14 \times (1.79 \times 10^{-27}) \times (8.6 \times 10^{6})}{1.6 \times 10^{-19}} = \mathbf{0.6043\ weber/m^2}$$

Example 2 *A cyclotron with a magnetic field* $B = 1.5$ weber/meter^2 *is used to accelerated proton. Calculate the frequency of the oscillator connected across the dees.*

Solution The frequency of the oscillator is given by

$\boxed{f = \frac{q.B}{2\pi m}}$

$$f_0 = \frac{q\,B}{2\,\pi\,m}$$

$$= \frac{(1.6 \times 10^{-19})\,(1.5)}{2 \times 3.14 \times (1.67 \times 10^{-27})}$$

$$= 22.87 \times 10^{6}\ \text{Hz} = \mathbf{22.87\ MHz}$$

Example 3 *A cyclotron in which the flux density is* 0.7 weber/m^2 *is used to accelerate deutrons. How rapidly should the electric field between the dees be reversed? Mass of deutron* $= 3.34 \times 10^{-27}$ kg.

Solution We know that $t_0 = \dfrac{2\,\pi\,m}{q\,B}$. Hence, time taken by deuteron to travel a semi circular path

$t = \frac{2\pi m}{2 q B}$

$$t = \frac{1}{2}\left[\frac{2\,\pi\,m}{q\,B}\right]$$

$$t = \frac{\pi\,m}{B\,q} = \frac{3.14 \times (3.34 \times 10^{-27})}{0.7 \times (1.6 \times 10^{-19})}$$

$$= \mathbf{9.372 \times 10^{-8}\ sec.}$$

Example 4 *A cyclotron having dees of radius* 40 cm *is adjusted for accelerating hydrogen nuclei. The polarity of dees is reversed* 30×10^{6} times/sec. *Find the energy of the issuing proton (Mass of proton* $= 1.67 \times 10^{-27}$ kg).

Solution The maximum linear velocity is given by

$V = \frac{2\pi r}{t} = 2\pi$

$$v_m = r\,\omega = r \times 2\,\pi\,n$$

$$= (0.40\ \text{m}) \times 2 \times 3.14 \times 30 \times 10^{6} = 7.536 \times 10^{7}\ \text{m/s}$$

$$\text{K.E. of proton} = \frac{1}{2}\,m \times v_m^{\;2} = \frac{1}{2} \times (1.67 \times 10^{-27}) \times (7.536 \times 10^{7})$$

$$= 4.742 \times 10^{-12}\ \text{joules}$$

$$= \frac{4.742 \times 10^{-12}}{1.6 \times 10^{-13}}\ \text{MeV} = \mathbf{29.63\ MeV}$$

Example 5 *In a certain cyclotron, the maximum radius that the path of deuteron may have before it is deflected out of the magnetic field is 20 cm.*

(i) Calculate the velocity of deuteron at this radius.
(ii) What is the energy of deuteron in MeV? Given magnetic field = 1.5 weber/m^2.
Mass of deuteron $= 3.34 \times 10^{-27}$ kg.

$r = 0.2\,m$
$V = 2\pi r$

Solution (i) The velocity of deuteron is given by

$$v = \frac{q\,B\,r}{m}$$

$\frac{mV^2}{r} = qVB$

$V = \frac{qBr}{m}$

Here, $q = 1.6 \times 10^{-19}$ coul., $m = 3.34$ k gm

$r = 20\text{ cm} = 0.2$ metre,

$B = 1.5\text{ weber/m}^2$

$$v = \frac{(1.6 \times 10^{-19}) \times 1.5 \times 0.2}{8.34 \times 10^{-27}} = \mathbf{1.437 \times 10^7\ m/sec.}$$

$V = \frac{1.6 \times 10^{-19} \times 1.5 \times 0.2}{3.3 \times 10^{-27}}$

(ii) Energy of the deuteron $= \frac{1}{2} m v^2$

$E = \frac{1}{2} mV^2$

$$= \frac{1}{2} \times 3.34 \times 10^{-27} \times (1.437 \times 10^7)^2$$

$$= 3.449 \times 10^{-13} \text{ joules}$$

$$= \frac{3.449 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV} = 2.156 \times 10^6 \text{ eV} = \mathbf{2.156\ MeV}$$

Example 6 *The magnetic induction in a cyclotron which is accelerating protons is* 1.5 weber/m^2. *How many times per second should the potential across the dees reverse? What is the maximum K.E. of the proton if the diameter of the cyclotron is* 1 m? *Mass of proton* $= 1.67 \times 10^{-27}$ kg.

$f = ?$ $B = 1.5$ $KE = ?$

Solution The frequency of the applied P.D. is given by

$f = \frac{Bq}{2\pi m}$

$$f_0 = \frac{B\,q}{2\,\pi\,m} = \frac{1.5 \times (1.6 \times 10^{-19})}{2 \times 3.14 \times (1.67 \times 10^{-27})}$$

$$= 22.88 \times 10^6 \text{ cycles/sec} = \mathbf{22.88\ mega\ cycles/sec.}$$

The velocity of the proton is given by

$$v = \frac{q\,B\,r}{m} = \frac{(1.6 \times 10^{-19}) \times (1.5) \times (0.5)}{1.67 \times 10^{-27}}$$

$$= 7.186 \times 10^7 \text{ m/sec}$$

$$\text{K.E.} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times (1.67 \times 10^{-27}) \times (7.186 \times 10^7)^2$$

$$= 4.312 \times 10^{-12} \text{ joules}$$

$$= \frac{4.312 \times 10^{-12}}{1.6 \times 10^{-19}} = 26.96 \times 10^6 \text{ eV}$$

$$= \mathbf{26.96\ MeV}$$

Example 7 *Protons are accelerated in a cyclotron with dees of radius 40 cm and the frequency of the alternating potential is 10 mega cycle per second at 10,000 volts. Calculate (a) the magnetic field B (b) speed and (c) K.E. of proton.*

10×10^6

Solution (a) We know that, $f_0 = \frac{q\,B}{2\,\pi\,m}$

$$\therefore \quad B = \frac{2\,\pi\,m\,f_0}{q} = \frac{2 \times 3.14 \times (1.67 \times 10^{-27})\,(10 \times 10^6)}{(1.6 \times 10^{-19})}$$

$$= \mathbf{0.66\ Weber/metre^2}$$

(b) The velocity v is given by

$$v = \frac{q\,B\,R}{m}$$

$$= \frac{(1.6 \times 10^{-19})\,(0.66)\,(0.40)}{1.67 \times 10^{-27}}$$

$$= \mathbf{25.29 \times 10^6\ m/s}$$

(c) K.E. $= \frac{1}{2} m v^2$

$$= \frac{1}{2}(1.67 \times 10^{-27})\,(25.29 \times 10^6)^2 \text{ joules}$$

$$= \frac{1}{2}\frac{(6.67 \times 10^{-27})\,(25.29 \times 10^6)^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 3.338 \times 10^6 \text{ eV} = \mathbf{3.338\ MeV}$$

Example 8 *A cyclotron with dees of radius 2 meter has a magnetic field of 0.75 weber/m². Calculate the maximum energies to which a proton can be accelerated.*

Solution Let, R be radius of the outermost orbit, then maximum energy E_m of the particle at the time of leaving the cyclotron is given by

$$E_m = \frac{R^2\,q^2\,B^2}{2\,m} \text{ joule}$$

$$E_n = \frac{q^2 B^2 R^2}{2m}$$

As 1 MeV $= 1 \times 10^6$ eV $= (1 \times 10^6)\,(1.6 \times 10^{-19})$ joule

$$= (1.6 \times 10^{-13}) \text{ joule}$$

$$\frac{q^2 B^2 R^2}{2m}$$

$$E_m = \frac{R^2\,q^2\,B^2}{2\,m \times (1.6 \times 10^{-13})} \text{ MeV}$$

For proton $m = 1.67 \times 10^{-27}$ kg and $q = 1.6 \times 10^{-19}$ coulomb

$$\therefore \quad E_m = \frac{(2)^2 \times (1.6 \times 10^{-19})^2 \times (0.75)^2}{2 \times (1.67 \times 10^{-27}) \times (1.6 \times 10^{-13})}$$

$$= \mathbf{108\ MeV}$$

Example 9 *The radius of the dees of cyclotron is* 0.5 m. *Find the required magnetic field and radio frequency to accelerate an* α*-particle to an energy of* 20 MeV. *(Mass of* α*-particle* $= 6.643 \times 10^{-24}$ kg *and charge on* α*-particle* $= 3.2 \times 10^{-19}$ coulomb)

Solution K.E. of α-particle $= \frac{1}{2} m v^2$

$E = \frac{1}{2} mv^2$

Given K.E. $= 20$ MeV $= (20 \times 10^6) \times (1.6 \times 10^{-19})$ joule

$20 \times 10^6 \; 1.6 \times 10^{-19} = \frac{1}{2}(6.6 \times 10^{-24}) v^2$

$$\therefore \quad (20 \times 10^6) \times (1.6 \times 10^{-19}) = \frac{1}{2} \times (6.643 \times 10^{-24})\, v^2$$

$$v^2 = \frac{2 \times (20 \times 10^6) \times (1.6 \times 10^{-19})}{6.643 \times 10^{-24}}$$

Solving for v, we get $\quad v = 9.815 \times 10^5$ m/sec.

$$v = \frac{R q B}{m} \quad \text{or} \quad B = \frac{v m}{R q}$$

$v = qB$

Substituting the given values, we have

$\frac{mv^2}{r} = qvB$

$v = \frac{qBr}{m}$

$$B = \frac{(9.815 \times 10^5)(6.643 \times 10^{-24})}{(0.5)(3.2 \times 10^{-19})}$$

$$= \mathbf{40.75\ Weber/m^2}$$

The frequency applied is given by

$f = \frac{qB}{2\pi m}$

$$f_0 = \frac{q B}{2 \pi m} = \frac{(3.2 \times 10^{-19})(40.75)}{2 \times 3.14 \times (6.643 \times 10^{-24})}$$

$$= \mathbf{3.125 \times 10^5\ Hz}$$

Example 10 *If an alternating e.m.f. of* 20 kV *peak value and* 5 MHz *frequency is used in a cyclotron, an emergent beam of ions of* 2 MeV *is obtained. Find the maximum number of revolutions the ions undergo inside the dees and the strength of the magnetic field, if (e/m) for the ions is* 0.9×10^4 emu/gm. *Also determine the radius of the dees.*

Solution The number of revolutions the ions undergo inside the dee

$$n = \frac{2 \times 10^6}{2 \times 10^4} = 100$$

Now
$$f_0 = \frac{q B}{2 \pi m} \quad \text{or} \quad B = \frac{2 \pi m f_0}{q} = \frac{2 \pi f_0}{(q/m)}$$

$$B = \frac{2 \times 3.14 \times (5 \times 10^6)}{0.9 \times 10^4} = \mathbf{3491\ Weber/m^2}$$

Further, $\frac{m v^2}{r_{max}} = B e v$ or $v = \frac{B e r_{max}}{m}$

$$\therefore \quad E = \frac{1}{2} m v^2 = \frac{1}{2} \frac{B^2 e^2 r_{max}^2}{m}$$

or $\quad r_{max}^2 = \frac{2 m E}{B^2 e^2}$ or $r_{max} = \frac{\sqrt{(2 m E)}}{B e}$

$$\therefore \quad r_{max} = \frac{\sqrt{2 \times (1.67 \times 10^{-24}) \times (3.2 \times 10^{-6})}}{3491 \times (1.6 \times 10^{-19})} = \mathbf{58.45\ cm}$$

3.6 SYNCHROCYCLOTRON OR FREQUENCY MODULATED CYCLOTRON

Principle

The resonance frequency of a cyclotron is given by

$$\boxed{f_0 = \frac{q B}{2 m}}$$

We know that the variation of mass with velocity is given by

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

If $v = 0.8\,c$, the $r/c = 0.8$ and $m = 1.66\, m_0$.

Thus, at high velocities due to relativistic increase in mass, the orbital frequency of the particle is decreased. The particle gradually goes out of phase with the applied a.c. voltage whose frequency is constant. It is, therefore, necessary to provide corrective measures to compensate the decrease in frequency and to keep the particle in phase with the applied a.c. voltage. The compensation for the decrease in frequency can be made in the following two ways :

(i) Keeping the value of B constant, the oscillator frequency is decreased in step with the decrease in orbital frequency of the particle. A device which adopts this principle is called ***Synchrocyclotron*** or ***frequency modulated cyclotron.***

(ii) The magnetic flux density B may be increased to that $B\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$ remains constant. The frequency of high frequency oscillator is kept fixed. A device which adopts this principle is called a ***Synchrotron.***

Advantages of Synchro-cyclotron over Cyclotron

Followings are the advantages of synchro-cyclotron over cyclotron :

~~(1)~~ A single dee may be used. Instead of second dee, an earthed sheath is used opposite the opening of the dee and a.c. potential is applied between single dee and earth.

(2) Comparatively small voltages may be needed across the dee gaps. Thus, greater efficiency of high frequency oscillator may be obtained.

(3) The gap between the magnetic pole pieces can be reduced thus permitting high flux densities to be obtained.

(4) The pole pieces can be suitably shaped to produce a flux density decreasing outwards from the centre, thus achieving good focussing of accelerated particles.

Disadvantages

Followings are the disadvantages of synchrocyclotron over cyclotron :

(1) Only a small number of particles can be brought to the orbit of maximum radius and energy.

(2) Output of the particles is in pulses.

Construction of Synchro-cyclotron

The Berkeley synchrocyclotron is shown in fig. (3). It has a huge magnet with a pole diameter of 4.7 m and weighing 4000 tons. The modulation frequency is 120 Hz.

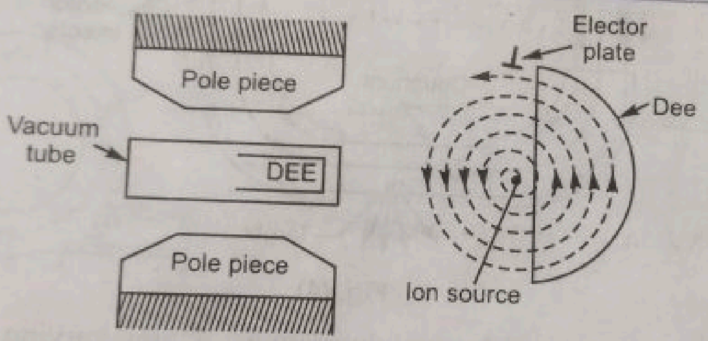


Fig. (3) Synchro-cyclotron

This is produced by a large rotating condenser that forms the part of capacitance in rf oscillator circuit. To reduce the scattering of ions by residual gas molecules, the pressure in the chamber is reduced to about 10^{-6} cm of mercury. Each ions makes over 10^4 revolutions before attaining maximum energy. Here a single dees is employed. This feature makes the ion beam much more accessible for experiment. The ejector plate with a high positive voltage pulse deflects the ion groups on to the target to be bombarded.

Synchro-cyclotron can accelerate deuterons to 200 MeV, α-particles to 400 MeV and protons to 350 MeV.

Difference between cyclotron and synchrocyclotron

The following are the basic differences between cyclotron and synchrocyclotron :

(i) There are two dees in cyclotron while in synchrocyclotron there is only one dee.

(ii) The oscillator frequency in cyclotron is fixed while it is variable in case of synchrocyclotron.

(iii) The cyclotron provides the continuous supply of accelerated particles. On the other hand, the particles come out in bursts of a few hundred per second in case of synchrocyclotron.

3.7 THE SYNCHROTRON

The proton synchrotron is a machine which accelerates protons to energy in GeV range (1 GeV = 1 billion electron volt). The device uses a ring shaped or doughnut vacuum tube. This tube is circular in the installation at Birmingham, England and has a race-track design in installation in America. Here we shall describe the American design.

Construction

It consists of a ring-shaped magnet which is in four quadrants as shown in fig. (4).

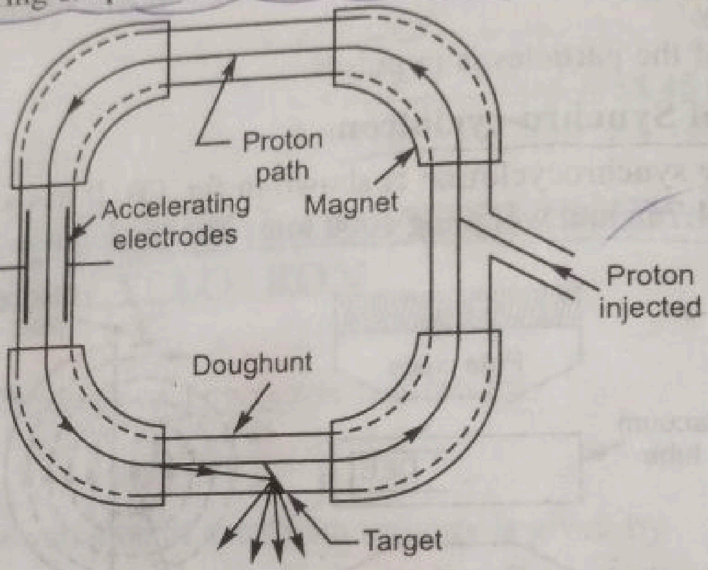


Fig. (4)

The four quadrants are spaced by straight sections. A time varying magnetic field is produced by applying a.c. voltage to the copper winding of the magnet. The protons are injected by a linear accelerator or Van de Graaff generator into the doughnut at energy between (1 MeV and 10 MeV). This injection is tangential to the orbit of ring shaped doughnut. A high frequency resonator cavity accelerator (two electrodes connected to high frequency generator) is used in one of the straight parts.

Operation

The pre-accelerated protons (1–10 MeV) are injected into doughnut with the help of linear accelerator. Under the influence of magnetic field, the protons are made to circulate in ring shaped vacuum tube. In each revolution the protons are accelerated once when they pass through electrodes connected to high frequency generator. The field strength and the frequency of the oscillator are simultaneously increased in such a way that the protons travel in a circular path of constant radius and arrive always at the electrodes when applied voltage is in right phase for acceleration. When protons reach their maximum energy, the oscillator frequency is distorted. Now the proton's orbit contracts or expands such that they strike the target.

The proton synchrotron can accelerate the protons to energies of the order of 10 GeV. The Berkeley machine can accelerate protons to 6 BeV and is called **bevatron**. The Brookhaven machine called as **cosmotron** can energise protons to 2.2 BeV. The machine is known as cosmotron because particles of such energies can only be found in energetic cosmic rays.

3.8 BETATRON

Betatron is a device for speeding up electrons to extremely high energies with the help of expanding magnetic field. The cyclotron principle was incapable for accelerating the electrons to high ènergies because of the large relativistic increase of mass at low energies. The first betatron was constructed by Kerst at the university of Illinois (U.S.A.) in 1940. This was capable to accelerate electrons to energies of 2 MeV. A model constructed in 1945 was capable of accelerating electrons to energies of 100 MeV. The betatron differs from cyclotron in the following two fundamental respects.

(i) In betatron, the electrons are accelerated by expanding magnetic field.

(ii) The circular orbit has a constant radius.

Construction

The construction of betatron is shown in fig. (5). It consists of highly evacuated tube known as *doughnut chamber*. This chamber is placed between the poles of an

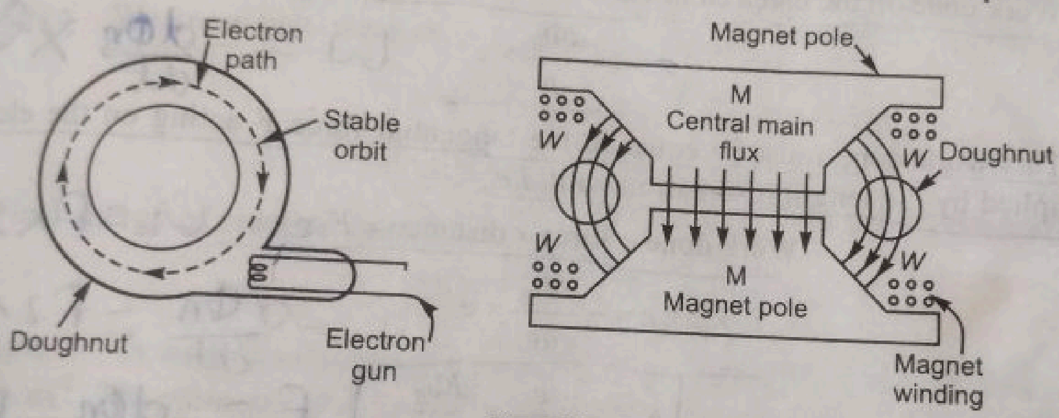


Fig. (5)

electromagnet excited by an alternating current usually of frequency 60 or 180 Hz. The poles of electromagnet are constructed in such a way as to provide a more stronger field in the central space. Electrons are produced by electron gun and are injected into doughnut at the beginning of cycle of alternate current in the electromagnet. The increasing magnetic flux gives rise to a voltage gradient (electric field) round the doughnut which accelerates the orbiting electrons. This increases the energy of the electrons.

Principle

The principle of betatron is the same as that of a transformer in which an alternating current applied to the primary coil induces an alternating current in the secondary. In betatron secondary coil is replaced by a doughnut vacuum chamber. When the electron is injected in doughnut, then alternating magnetic field has two effects : (i) an electromotive force is produced in the electron orbit by changing magnetic flux that gives an additional energy to the electrons and (ii) a radial force is produced by the action of magnetic field whose direction is perpendicular to the electron velocity which keeps the electrons moving in a circular path.

Operation

Electrons from the electron gun are injected into doughnut shaped vacuum chamber when the magnetic field is just rising from its zero value in the first quarter cycle as shown in fig. (6). The electrons now make several thousand revolutions and gain energy. When the magnetic field has reached its maximum value, the electrons are pulled out from their orbit. Either they strike a target and produce X-rays or emerge from the apparatus through a window.

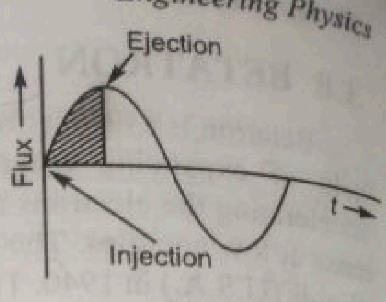


Fig. (6)

Betatron Condition : Consider an electron moving in a circular orbit of radius r in the magnetic field. Let at any instant, B be the magnetic field at this orbit and the total magnetic flux through the orbit is Φ_B. The flux Φ_B increases at the rate of $d\Phi_B/dt$ and the induced e.m.f. in the orbit is given by

$$\text{induced e.m.f.} = -\frac{d\Phi_B}{dt} \qquad \text{emf} = -\frac{d\Phi_B}{dt} \quad ...(1)$$

Work done on the electron in one revolution = induced e.m.f. × charge

$$= -\frac{d\Phi_B}{dt} \times e \qquad W = -\frac{d\Phi_B}{dt} \times e$$

This work done must be equal to the tangential force **F** acting on the electron multiplied by the length of the orbit path, *i.e.*,

$$\text{Work done} = \text{force} \times \text{distance} = F \times 2\pi r \qquad W = F \times 2\pi r$$

$$\therefore \quad F \times 2\pi r = -\frac{d\Phi_B}{dt} \times e \qquad -\frac{d\Phi_B}{dt} = F 2\pi r$$

or

$$\boxed{F = -\frac{e}{2\pi r} \times \frac{d\Phi_B}{dt}} \qquad F = -\frac{d\Phi_B}{dt} \times \frac{1}{2\pi r} \quad ...(2)$$

This force F will increase the electron energy and which in turn would tend to increase the orbit of large radius. In order to maintain the radius of the orbit, the force experienced by the electron must be counteracted. Suppose the velocity of the electron is v and its mass is m. When the electron moves in an orbit of radius r under the action of field of magnetic induction B, the inward radial force $B\ e\,v$ is to be equal to the outward centrifugal force $m v^2/r$.

$$\therefore \quad B\,e\,v = \frac{m v^2}{r} \qquad Bev = \frac{mv^2}{r}$$

or

$$m v = B\,e\,r \qquad Mv = Ber \quad ...(3)$$

According to Newton's-second law, the force is defined as the rate of change of momentum ($p = m v$), *i.e.*,

$$F = \frac{d}{dt} m v = \frac{d}{dt} B\,e\,r \qquad F = \frac{d}{dt}(mv)$$

or

$$\boxed{F = e r \frac{dB}{dt}} \qquad F = \frac{d}{dt}(Ber) \quad ...(4)$$

$$F = er\left(\frac{dB}{dt}\right)$$

To maintain the radius constant, the value of F given in equation (3) and equation (4) should numerically be equal, hence

$$\frac{\cancel{e}}{2\pi r}\frac{d\Phi_B}{dt} = r\frac{dB}{dt}, \cancel{e}$$

$$\frac{e}{2\pi r}\frac{d\Phi_B}{dt} = er\frac{dB}{dt}$$

or $$\int \frac{d\Phi_B}{dt} = 2\pi r^2 \frac{dB}{dt}$$

$$\Phi_B = 2\pi r^2 B$$

Integrating, we get $$\boxed{\Phi_B = 2\pi r^2 B}$$

This is known as betatron condition. ...(5)

Numerical Examples

Example 1 *The magnetic flux within a stable orbit in a betatron changes at constant rate of 15 Wb/s. What would be the energy of an electron which undergoes 10^6 revolutions?*

$\frac{d\Phi}{dt} = 15$ Wb/s

$E_n = \frac{d\Phi}{dt} \times e$ eV

Solution We know that increases in energy per revolution

$$= e\frac{d\Phi_B}{dt} \text{ joules} = \frac{d\Phi_B}{dt} \text{ electron-volt}$$

$$= 15\,\text{eV} = 15 \times 10^{-6}\ \text{MeV}$$

Final energy of electron after 10^6 revolutions

$$= 15 \times 10^{-6} \times 10^6 = \mathbf{15\ MeV}$$

Example 2 *In a certain betatron, the maximum magnetic field at the electron orbit is 0.5 Wb/m². The diameter of the stable orbit is 1.5 m. If the frequency of the alternating current through electromagnet coils is 59 Hz, calculate the electrons;*

(i) final energy

(ii) average energy gained per revolution.

Solution (i) In betatron, the acceleration time is equal to quarter time period, *i.e.*, $1/4f$ or $\pi/2\omega$ second. The total distance travelled by the electron is $c \times \pi/2\omega$ because the electron velocity is nearly equal to velocity of light c. If R is the radius of the stable orbit, then number of revolutions N made is given by

$$N = \frac{c\,\pi/2\omega}{2\pi R} = \frac{c}{4\omega R} = \frac{3 \times 10^8}{4 \times (2\pi \times 50) \times 1.5} = 3.1 \times 10^5$$

The momentum of electron $= E/c$ (treated relativistically)

$$mv = E/c$$

Moreover, $$\frac{mv^2}{R} = Bev$$

$\therefore$ $$mv = BeR \text{ or } E/c = BeR \text{ or } E = BeRc$$

or $$E = \frac{0.5 \times 1.6 \times 10^{-19} \times 0.75 \times 3 \times 10^8}{1.6 \times 10^{-13}}\ \text{MeV} = \mathbf{112.5\ MeV}$$

(ii) The average energy per revolution is $= 112.5 \times 10^6 / 3.1 \times 10^5 = \mathbf{363\ eV}$

3.9 NUCLEAR DETECTORS

The instruments which are used to detect nuclear radiations are called nuclear detectors. A list of nuclear detectors is given below :

1. *Ionization chamber*
2. *Proportional counter*
3. *Bubble chamber*
4. *Cloud chamber*
5. *Scintillation counter*
6. *Semiconductor counter*
7. *Photographic emulsion*
8. *Spark chamber*
9. *Geiger-Muller counter, etc.*

Here, we shall discuss only Geiger-Muller counter.

3.10 GEIGER-MULLER COUNTER

The Geiger-Muller counter consists of a fine wire (usually tungsten) placed along the axis of a hollow metal-cylinder electrode (cathode) enclosed in a thin glass tube. The tube contains a mixture of 90% argon at 10 cm pressure and 10% ethyl alcohol vapour at 1 cm pressure. Different mixtures of gases at different pressure are used in different designs. At one end of the tube, a window covered with thin mica sheet (Fig. 7) is provided through which the ionizing particles or radiations may enter the tube. A d.c. potential of about 1200 volts is applied between the cathode and the wire

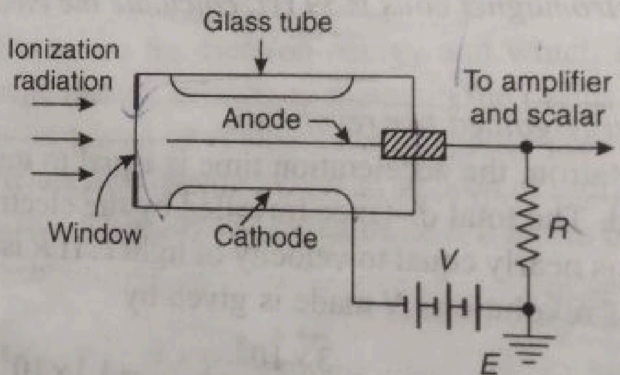


Fig. (7) G.M. Counter.

which acts as an anode. The value of the voltage is adjusted to be somewhat below the breakdown voltage of the gaesous mixture. A high resistance R is connected in series with battery.

When a charged particle passes through the counter, it ionizes the gas molecules. The central wire attracts the electrons while the cylindrical electrode attracts the positive ion. This causes an ionization current which depends upon the applied voltage. At sufficiently high voltages, the electrons gain high kinetic energy and cause further ionization of argon atoms. Thus, the larger number of secondary electrons are produced. The number of secondary electrons is independent of the number of primary ions produced by incoming particle due to the following

(i) The production of secondary electrons is not confined to the region near the primary electrons but it takes place all along the length of the wire as their number is extremely large ($\approx 10^8$).

(ii) The production of secondary electrons at one point affects the production at other points.

The incoming particle serves the purpose of triggering the release of an *avalanche* of secondary electrons. The electrons quickly reach the anode and cause ionization current. The positive ions move more slowly away from anode and they form a sheath around the anode for a short while. They reduce the potential difference between the electrodes to a very low value because ion sheath depresses electric field near anode. The current therefore, stops. In this way a brief pulse of current flows through resistance R. This current creates a potential differences across R. The pulse is amplified and fed to counter circuit. As each incoming particle produces a pulse, hence, the number of incoming particles can be counted.

The successful operation of G.M. Counter depends upon the proper voltage to the electrodes. Fig. (8) represents the counts per minute as a function of voltage. It is

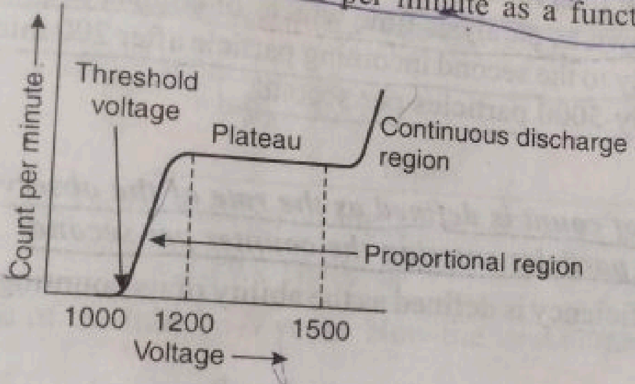


Fig. (8)

obvious from the figure that if the voltage is less than 1000 volt, there is no discharge, i.e., no secondary ionization. When the voltage is increased, secondary ionization takes place. Now the number of impulses increases almost linearly with applied voltage. As already discussed, this region is most suitable for proportional counters. As the applied voltage is further increased to about 1200 volts, the number of impulses remains constant over a certain region known as plateau. In this region, the magnitude of impulses becomes independent of the amount of original ionization and is a function of potential, nature of gas, resistance R and geometrical condition of apparatus. This region is most suitable of G.M. Counter. If the voltage is increased above this region, a continuous discharge will take place. This is undesirable and hence, avoided.

Quenching

When the positive ions reach the cathode, they detach secondary electrons from the cathode because they have acquired large kinetic energies. These electrons travel to the anode and produce fresh avalanches, *i.e.*, unwanted pulse. The counter is now kept in the state of continuous avalanching. If in this state, measurements are made, the counter will confuse the two pulses, one due to continuous avalanching and the other due to fresh event. The process of preventing the continuous avalanching is known as quenching. The self-quenching is obtained by adding a quenching agent like alcohol vapour in the tube. Alcohol has low ionization potential (11.3 eV). The argon ions (having ionization potential 15.7 eV) on their journey to the cathode are practically all neutralized* by acquiring an electron from the alcohol molecules. Now only alcohol ions reach the cathode. They acquire electrons from cathode and becomes neutral alcohol molecules. Though the alcohol molecules still possess kinetic energy yet this is used for dissociation into alcohol atoms and not for the production of secondary electrons by colliding with cathode. Thus, the possibility of continuous avalanching is removed.

Counting rate

The G.M. Counter can count about 500 particles per second. The counting rate depends upon (i) dead time, (ii) recovery time and (iii) paralysis time. The slowly moving positive ions take about 100 microseconds to reach the cathode. This time is known as dead time. If a second particle enters the tube during this time, it will not be recorded as the potential difference across the electrodes is very low. After dead time, the tube takes nearly 100 microseconds before it regains its original working conditions. This time interval is known as recovery time. The sum of dead time and recovery time is known as paralysis time which, of course, is 200 microseconds. The tube can respond fully to the second incoming particle after 200 microsecond. Thus, the tube can count nearly 5000 particles per second.

Efficiency

The efficiency of count is defined as the rate of the observed counts/sec to the number of ionizing particles entering the counter per second.

The counting efficiency is defined as the ability of its counting, if atleast an ion-pair is produced in it.

Counting efficiency $\eta = 1 - \exp.(s\,p\,l)$

where s = specific ionization at one atmosphere

p = pressure in atmosphere

l = length of the ionization particle in the counter

The G.M. Counter is very useful for counting β-particles. It can also be used for measuring γ-rays intensities. It cannot be used for counting α-particles due to their low energy as the window can not be made thin enough to pass them.

NUMERICAL EXAMPLES

Example 1 *A G.M. Counter wire collects* 10^8 *electrons per discharge. When the counting rate is* 500 counts/minute, *what will be the average current in the circuit?*

Solution Counting rate = 500 counts/min.

The wire collects 10^8 electrons per discharge

*The positive argon ions travelling towards cathode collide with alcohol molecules. They ionize them and transfer their kinetic energy. The argon ion acquire th... and becomes neutral argon atoms. Th...

$\therefore$ Total number of electrons collected in one min.

$$n = 500 \times 10^8 = 5 \times 10^{10}$$

$$\text{Charge/min} = n\,e = (5 \times 10^{10}) \times (1.6 \times 10^{-19}) \text{ coul/min.}$$

$q = ne$

$I = \frac{q}{t} = \frac{ne}{60s}$

$$\text{Charge/second} = \frac{(5 \times 10^{10})(1.6 \times 10^{-19})}{60} = 1.33 \times 10^{-10} \text{ Amp.}$$

Now, charge/second gives the average current

$\therefore$ Average count $= \mathbf{1.33 \times 10^{-10}}$ **Amp.**

Example 2 *A self-quenched G.M. Counter operates at* 1000 volts *and has a wire of diameter* 0.2 mm. *The radius of the cathode is* 2 cm *and the tube has a guaranteed life time of* 10^9 *counts. What is the maximum radial field and how long will the counter last if it is used on an average for* 30 hours *per week at* 3000 counts per minute? *Assume* 50 *weeks to a year.*

Solution The radial field at the centre is given by

$$E_{\max} = \frac{V}{r \log_e (b/a)}$$

$$= \frac{1000}{0.02 \times [2.3 \log_{10} (2 \times 10^{-2}/10^{-4})]}$$

$$= \mathbf{1.89 \times 10^6\ V/m}$$

Let, the life time of the tube be N years. Now the total number of counts recorded will be

$$N \times 50 \times 30 \times 60 \times 3000 = 2.7 \times 10^8\ N$$

According to given problem

$$2.7 \times 10^8\ N = 10^9$$

$$N = \frac{10^9}{2.7 \times 10^8}$$

$$= \mathbf{3.7\ years}$$

3.11 MOTION OF CHARGED PARTICLES IN ELECTRIC FIELD

When a charged particle moves in an electric field, the following important points are observed :

(1) When the charged particle moves perpendicular to the electric field, it follows a parabolic path.

(2) If the charged particle moves in the direction of electric field, it experiences a force $q\,\mathbf{E}$ in the direction of electric field and hence, accelerated in the same direction.

(3) If the charged particle moves in opposite direction of electric field, it experiences a force $-q$ **E** and hence, decelerated.

(4) When the particle leaves the plates, it follows a straight line path which is tangent to the parabola. The angle is given by

$\tan\theta = \frac{dy}{dx}$

$$\tan\theta = \left(\frac{dy}{dx}\right)_{x=l} = \left[\frac{qE}{2mv_x^2}\times 2x\right]_{x=l}$$

or

$$\boxed{\tan\theta = \frac{qEl}{mv_x^2}}$$

$\tan\theta = \frac{qEl}{mv_x^2}$

This principle is used in cathode ray tubes.

We shall consider these points in detail.

(1) Motion in Uniform Electric Field.

Consider the case of a charged particle of mass m and having a charge q in uniform electric field of intensity **E**. This experiences a force **F** given by q **E**. The acceleration **a** of the particle in the direction of **F** is given by

$$\boxed{\mathbf{a} = \frac{\text{Force}}{\text{mass}}} \qquad (\because \text{ Force} = \text{mass} \times \text{acceleration})$$

or

$$\boxed{\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\,\mathbf{E}}{m}} \qquad \ldots(1)$$

The velocity **v** of the particle can be obtained by integrating the above expression. Thus,

$$\mathbf{v} = \frac{q}{m}\mathbf{E}\,t + C_1 \qquad \ldots(2)$$

$\frac{V}{a} = \frac{qEt + C_1}{m}$

where C_1 is constant of integration. To obtain the value of C_1, we apply the condition that at $t = 0$, the velocity v of the particle is its initial velocity u, *i.e.*, $\mathbf{v} = \mathbf{u}$. Hence, $\mathbf{u} = C_1$. Substituting the value of C_1 in eq. (2), we get

$$\mathbf{v} = \frac{q}{m}\mathbf{E}\,t + \mathbf{u} \qquad \ldots(3)$$

Further, integration of eq. (3) w.r.t. time gives the value of position vector **r** because $\mathbf{v} = \left(\frac{d\mathbf{r}}{dt}\right)$. Hence,

$$\mathbf{r} = \frac{q}{m}\mathbf{E}\frac{t^2}{2} + \mathbf{u}\,t + C_2 \qquad \ldots(4)$$

where C_2 is another constant of integration. The value of C_2 can be obtained by applying the condition that at $t = 0$, $\mathbf{r} = \mathbf{r}_0$ (initial position). Thus,

Thus,

$$\mathbf{r}_0 = 0 + 0 + C_2 \quad \text{or} \quad C_2 = \mathbf{r}_0$$

$\therefore$

$$\boxed{\mathbf{r} = \frac{q}{m}\mathbf{E}\frac{t^2}{2} + \mathbf{u}\,t + \mathbf{r}_0} \qquad \ldots(5)$$

This relation gives the path of the particle in a uniform electric field provided the initial velocity **u** is also along **E**. This is the case of longitudinal field. If $\mathbf{r}_0 = 0$ and $\mathbf{u} = 0$, then

$$\boxed{\mathbf{r} = \frac{1}{2}\left(\frac{q}{m}\right)\mathbf{E}\,t^2} \qquad ...(6)$$

$r = \frac{1}{2}\frac{qEt^2}{m}$

(2) Motion in a Perpendicular Electric Field.

If the direction of electric field is perpendicular to the direction of motion of the charge, the field is known as transverse electric field. Consider the case of a charged particle of mass m and charge q enters between two parallel plates A and B [see fig. (9)]

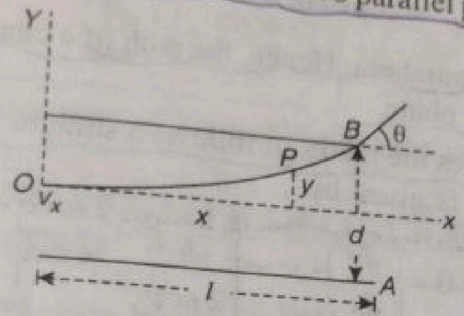


Fig. (9)

along X-axis with initial velocity v_x. Let the plates are of length l, separated at a distance d and kept at a potential difference of V volt. The electric field $E = \left(\frac{V}{d}\right)$ acting along Y-axis. Thus, a force $q\,\mathbf{E}$ acts along Y-axis.

As there is no force along X-axis, the time taken by the particle to pass through the plates will be $\frac{l}{v_x}$. Thus,

$$\boxed{t = \left(\frac{l}{v_x}\right)} \qquad ...(1)$$

Now, we shall consider the motion of the particle along Y-direction. The acceleration a of the particle along Y-direction is given by

$$a = \frac{\text{force}}{\text{mass}} = \frac{q\,E}{m}$$

$F = ma \quad a = \frac{F}{m} = \frac{qE}{m}$

The velocity of the particle in Y-direction at any time t' after entering the field

$$v_y = 0 + a\,t' = 0 + \left(\frac{q\,E}{m}\right)t' \qquad ...(2)$$

($\because$ Initial velocity of particle along Y-axis is zero)

Substituting the value of t' using eq. (1) in eq. (2), we get

$$v_y = \left(\frac{q\,E}{m}\right)\left(\frac{x}{v_x}\right) \qquad (l = x,\ \text{when } t = t') \qquad ...(3)$$

Displacement in Y-direction is given by

$$y = \frac{1}{2} \times \text{acceleration} \times (t')^2$$

or

$$y = \frac{1}{2} \times \left(\frac{qE}{m}\right)\left(\frac{x}{v_x}\right)^2$$

or

$$\boxed{y = \frac{qE}{2 m v_x^2} \times x^2}$$

$$= \text{constant} \times x^2 \qquad \ldots(4)$$

This is an equation of parabola. Hence, the path of a charged particle between the plates is a parabola in X-Y plane.

When the particle leaves the plates, it follows a straight line path which is tangent to the parabola. The angle is given by

$$\tan\theta = \left(\frac{dy}{dx}\right)_{x=l} = \left[\frac{qE}{2 m v_x^2} \times 2x\right]_{x=l}$$

or

$$\boxed{\tan\theta = \frac{qEl}{m v_x^2}} \qquad \ldots(5)$$

$\tan\theta = \frac{qEl}{mv_x^2}$

This principle is used in cathode ray tubes.

(3) Motion in Alternating Electric Field.

The alternating field is represented by

$$\mathbf{E} = \mathbf{E}_0 \sin \omega t$$

Force on the particle = mass × acceleration

$$\mathbf{F} = m \times \frac{d^2\mathbf{r}}{dt^2}$$

$qE_0 \sin\omega t = m \frac{d^2x}{dt^2}$

$$q\,\mathbf{E}_0 \sin\omega t = m \times \frac{d^2\mathbf{r}}{dt^2} = m\frac{d\mathbf{v}}{dt} \qquad \ldots(1)$$

$qE \sin\omega t = m\frac{dv}{dt}$

Integrating eq. (1), we get velocity

$$\mathbf{v} = -\frac{q\,\mathbf{E}_0}{m}\frac{\cos\omega t}{\omega} + C_1$$

$-\frac{qE_0 \cos\omega t}{m\omega} + C_1$

$x = 0, v = 0$

where C_1 is constant of integration. The value of C_1 can be obtained by applying the condition that when $t = 0$, $v = 0$. Thus,

$$\mathbf{v} = -\frac{qE_0}{m\omega} + C_1 \quad \text{or} \quad C_1 = \frac{qE_0}{m\omega}$$

$C_1 = \frac{qE_0}{m\omega}$

Substituting the value of C_1, we get

$v = -\frac{qE_0}{m}\frac{\cos\omega t}{\omega} + \frac{qE_0}{m\omega}$

$$\mathbf{v} = -\frac{q\,\mathbf{E}_0}{m\omega}\cos\omega t + \frac{q\,\mathbf{E}_0}{m\omega} = \frac{q\,\mathbf{E}_0}{m\omega}[1 - \cos\omega t] \qquad \ldots(2)$$

Further, integration of eq. (2) gives the position vetor **r** because $\mathbf{v} = \left(\frac{d\mathbf{r}}{dt}\right)$. Hence,

$$\mathbf{r} = \frac{q\,\mathbf{E}_0}{m\,\omega}\int (1-\cos\omega t)\,dt$$

$r = \frac{qE}{m\omega}\int(1-\cos\omega t)\,dt$

or

$$\mathbf{r} = \frac{q\,\mathbf{E}_0}{m\,\omega}\left[t - \frac{\sin\omega t}{\omega}\right] + C_2$$

where C_2 is another constant of integration. The value of C_2 can be obtained by applying the condition that at $t = 0$, $\mathbf{r} = 0$. This gives $C_2 = 0$. Hence,

$$\boxed{\mathbf{r} = \frac{q\,\mathbf{E}_0}{m\,\omega}\left[t - \frac{\sin\omega t}{\omega}\right]}$$

$r = \frac{qE_0}{m\omega}\left(t - \frac{\sin\omega t}{\omega}\right)$

Eq. (3) gives the displacement of the particle in time t.

3.12 MOTION OF CHARGED PARTICLE IN MAGNETIC FIELD

When a charged particle having charge q travels with velocity **v** in magnetic field **B**, it experiences a force **F** given by

$$\mathbf{F} = q\,(\mathbf{v}\times\mathbf{B}) \quad \text{or} \quad F = q\,v\,B\sin\theta \qquad \ldots(1)$$

Now we consider the following cases :

(1) If the particle is at rest in magnetic field (*i.e.*, $v = 0$), *then it will experience no force.*

(2) If the particle is moving along the line of magnetic field, then **v** and **B** are parallel (*i.e.*, $\theta = 0$). *Hence, the particle will not experience any force.*

(3) If the particle is moving perpendicular to magnetic field, it experiences a maximum force denoted by $\mathbf{F}_m$.

Now, we shall describe the motion of charged particle in a magnetic field. Consider the case of a charged particle having charge q enters in a magnetic field at right angle to the direction of field as shown in fig. (10).

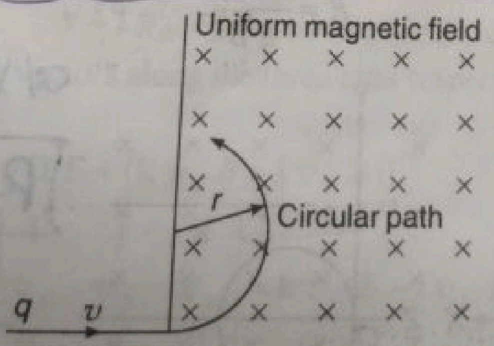


Fig. (10)

As the force due to magnetic field on the charged particle is perpendicular to the direction of motion of particle and direction of field, hence, no work is done by magnetic field on the charged particle. This shows that the particle does not gain kinetic energy. Thus, the velocity of particle remains unchanged, of course, direction is changed. As **v** and **B** are constant, **F** is also constant and is maximum which is represented by $\mathbf{F}_m$. The force provides the necessary centripetal force. Hence, the path traversed by the particle will be circular.

Now,
$$F_m = q\,v\,B \quad ...(2)$$

Let a be the acceleration produced by this force, then

$$\mathbf{F}_m = m\,\mathbf{a} = m\frac{v^2}{r} = q\,v\,\mathbf{B}. \quad ...(3)$$

$ma = \frac{mv^2}{r}$

$a = \frac{v^2}{r}$

From eq. (3)
$$m\mathbf{v} = q\,\mathbf{B}\,r \text{ or } \mathbf{r} = \frac{m\,v}{q\,\mathbf{B}} \quad ...(4)$$

$qvB = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$

The angular velocity ω is given by

$$\boxed{\omega = \frac{q\,B}{m}} \text{ or } \boxed{f = \frac{\omega}{2\pi} = \frac{q\,B}{2\pi m}} \quad ...(5)$$

$\omega = \frac{qB}{m}$

The frequency is dependent of v. This frequency is known as **cyclotron frequency**.

Here, it should be remembered [eq. (4)] that faster particles move in bigger circle and slower particles move in smaller circle such that time period T is the same.

The time period T is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{q\,B} \quad ...(6)$$

$\boxed{T = \frac{2\pi m}{qB}}$

(4) Here we shall consider a general case in which the charged particle makes an angle θ with the direction of **B** as shown in fig. (11a). In this case we resolve the velocity in two components, *i.e.*, v_x along **B** and v_y normal to **B**. As v_x is along **B**, *i.e.*, angle between v_x and B is zero hence v_x is unaffected by the field. As a result the particle will move in the direction of **B** with constant velocity. Since v_y is perpendicular to **B**, the particle will describe a circular path in Y-Z plane [fig. (11b)]. The radius of the circle is given by the expression

$$R = \frac{m\,v_y}{q\,B}.$$

$R = \frac{mv}{qB}$

$qvB = \frac{mv^2}{R}$

$\boxed{R = \frac{mv}{qB}}$

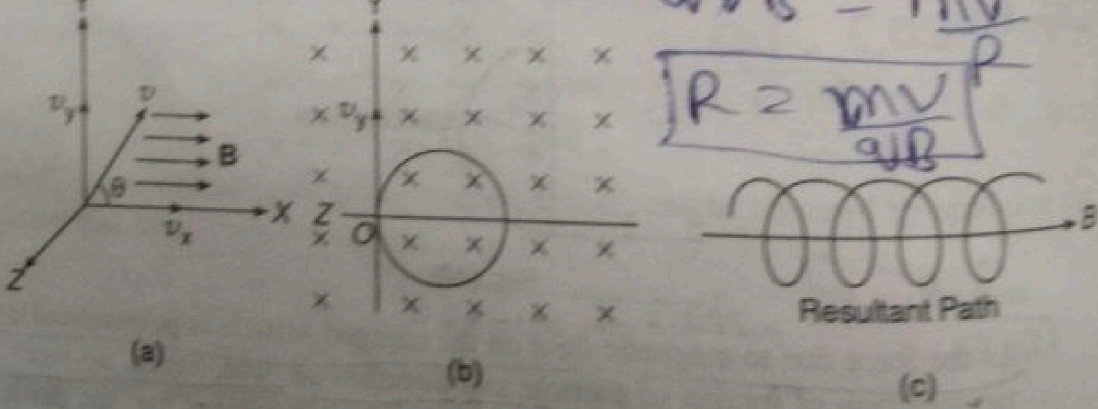


Fig. (11)

The resultant path of the particle is a helix as shown in fig. (11c). This is superposition of two motions, one of constant velocity v_x along **B** and the second a circular motion perpendicular to **B**. The pitch of the helix is defined as the distance between two successive circular paths. Hence,

Pitch p = velocity × time to describe a circular path

$$= v_x \times \frac{2\pi m}{qB} = \frac{2\pi m v_x}{qB}$$

Pitch = Velo x Time
P = Vx x 2πm

The component of Lorentz force in the direction of field is zero because VB

$$\mathbf{F}_x = q\,(\mathbf{v}_x \times \mathbf{B}) = 0$$

so v_x remains constant

Further,

$$\mathbf{F}_y = \frac{dv_y}{dt} = q\,(\mathbf{v}_y \times \mathbf{B})$$

The angle between $\mathbf{v}_y$ and **B** remains constant throughout the motion and is equal to $\frac{\pi}{2}$.

3.13 MOTION OF CHARGED PARTICLES IN CROSSED ELECTRIC AND MAGNETIC FIELDS

Here we shall consider the motion of a particle of charge q and mass m placed in electric and magnetic fields, *i.e.*, both fields act simultaneously on the particle. Suppose the electric field **E** is along X-axis while the magnetic field **B** is along Y-axis. Due to electric field a force q **E** acts on the particle along X-axis. The force due to magnetic field will be $q\,(\mathbf{v} \times \mathbf{B})$, where **v** is the velocity of the particle. The direction of this force will be perpendicular to **B** as well as **v**. The resultant force known as ***Lorentz force*** acting on the particle is given by

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad \text{...(1)}$$

F = q(E + VxB)

Here $\mathbf{E} = \mathbf{i}\,E, \quad \mathbf{B} = \mathbf{j}\,B$

and $\mathbf{v} = \mathbf{i}\,v_x + \mathbf{j}\,v_y + \mathbf{k}\,v_z$

where **i**, **j** and **k** are the unit vectors along the three axes respectively. Substituting this values, we get

$$\mathbf{F} = q\,[\mathbf{i}\,E + (\mathbf{k}\,v_x B - \mathbf{i}\,v_z B)]$$

$$\because \quad \mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ 0 & B & 0 \end{vmatrix} = \mathbf{k}\,v_x B - \mathbf{i}\,v_z B$$

$$\therefore \quad \mathbf{F} = \mathbf{i}\,q\,[E - v_z B] + \mathbf{k}\,q\,v_x B$$

or

$$m\frac{d\mathbf{v}}{dt} = \mathbf{i}\,q\,[E - v_z B] + \mathbf{k}\,q\,v_x B \qquad \text{...(2)}$$

$$F_x = m\frac{dv_x}{dt} = q\,(E - v_z B)$$

and

$$F_z = m\frac{dv_z}{dt} = q\,v_x B$$

Differentiating w.r.t. time, we get

$$m\frac{d^2 v_x}{dt^2} = -q\,B\,\frac{dv_z}{dt}$$

$m\frac{d^2v}{dt} = -qB - \frac{dv}{dt}$

$$= -q\,B\left(\frac{q\,v_x B}{m}\right) \qquad \left[\because \frac{dv_z}{dt} = q\,v_x B/m\right]$$

or

$$\frac{d^2 v_x}{dt^2} + \frac{q^2 B^2}{m^2}\,v_x = 0 \qquad ...(3)$$

$\frac{d^2v}{dt^2} + \frac{q^2B^2}{m^2}v_x = 0$

This is a general equation of simple harmonic motion where

$$\boxed{\omega^2 = q^2 B^2/m^2} \text{ or } \boxed{\omega = q\,B/m}$$

The solution of eq. (3) is given by

$$v_x = A\,\sin(\omega t + \phi)$$

where A is the amplitude and ϕ is the phases of S.H.M. At $t = 0, v_x = 0$, hence, $\phi = 0$. So

$$v_x = A\,\sin\omega t \qquad ...(4)$$

To obtain the value of A, we differentiate eq. (4)

$$\therefore \qquad \frac{dv_x}{dt} = A\,\omega\cos\omega t$$

At time $t = 0$, $\frac{dv_x}{dt} = A\,\omega$. But this is same as $q\,E/m$

$$\therefore \qquad A\,\omega = q\,E/m \quad \text{or} \quad A = q\,E/m\,\omega = E/B$$

Hence, the solution of eq. (3) becomes

$$\boxed{v_x = (E/B)\sin\omega t} \qquad ...(5)$$

Now $\quad F_x = m\frac{d}{dt}v_x = m\left(\frac{E}{B}\right)\omega\cos\omega t = q\,(E - v_z B)$

or

$$m\left(\frac{E}{B}\right)\omega\cos\omega t = q\,E - q\,v_z B$$

or

$$m\left(\frac{E}{B}\right)\left(\frac{q\,B}{m}\right)\cos\omega t = q\,E - q\,v_z B$$

$$\boxed{v_z = \frac{E}{B}[1 - \cos\omega t]} \qquad ...(6)$$

Integrating eq. (5), we get

$$x = \frac{E}{B\,\omega}(-\cos\omega t) + C_1$$

Applying the boundary condition that at $t=0, x=0$, we get

$$0=-\frac{E}{B\omega}+C_1 \quad \text{or} \quad C_1=\frac{E}{B\omega}$$

$$\therefore \quad \boxed{x=\frac{E}{B\omega}[1-\cos\omega t]} \quad \ldots(7)$$

Similarly, integrating eq. (6) and applying the boundary condition that at $t=0$, $z=0$, we get

$$\boxed{z=\frac{E}{B\omega}[\omega t-\sin\omega t]} \quad \ldots(8)$$

Eqs. (7) and (8) gives the displacements along x and z axes.

Important Points.

(1) When a charged particle is subjected simutlaneously to both the electric field **E** and magnetic field **B**, it experiences electric force $q\,\mathbf{E}$ and magnetic force $q\,(\mathbf{v}\times\mathbf{B})$. The resultant force **F** is givenby

$$F=q\,[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]$$

$F = q(E + V \times B)$

This is known as ***Lorentz-force.***

(2) Depending on the direction of **v, E** and **B**, a number of situations are possible. Here we consider the following special cases :

(i) **v, E** *and* **B** *all are collinear.*

In this situation, the particle is either moving parallel or antiparallel to the fields. The magnetic force on the charged particle will be zero. There will be an electric force on the particle which produces an acceleration **a** given by

$$\mathbf{a}=q\,\mathbf{E}/m$$

The direction of **a** is along the electric field. So, in this situation speed, velocity, momentum and kinetic energy all will change, of course, the direction of motion remains the same as shown in fig. (12).

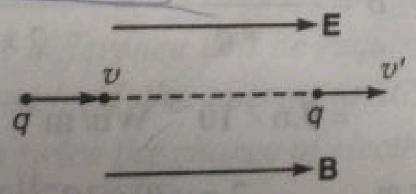


Fig. (12)

(ii) **v, E** *and* **B** *all are mutually perpendicular.*

The situation is shown in fig. (13). In this situation **E** and **B** are such that

$$\mathbf{F}=\mathbf{F}_e+\mathbf{F}_m=0,$$

i.e., acceleration, $\mathbf{a}=(\mathbf{F}/m)=0$

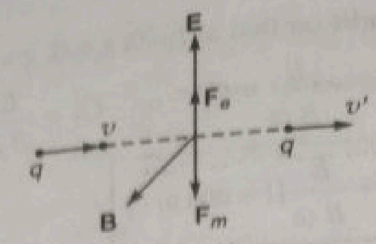


Fig. (13)

Therefore, the particle will pass through the fields without any change in its velocity.

As $$\mathbf{F}_e = \mathbf{F}_m \quad \text{so} \quad q\,E = q\,v\,B$$

$\therefore$ $$v = E/B$$

This principle is used in ***velocity selector*** to get a charged beam with a specific velocity.

NUMERICAL EXAMPLES

Example 1 *An electron moving with a velocity* 10^8 metre per second *enters a magnetic field at an angle of* $20°$ *to the direction of the field. Calculate*

(a) the value of magnetic induction so that the helical path radius be 2 metre.

(b) the time required to execute one revolution on the helical path.

(c) the pitch of the spiral.

Solution (a) The force acting on a charged particle in magnetic field is given by

$F_m = q\,v\,B = \text{mass} \times \text{acceleration} = m_e \times a$

$\therefore$ $$a = \frac{v^2 \sin^2\theta}{r}$$

$\therefore$ $$m_e a = m_e v^2 \sin^2\theta / r = q\,v\,B \sin\theta$$

or $$B = \frac{m_e v \sin^2\theta}{r\,q} = \frac{9\times10^{-31}\times10^{8}\times\sin 20}{2\times(1.6\times10^{-19})}$$

$$= \mathbf{9.6\times10^{-5}\ Wb/m^2}$$

(b) $$T = \frac{2\pi m_e}{B\,q} = \frac{2\pi\times(9\times10^{-31})}{(9.6\times10^{-5})(1.6\times10^{-19})}$$

$$= \mathbf{3.67\times10^{-7}\ sec.}$$

(c) Velocity component parallel to magnetic field

$$v_y = v\cos 20° = 10^8 \times 0.9397$$

$$= 9.397\times10^{7}\ \text{m/sec}$$

$\therefore$ Pitch of the spiral =

$= 9.397 \times 10^7 \times 3.97 \times 10^{-7}$

$=$ **34.51 metre.**

...ample 2 *A* 15000 eV *electron is describing a circle in a uniform field of magnetic ...ction of* 250 gauss *acting at right angle to it. Compute the radius of the circle.* (m = 9.1 × 10^{-31} kg, $e = 1.6 \times 10^{-19}$ coulomb, 1 eV = 1.6×10^{-19} joule).

...ution Given that

Kinetic energy of electron = 15000 eV

$$\therefore \quad \frac{1}{2} m v^2 = 15000 \text{ eV}$$

$$= 15000 \times (1.6 \times 10^{-19}) \text{ joule}$$

$$= 2.4 \times 10^{-15} \text{ joule}$$

$$v = \sqrt{\left(\frac{2 \times (2.4 \times 10^{-15})}{9.1 \times 10^{-31}}\right)}$$

$$= 7.3 \times 10^7 \text{ m/sec}$$

We know that $\quad \mathbf{F} = e\,\mathbf{v} \times \mathbf{B}$

or $\quad F = e v B \sin\theta = e v b \qquad (\because \theta = 90°)$

Now $\quad F = e v B = \dfrac{m v^2}{r} \quad$ or $\quad r = \dfrac{m v}{e B}$

or
$$r = \frac{(9.1 \times 10^{-31})(7.3 \times 10^7)}{(1.6 \times 10^{-19}) \times (250/10^4)} \qquad \left(\because 1 \frac{\text{weber}}{\text{metre}^2} = 10^4 \text{ gauss}\right)$$

$$r = 0.0166 \text{ metre} = \mathbf{1.66\ cm}$$

...ample 3 *An* α*-particle is describing a circle of radius* 0.45 metre *in a field of ...netic induction of* 1.2 weber/metre^2. *Find its speed, frequency of rotation and ...etic energy. What potential difference will be required which will accelerate the ...ticle so as to give this much of the energy to it? The mass of* α*-particle is* ...$\times 10^{-27}$ kg *and its charge is twice the charge of electron, i.e.,* 3.2×10^{-19} coulomb.

...ution We know that

$$F = e v B = \frac{m v^2}{r} \quad \text{or} \quad v = \frac{e B r}{m}$$

Substituting the given values, we get

$$\frac{(3.2 \times 10^{-19} \text{ coulomb})(1.2 \times \text{weber/m}^2)(0.45 \text{ metre})}{(\ldots \times 10^{-27} \text{ kg})}$$

The frequency of rotation is given by

$$\nu = \frac{v}{2\pi r} = \frac{2.6 \times 10^7 \text{ m/sec}}{2 \times 3.14 \times (0.45 \text{ m})}$$

$$= \mathbf{9.2 \times 10^6 \text{ per sec.}}$$

The kinetic energy of α-particle is given by $K = \frac{1}{2} m v^2$

$$= \frac{1}{2}(6.8 \times 10^{-27} \text{ kg}) \times (2.6 \times 10^7 \text{ m/sec}^2)$$

$$= 2.3 \times 10^{-12} \text{ joule}$$

$$= \frac{2.3 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \qquad (\because 1\text{eV} = 1.6 \times 10^{-19} \text{ joule})$$

$$= 14 \times 10^6 \text{ eV} = \mathbf{14 \text{ MeV}}.$$

The electron will acquire this amount of energy (14 MeV) when it is accelerated through a potential difference of 14×10^6 volt. Since, α-particle carries a charge twice that of an electron it will acquire this energy when accelerated through half of the potential difference of electron, *i.e.*, through $\mathbf{7 \times 10^6}$ **volt.**

Example 4 *A proton (charge* 1.6×10^{-19} C, *mass* $m = 1.67 \times 10^{-27}$ kg) *is shot with a speed* 8×10^6 m/s *at an angle of* 30° *with the X-axis. A uniform magnetic field* $B = 0.30$ T *exists along the X-axis. Show that path of the proton is a helix. Find the radius and pitch of the helix.*

Solution The situation is shown in fig. (14)

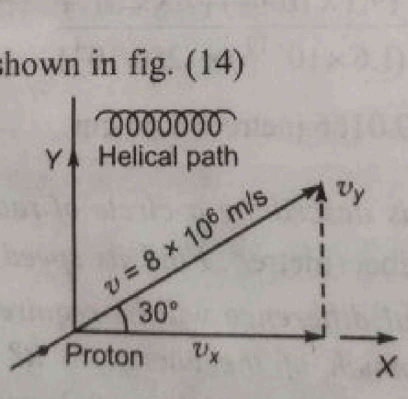


Fig. (14)

$$v_x = v \cos 30° = (8 \times 10^6)(0.866) = 6.93 \times 10^6 \text{ m/s}$$

$$v_y = v \sin 30° = (8 \times 10^6) \times (0.5) = 4.0 \times 10^6 \text{ m/s}$$

the magnetic force due to v_x

$F = q v_x B \sin 0° = 0$ (B and v_x are along the same direction)

Due to combined action of v_x, v_y and B, the proton moves in a helical path.

The magnetic force due to v_y has no x-component. So, the motion along x-axis is uniform at speed v_x.

The radius of circular transverse motion is

$$r=\frac{m v_y}{qB}=\frac{(1.67\times10^{-27})(4\times10^{6})}{(1.6\times10^{-19})\times0.3}=0.139\text{ m}$$

∴ Radius of the helix = **0.139 m**

Time taken to complete one circle

$$T=\frac{2\pi r}{v_y}=\frac{2\times3.14\times0.139}{4\times10^{6}}=2.19\times10^{-7}\text{ sec}$$

The pitch of the helix = Distance travelled by the proton along X-axis in time T

$$=v_x\times T$$

$$=(6.93\times10^{6})\times(2.19\times10^{-7})$$

$$=\mathbf{1.515\ m}$$

Example 5 *A particle of mass* 1×10^{-26} kg *and charge* $+1.6\times10^{-19}$ C *travelling with velocity* 1.28×10^{6} m/s *in the* $+x$ *direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that* $E_x=E_y=0$, $E_z=-102.4$ kV/m *and* $B_x=B_z=0$, $B_y=8\times10^{-2}$ Wb m^{-2}. *The particle enters this region at the origin at time* $t=0$. *Determine the location x, y and z coordinates of the particle at* $t=5\times10^{-6}$ s. *If the electric field is switched off at this instant (with magnetic field still present), what will be the position of the particle at* $t=7.45\times10^{-6}$ s?

Solution Velocity of particle in $+x$ direction $=1.28\times10^{6}$ m/sec.

$$E_x=E_y=0,\ E_z=-102.4\text{ kV/m}$$

$$B_x=B_z=0,\ B_y=8\times10^{-2}\text{ Wb/m}^2$$

These are shown in fig. (15)

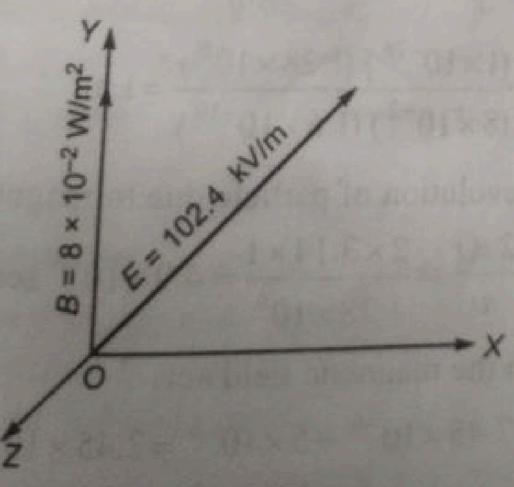


Fig. (15)

Electric force $\mathbf{F}_e$ on the charge is given by

$$(1.6\times10^{-19})(102.4\times10^{3})$$

$$= 163.84 \times 10^{-16} \text{ along } -z \text{ direction}$$

Magnetic force $\mathbf{F}_m$ on the charge is given

$$|\mathbf{F}_m| = q\,v\,B$$
$$= (1.6 \times 10^{-19})\,(1.28 \times 10^{6})\,(8 \times 10^{-2})$$
$$= 163.84 \times 10^{-16} \text{ along } +z \text{ direction}$$

Thus, there are two equal, opposite and collinear force acting on the particle along z-axis. Hence, the resultant force on particle is zero. The particle moves thus along the X-axis without deflection.

(a) At time $t = 5 \times 10^{-6}$ second, the distance x travelled by the particle.

$$x = v\,t = (1.28 \times 10^{6})\,(5 \times 10^{-6}) = 6.4 \text{ m}$$

$\therefore$ Coordinated of the particle = **(6.4, 0, 0)**

(b) When the electric field is switched off, there will be a force 163.84×10^{-16} along $+z$ axis acting on the particle. The particle moves in uniform magnetic field and describes a circular path in X-Z plane as shown in fig.(16). We know that when a particle of mass m and charge q is subjected to a magnetic field B acting perpendicular

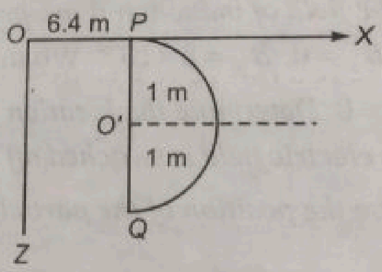


Fig. (16)

to it, the particle moves with velocity v in a circular trajectory whose radius r is given by

$$B\,q\,v = \frac{m\,v^2}{r} \quad \text{or} \quad r = \frac{m\,v}{B\,q}$$

$$\therefore \quad r = \frac{(1 \times 10^{-26})\,(1.28 \times 10^{6})}{(8 \times 10^{-2})\,(1.6 \times 10^{-19})} = 1 \text{ m}$$

The time period t of revolution of particle due to magnetic field is given by

$$t = \frac{2\,\pi\,r}{v} = \frac{2 \times 3.14 \times 1}{1.28 \times 10^{6}} = 4.9 \times 10^{-6} \text{ sec}$$

The time during which the magnetic field acts

$$= 7.45 \times 10^{-6} - 5 \times 10^{-6} = 2.45 \times 10^{-6} \text{ sec.}$$

As the time period is 4.9×10^{-6} sec, and the magnetic field acts only for a time 2.45×10^{-6} sec, *i.e.*, for half the time period hence the particle covers a distance PQ, *i.e.* 2 metre along Z direction.

The coordinates of particle are (6.4 m, 0, 2 ...

Example 6 *A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 120$ kV/m and $B = 50$ mT. Then the beam strikes a ground target. Find the force with which the beam acts on the target if the beam current is equal to $i = 0.8$ mA.*

Solution Here the proton beam passes without deviation, hence

$$B\,e\,v = e\,E$$

$$\therefore \quad v = \frac{E}{B} = \frac{120 \times 10^{3}}{50 \times 10^{-3}} = 2.4 \times 10^{6} \text{ m/s}$$

number of protons reaching per second

$$\frac{n\,q}{t} = 0.8 \times 10^{-3}$$

or
$$\frac{n \times 1.6 \times 10^{-19}}{1} = 0.8 \times 10^{-3}$$

$$n = 5 \times 10^{15}$$

Total mass of proton

$$= (5 \times 10^{15}) \times (1.67 \times 10^{-27}) = 8.35 \times 10^{-12}$$

$$\text{Force} = \frac{\text{Change in momentum}}{\text{sec}}$$

$$= [(8.35 \times 10^{-12})\,(2.4 \times 10^{6})]/1$$

$$= 20.04 \times 10^{-6} \text{ N} = \mathbf{20.04\,\mu\,N}$$

Example 7 *Uniform electric and magnetic fields with strength E and induction B respectively are directed along Y-axis as shown in fig. (17). A particle with specific charge q/m leaves the origin O in the direction of X-axis with an initial non-relativistic velocity v_0. Find*

(i) the coordinate y_n of the particle when it crosses the Y-axis for the nth time;

(ii) the angle α between the particles' velocity vector and Y-axis at that moment.

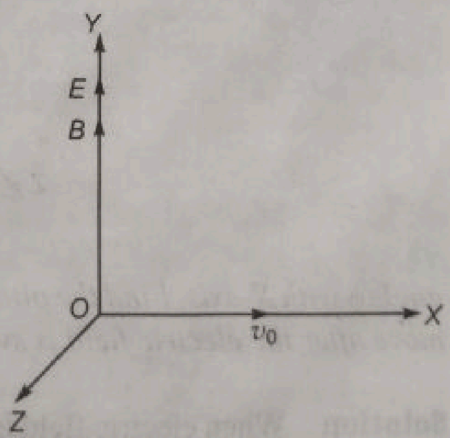


Fig. (17)

Solution (i) There will be an acceleration a_y on the particle due to electric force which acts along positive Y-direction.

Acceleration
$$a_y = \frac{q\,E}{m}$$

Let, the particle crosses Y-axis nth time after t second. Hence

$$y_n = \frac{q}{2}\frac{E}{m}t^2 \qquad ...(1)$$

If n be the number of rotaton, then

$$t = n \times T = \frac{2\pi m n}{q B} \qquad ...(2)$$

From eqs. (1) and (2), we get

$$y_n = \frac{q}{2}\frac{E}{m} \times \left(\frac{2\pi m n}{q B}\right)^2 = \frac{2\pi^2 m n^2 E}{q B^2}$$

(ii)

$$v_y = a_y \times t$$

$$= \left(\frac{q E}{m}\right) \times \left(\frac{2\pi m n}{q B}\right) = \frac{2\pi n E}{B}$$

$$\tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi n E}$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi n E}\right)$$

Example 8 *Non-relativistic protons move rectilinearly in the region of space where there are uniform mutually perpendicular electric and magnetic fields with E and B. The trajectory of the protons lie in the plane X-Y as shown in* fig. (18) *and forms an*

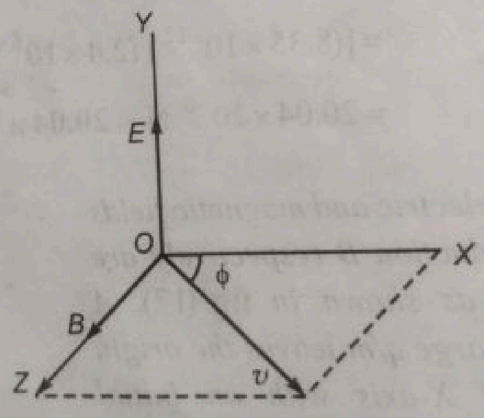


Fig. (18)

angle ϕ with X-axis. Find the pitch of the helical trajectory along which the protons will move after the electric field is switched off.

Solution When electric field is switched off, the path followed by the particle will be helical.

$$\text{Pitch} = v_{\parallel} T \qquad ...(1)$$

where $v_{\parallel}$ = velocity of the particle parallel to **B**, and

$$T = \text{time of revolution} = \left(\frac{2\pi m}{q B}\right)$$

Here $v_{\parallel} = v \cos(90 - \phi) = v \sin \phi$

Hence from eq. (1)

$$\text{Pitch} = (v \sin \phi)\left(\frac{2\pi m}{q B}\right) \qquad ...(2)$$

When both fields are present

$$q E = q v B \sin(90 - \phi) = q v B \cos \phi$$

or

$$v = \frac{E}{B \cos \phi} \qquad ...(3)$$

Substituting the value of v from eq. (3) in eq. (2), we get

$$\text{Pitch} = \left(\frac{E}{B \cos \phi} \times \sin \phi\right)\frac{2\pi m}{q B} = \frac{2\pi m E}{q B^2} \tan \phi$$

3.14 ASTON'S MASS SPECTROGRAPH

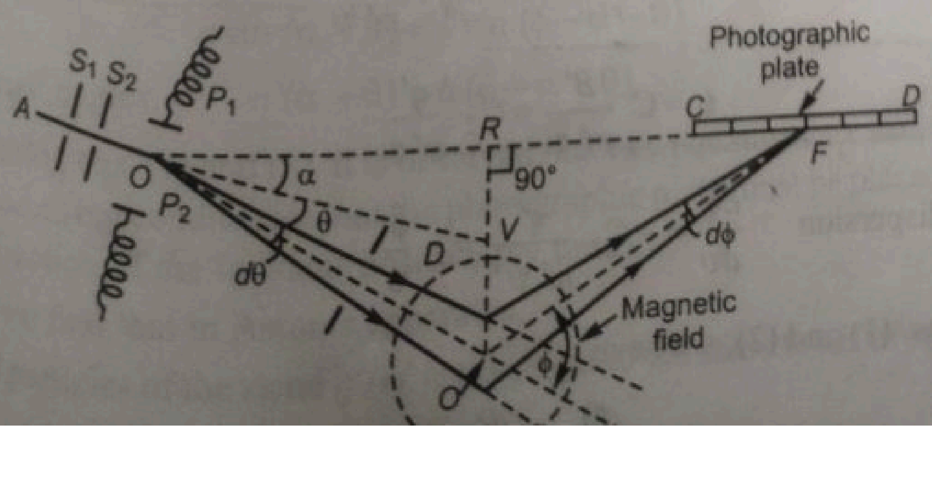
Aston's mass spectrograph is an apparatus of high accuracy designed by Aston of Cambridge University, which enables the measurement of the mass of single ions and is useful for the investigation of isotopes. This method is an improvement on J.J. Thomson's method.

Principle

The positive rays emerging from perforated cathode are made into a fine pencil by using slits. They are then subjected to an electrostatic field in a direction perpendicular to the direction of rays with the help of electrically charged plates P_1 and P_2. The beam is not only deflected but also dispersed because the particles are having different velocities. The dispersed beam is then subjected to a magnetic field whose direction is perpendicular to the direction and deviation in an opposite direction but in the same plane. If a photographic plate is held in the direction of deflected beam, line images are obtained. Each line corresponds to particular value of q'/m'. The number of lines correspond to the number of isotopes present in the element.

Theory

The different parts of the Aston's mass spectrograph are shown in fig. (19). AO is the direction of positive rays before entering the electrostatic field. S_1 and S_2 are slits



which provides a fine pencil of positive rays. The electrostatic field is maintained by plates P_1 and P_2 in a direction perpendicular to the beam. As all the positive ions have same specific charge (q'/m'), they are deflected towards the negative plate P_2. Let, θ and $d\theta$ be the angles of deviation and dispersion (due to different velocities). Using a diaphram D some of the rays are selected and are allowed to pass between the poles of an electromagnet. The magnetic field being perpendicular to the plane of the paper and inward. According to the Fleming's left hand rule, the beam will be deflected upwards. This magnetic field annuls the dispersion produced by electric field and recombines the particle which are brought to focus in the form of sharp lines on a photographic plate CD. The lines are similar to those spectral lines.

Let, q' = charge on positive ray particle,

m' = mass of each particle,

E = electrostatic field,

B = magnetic field strength,

v = velocity of each particle,

ϕ = angle of deviation produced by magnetic field,

$d\phi$ = angle of dispersion produced by magnetic field,

Considering that the deflection in electrostatic field is small, the curve near the vertex may be considered as circular of radius r, we have

$$E\,q' = \frac{m'\,v^2}{r} \quad \text{or} \quad \frac{1}{r} = \frac{E\,q'}{m'\,v^2}$$

Hence, the deflection θ, which is proportional to $1/r$ is given by

$$\theta = C\,\frac{E\,q'}{m'\,v^2} = C_1\,\frac{q'}{m'\,v^2} \qquad \left(\begin{array}{l}\text{where } C_1 = C\,E \\ \text{because } E = \text{constant}\end{array}\right)$$

$$\therefore \text{ Dispersion} \qquad \frac{d\theta}{dv} = -2\,C_1\,\frac{q'}{m'\,v^3} = -2\,\frac{\theta}{v} \qquad \text{...(1)}$$

If r' be the radius of curvature in magnetic field, then

$$B\,q'\,v = \frac{m'\,v^2}{r'} \quad \text{or} \quad \frac{1}{r'} = \frac{B\,q'}{m'\,v}$$

$$\therefore \qquad \phi = C'\,\frac{B'}{m'\,v} = C_2\,\frac{q'}{m'\,v} \qquad (\because B \text{ is constant})$$

$$\text{Again dispersion} \qquad \frac{d\phi}{dv} = -C_2\,\frac{q'}{m'\,v^2} = -\frac{\phi}{v} \qquad \text{...(2)}$$

From eqs. (1) and (2), we have

$$\frac{d\theta}{\theta} = 2\,\frac{d\phi}{\phi} \qquad \text{...(3)}$$

Thus, for a given deflection, the dispersion due to the electric field is twice that due to magnetic field. The small changes d and $d\phi$ refer to the particles with identical mass and charge but possessing velocities differing by dv.

In the absence of magnetic field, the dispersion produced in the beam for a distance $(a+b)$ is given by

$$= (a+b)\, d\theta$$

where a = distance OO' and b = distance $O'F$ ...(4)

The magnetic field acts in a direction perpendicular to the electric field and produces the same dispersion in a distance b but in the opposite direction.

Dispersion produced by the magnetic field $= b\, d\phi$ ...(5)

As all the ions are focussed to the same position

$$(a+b)\, d\theta = b\, d\phi$$

and

$$\frac{d\theta}{d\phi} = \frac{b}{(a+b)} \quad ...(6)$$

$\frac{d\theta}{\theta} = \frac{2\,d\phi}{\phi}$

From Eq. (3)

$$\frac{d\theta}{d\phi} = \frac{2\theta}{\phi}$$

$\frac{d\phi}{d\theta} = \frac{2\theta}{\phi}$

$$\therefore \quad \frac{b}{(a+b)} = \frac{2\theta}{\phi} \quad \text{or} \quad b\phi = (a+b)\,2\theta$$

$\frac{2\theta}{\phi} = \frac{b}{(a+b)}$

$$b(\phi - 2\theta) = 2a\theta$$

or

$$\boxed{(b/a) = \{2\theta/(\phi - 2\theta)\}} \quad ...(7)$$

$b\phi = (a+b)2\theta$

This is the condition of focussing.

Let, $O'R$ be the perpendicular to the line CD produced and $\angle ROV = \alpha$. Then from $\Delta ROO'$, we have

$b(\phi - 2\theta) = 2a\theta$

$$RO' = OO' \sin(\alpha + \theta) = a \sin(\alpha + \theta)$$

$\frac{b}{a} = (2\theta/\phi - 2\theta)$

In $\Delta RO'F$,
$$RO' = O'F \sin(RFO')$$
$$= b \sin[180° - (\phi - \alpha - \theta)] = b \sin(\phi - \alpha - \theta)$$

$$\therefore \quad a \sin(\alpha + \theta) = b \sin(\phi - \alpha - \theta) \quad ...(8)$$

For small angles, $\quad a(\alpha + \theta) = b(\phi - \alpha - \theta)$

Comparing eqs. (7) and (8), it is observed that two equations are same when $\alpha = \theta$. Thus, the focussing condition is that the photographic plate must be placed at an angle θ with the direction of the incident positive ray beam.

Thus, we find that in Aston's apparatus,

(i) All particles of the same q'/m' are brought to the same focus irrespective of the velocities.

(ii) P... are brought to different foci.

(iii) The intensity of a line in the mass spectrograph is proportional to the number of particles. Hence, ***the relative abundance of the various isotopes can be estimated by this method.***

(iv) The velocity dispersion of the ionic beam is greater in Aston's mass spectrograph than in Thomson method.

Detection of Isotopes

When a beam of positive rays contains ions having different values of e/m, then a number of sharp images of the slit are obtained on the plate CD. Each image corresponds to a particular value of e/m. Thus, the various isotopes present could be detected.

3.15 BAINBRIDGE MASS SPECTROGRAPH

A recent type of mass spectrograph, having high resolving power is due to Bainbridge. The Bainbridge mass spectrograph is shown in fig. (20).

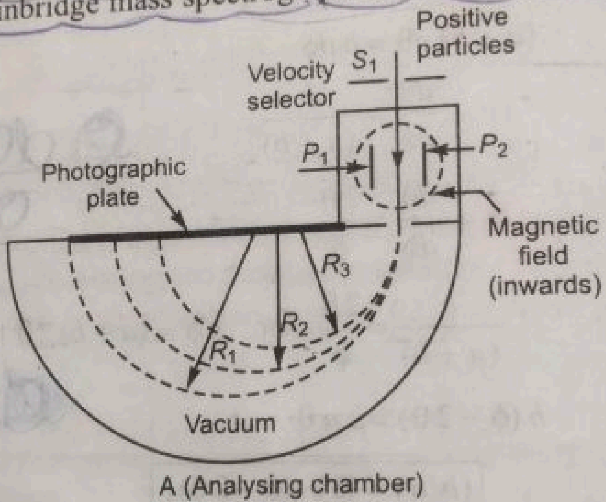


Fig. (20)

Construction and Working

In this mass spectrograph, a beam of positive particles from a discharge tube, obtained in the form a fine pencil of beam, are allowed to pass through a velocity selector. The velocity selector is a device in which electric and magnetic fields are applied in the same region. The electric fields is in the plane of the paper while the magnetic field is at right angle to it but in the plane of paper. The positive particles tend to be deflected towards the negative plate by the electric field while towards the positive plate due to magnetic field. If E is the electric field strength and B be magnetic field strength, then force experienced by the positive particle due to electric field $= E\,q'$ (q' = charge on the particle) and force experienced by it due to magnetic field $= B\,q'v$ (where v = velocity of the particle).

As these forces are equal and opposite

$$B\,q'v = E\,q' \quad \text{or} \quad \boxed{v = \frac{E}{B}} \qquad ...(1)$$

Thus, the positive particles having this velocity will not be deviated while passing through the velocity selector. All other particles will be deflected to one or the other side and will be removed from the stream. Now the particles having the same velocity emerge from velocity selector through a slit and enter into an analysing chamber A. Here again a magnetic field of strength B' is applied perpendicular to the plane of paper and outwards. Due to this magnetic field, the particles emerge along circular paths and fall on the photographic plate. The trace lines on photographic plate called the mass spectrum. All the particles having same value of q'/m' traverse a semi-circular paths of radius r is given by

$$\frac{m' v^2}{r} = B' q' v \quad \text{or} \quad r = \frac{m' v^2}{B' q' v}$$

$$r = \frac{m' v}{B' q'}$$

Here, B' and v are constants, hence ...(2)

$$r = K\, m' \text{ (for particles have same charge } q')$$

Uses

Thus, the positive particles having same value of q'/m' are focussed on the same line while the positive particles having different values of q'/m' are focussed on different positions of photographic plate. Also, by comparing the positions of the line produced by a particle of unknown mass, with the position of the trace produced by a particle of a known mass, the unknown mass can be determined.

Numerical Examples

Example 1 *The distance between the traces corresponding to mass 12 and 16 in Aston's mass spectrograph is 4.8 cm. Calculate the mass of the particle whose trace is at a distance of 8.4 cm from the trace of mass 16.*

Solution Let, x be the linear distance of the trace from the fiduciary mark. Then for M.

$$x \propto 16 \quad \text{and} \quad x - 4.8 \propto 12$$

So

$$\frac{x}{x-4.8} = \frac{16}{12} \quad \text{or} \quad x = 19.2\,\text{cm}$$

Let, M be the mass of the particle whose trace is at a distance of 8.4 cm from the trace of 16, we have

$$19.2 \pm 8.4 \propto M$$

Further

$$\frac{19.1 \pm 8.4}{x} = \frac{M}{16} \quad \text{or} \quad \frac{19.2 \pm 8.4}{19.2} = \frac{M}{16}$$

or

$$\frac{M}{16} = 1 \pm \frac{8.4}{19.7} \quad \text{or} \quad M = 16 \pm 7$$

Example 2 *In an Aston's mass spectrograph, the trace of beryllium of mass 9 is obtained with a potential of 450 V between the plates. What should be the potential that should be applied if the trace of lithium of mass 7 be formed in the same position when the magnetic field is kept the same?*

Solution In this case, we have $x_1 \propto 9$ and $x_2 \propto 7$

$$\therefore \qquad \frac{x_2}{x_1} = \frac{7}{9} \qquad \ldots(1)$$

When the distance between the plates is constant, the linear distances are proportional to the electric field intensities, *i.e.*,

$$\frac{x_2}{x_1} = \frac{V_2}{V_1} \qquad \ldots(2)$$

From Eqs. (1) and (2), we get

$$\frac{V_2}{V_1} = \frac{7}{9} \quad \text{or} \quad V_2 = \frac{7}{9} V_1 = \frac{7}{9} \times 450 = \mathbf{350\ V}$$

Hence, to get the trace at the same point, the P.D. between the plates must be 350 V.

Example 3 *In a Bainbridge mass spectrograph, single ionised atoms of* Ne^{20} *pass into the deflection chamber with a velocity of* 10^5 m/sec. *If they are deflected by a magnetic field of flux density* 0.08 tesla, *calculate the radius of their path. Where would the* Ne^{22} *ions would fall, if they had the same initial velocity?*

Solution We know that $B\,q'\,v = m\,v^2/r$

For the ion of Ne^{20}

$$B\,e\,v = \frac{m\,v^2}{r_{20}} \quad \text{or} \quad r_{20} = \frac{m\,v}{B\,e}$$

Substituting the given values, we have

$$r_{20} = \frac{(20/6.023 \times 10^{26})\ \text{kg}\ (10^5)\ \text{m/sec}}{0.08\ \text{tesla} \times 1.602 \times 10^{-19}\ \text{coulomb}}$$

$$= 0.26\ \text{metre}$$

Now, r_{22} is given by

$$\frac{r_{22}}{r_{20}} = \frac{22}{20} \quad \text{or} \quad r_{22} = \frac{22}{20} \times r_{20}$$

$$\therefore \qquad r_{22} = \frac{22}{20} \times 0.26 = \mathbf{0.286\ metre}$$

Further $\qquad 2\,r_{22} - 2\,r_{20} = 2\,(0.286 - 0.26) = 2 \times 0.026$

$$= \mathbf{0.052\ m}$$

Hence, the Ne ions would fall 0.052 metre of 5.2 cm away from Ne^{20} ions.