Regular Expression

* Definition: An α-numeric or algebraic format for which some set of strings are possible is known as regular expression (RE).

Regular set:

set of values derived from any R.E.

Properties of R.E.:-

1) Closure property: (Closure means repeatation)

(i) Kleene closure (*): all combination of i/p string which contain ε.

format:

$$a^* = \sum_{i=0}^{n} a^i$$

Kleene closure of a is a^*

$$a^* = \{a^0, a^1, a^2 \, ---- \, a^n\}$$
$$a^* = \{\epsilon, a, aa, \, --- \, a^n\}$$

(ii) Positive closure (+): All combination of i/p string which does not contain ε.

$$\boxed{a^+ = \sum_{i=1}^{n} a^i}$$

$$a^+ = \{a^1, a^2, a^3 \, --- \, a^n\}$$
$$a^+ = \{a, aa, aaa \, --- \, a^n\}$$

2) Association Property:

$a \cup b$
a , b
$a + b$
a / b

for ex:- $0 + 1 = \{0, 1\}$

3) Concatenation : (Multiplication)

$a \cap b$
$a \cdot b$
ab

for ex:- $0 \cdot 1 = \{01\}$

4) (change) Commutative : Association is commutative but concatenation is non - commutative.

$a + b = b + a$
$ab \neq ba$

for ex:- $0 + 1 = 1 + 0 = 1$
$01 \neq 10$

5) $\boxed{aa^* = a^+ = a^* a}$

Proof: L.H.S. $= aa^*$

$= \{a\} \cap \{\epsilon, a, aa, aaa, ---, a^n\}$
$= \{a, aa, aaa, -----, a^n\}$
$= a^+$

$$L.H.S. = R.H.S.$$

$$\therefore \boxed{aa^* = a^+}$$

6) $\boxed{a\epsilon = a}$

$\boxed{\epsilon a = a}$

$\boxed{a + \phi = a}$

7) $\boxed{a^+ + \epsilon = a^*}$

Proof:

$$L.H.S = a^+ + \epsilon$$
$$= \{a, aa, aaa, ---\} \cup \{\epsilon\}$$
$$= \{\epsilon, a, aa, aaa, ---\}$$
$$= a^*$$

$$L.H.S = R.H.S.$$

Hence Proofed.

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$$0.1 + 1.0 + 1.1$$

$$= \{01, 10, 11\}$$

Conversion of Regular Expression

① Finite Automata to R.E.

Arden's equation:

$$\boxed{R = Q + RP}$$

Arden's Result:-

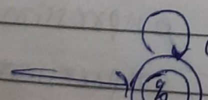
$$\boxed{R = QP^*}$$

$R = Q + RP = Q + QP + RPP$
$= Q + QP + QPP + RPPP$
$= \{Q, QP, QPP \ldots\}$
$= Q.P^*$

Steps & Rules : FA → RE

1) For given automata check all inputs on each state individually.
2) Add ϵ input on initial state.
3) For find regular expression of each state try to form arden's eqⁿ & apply arden's result.
4) To create arden's eqⁿ try to subtitute value of one eqⁿ into other eqⁿ.
5) Final answer will be regular expression of final state.
6) If more than one final state are available than final answer will be union of regular expression of each final state.

Que1:-



$q_0 = \epsilon + q_0 a$

apply arden's eqⁿ

$R = q_0$, $Q = \epsilon$, $P = a$

apply arden's result

$R = QP^*$

$\therefore q_0 = \epsilon a^*$

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$\therefore a\epsilon = a$

$$\boxed{q_0 = a^*}$$ Ans.

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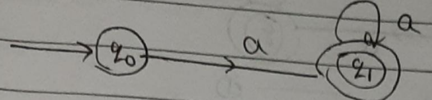
Que 2 :-



Sol :-
$q_0 = \epsilon$ —— ①

$q_1 = q_0 a + q_1 a$ —— ②

Substituting the value of q_0 form eq^n ① into ②

$q_1 = \epsilon \cdot a + q_1 a$

$q_1 = a + q_1 a$ —— ③

apply arden's eq^n into eq^n ③

$R = q_1$, $Q = a$, $P = a$

applying arden's result

$\therefore q_1 = aa^*$

$$\boxed{q_1 = a^+}$$ Ans.

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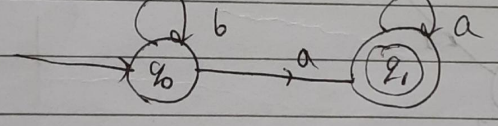
Que 3 :-



Sol :-
$q_0 = \epsilon + q_0 b$ —— ①

$q_1 = q_0 a + q_1 a$ —— ②

Applying arden's eq^n in eq^n ①

$R = q_0$, $Q = \epsilon$, $P = b$

$q_0 = \epsilon b^*$

$q_0 = b^*$ —— ③

Substitute the value of q_0 from eq^n ③ into eq ②

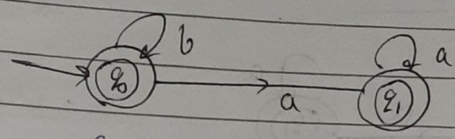
$q_1 = b^* a + q_1 a$

Apply arden's eq^n

$q_1 = b^* a \cdot a^*$

$\boxed{q_1 = b^* a^+}$ Ans.

Que 4:-



Sol:- $q_0 = \epsilon + q_0 b$ ——— ①

$q_1 = q_0 a + q_1 a$ ——— ②

Applying arden's eq^n in ①

$\therefore q_0 = \epsilon b^* = b^*$

Similarly applying eq^n in ②

$\boxed{q_1 = b^* a^+}$

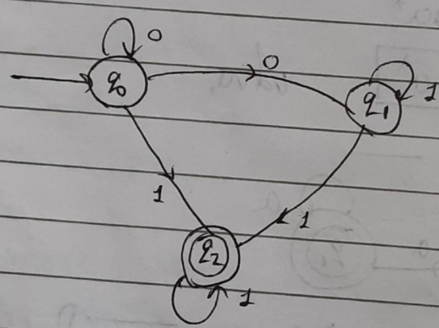
$\because$ Both states are final state, hence

$q_0 + q_1 = b^* + b^* a^+$

$= b^*(\epsilon + a^+)$

$\boxed{q_0 + q_1 = b^* a^*}$ Ans.

Ques 5:-



Sol:- $q_0 = \epsilon + q_0 0$ ——— ①

$q_1 = q_0 0 + q_1 1$ ——— ②

$q_2 = q_1 1 + q_2 1 + q_0 1$ ——— ③

from eq^n ①

$q_0 = \epsilon \cdot 0^*$

$q_0 = 0^*$

put value in eq^n ②

$q_1 = 0^* 0 + q_1 1$

$= 0^+ + q_1 1$

by arden's eq^n

$q_1 = 0^+ 1^*$

Put values of q_0 & q_1 in eq^n ③

$q_2 = 0^* 1 + 0^+ 1^* 1 + q_2 1$

$q_2 = \underbrace{0^* 1 + 0^+ 1^+}_{Q} + \underbrace{q_2 1}_{RP}$

$q_2 = (0^* 1 + 0^+ 1^+) 1^*$

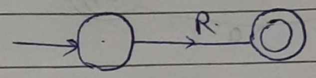
$= 0^* 1 . 1^* + 0^+ 1^+ 1^*$

$\boxed{q_2 = 0^* 1^+ + 0^+ 1^+ 1^*}$

RE to Finite Automata :-

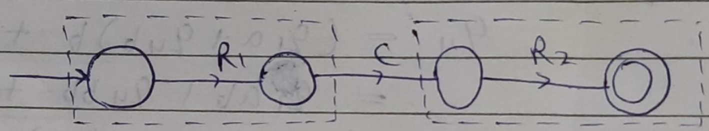
Rule :

1) If R is any regular expression then its FA M will be

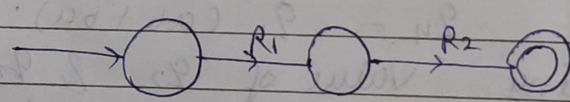


2) If R_1 & R_2 are two different RE whose FA are M_1 & M_2 then joint FA for R_1R_2 will be

$R_1 \cdot R_2$ (AND)

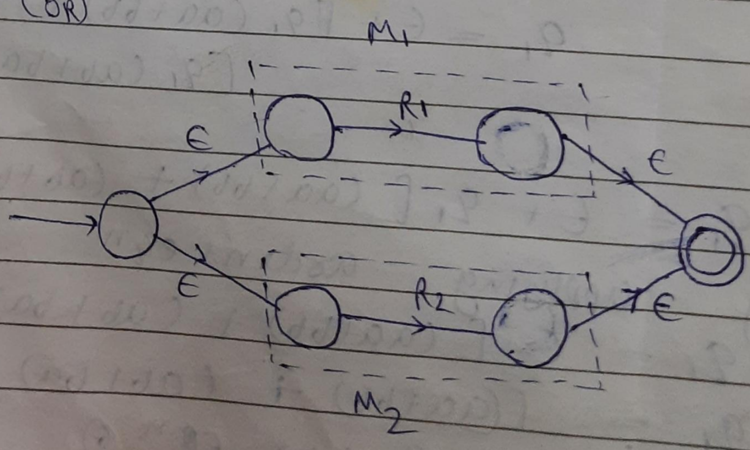


or



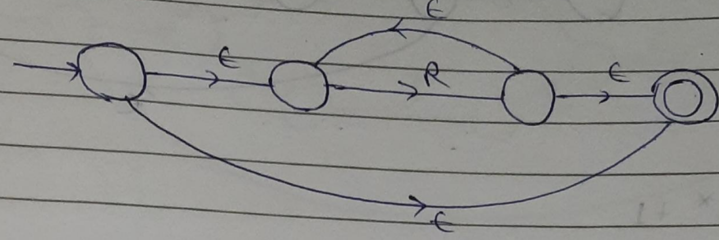
3) If R_1 & R_2 are two different RE where FA are M_1 & M_2 then joint FA for R_1+R_2 will be

R_1+R_2 (OR)



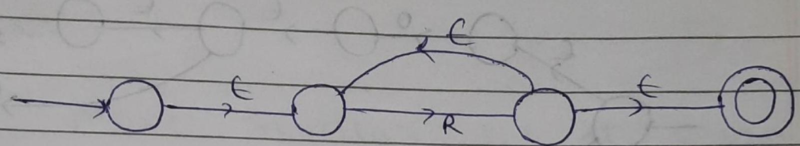
4) If R will be is any regular expression then FA for R^* will be

$\underline{\underline{R*}}$



5) If R is any regular expression then FA for R^+ will be

$\underline{\underline{R^+}}$



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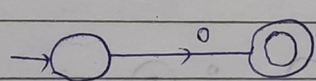
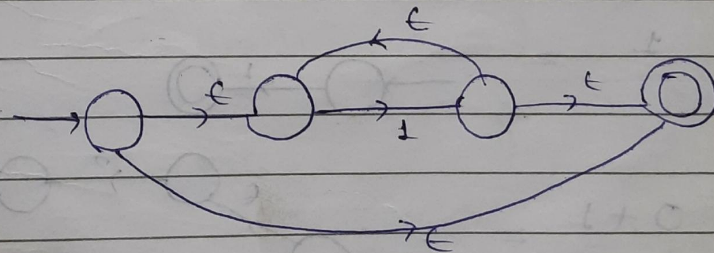
Que:- Construct FA for $\underline{01^* + 1}$

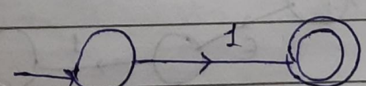
Sol:- $0 =$



$1^* =$



$1 =$



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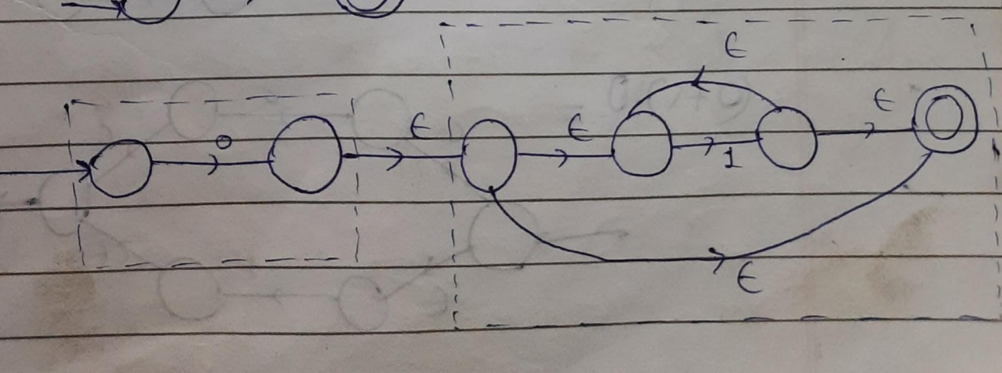
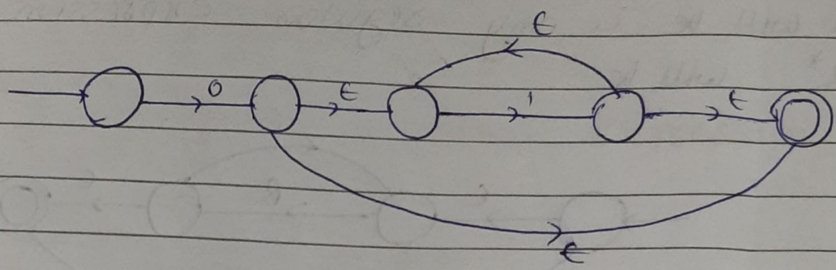
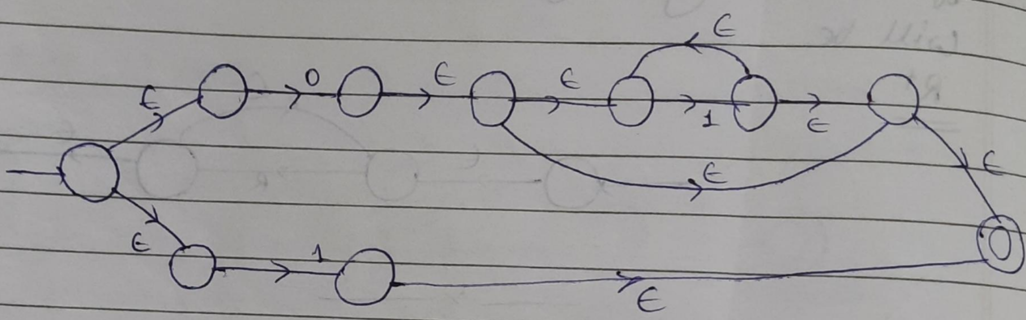
$01^* =$



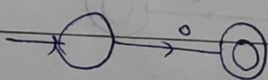


$\underbrace{01^*}_{R_1} \; \underbrace{+1}_{R_2}$

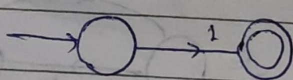


Que: $[(0+1)0]^*$

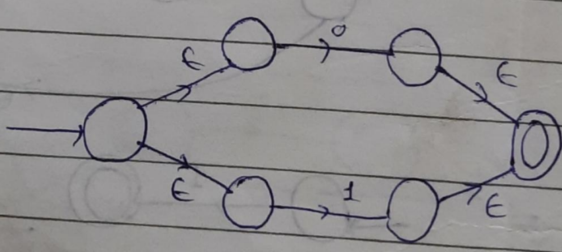
0 =



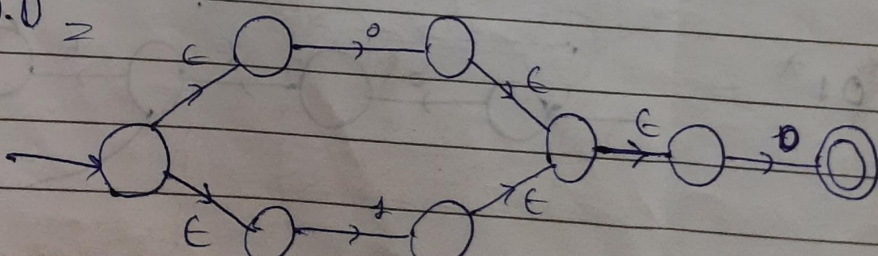
1 =



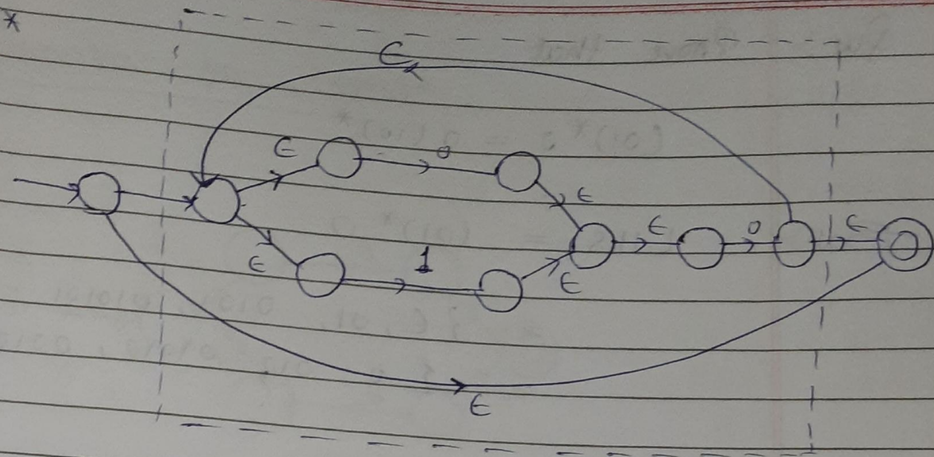
$0+1$ =



$(0+1).0$ =



$[(0+1)0]^*$



Regular set

Qu: $01^* + 1$, Make regular set of given RE.

Sol: $0 = \{0\}$

$1^* = \{\epsilon, 1, 11, 111, ----\}$

$1 = \{1\}$

$01^* = \{0\} \cap \{\epsilon, 1, 11, 111, ----\}$

$01^* = \{0, 01, 011, 0111, ---\}$

$01^* + 1 = \{0, 01, 011, 0111, ---\} \cup \{1\}$

$01^* + 1 = \{0, 1, 01, 011, 0111 ---\}$ Ans

Que:- Prove that

$$(01)^* 0 = 0(10)^*$$

Sol:- L.H.S. = $(01)^* 0$

$= \{\epsilon, 01, 0101, 010101 \ldots\} \cap \{0\}$
$= \{0, 010, 01010, 0010100 \ldots\}$

$0 = \{0\}$
$1 = \{1\}$

$01 = \{0\} \cap \{1\} = \{01\}$

$(01)^* = \{\epsilon, 01, 0101, 010101, \ldots\}$

R.H.S. $= 0(10)^*$

$= \{0\} \cap \{\epsilon, 10, 1010, \ldots\}$

$= \{0, 010, 01010 \ldots\}$

$\therefore$ L.H.S. = R.H.S.

Hence Proved.

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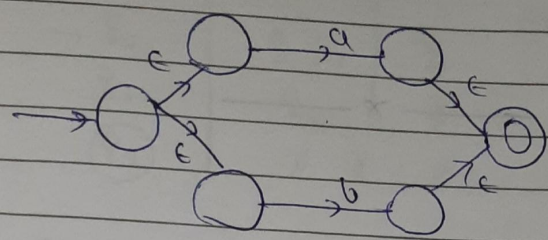
Universal Regular Expression: The expression which accepts all possible values of alphabets for which it is define.

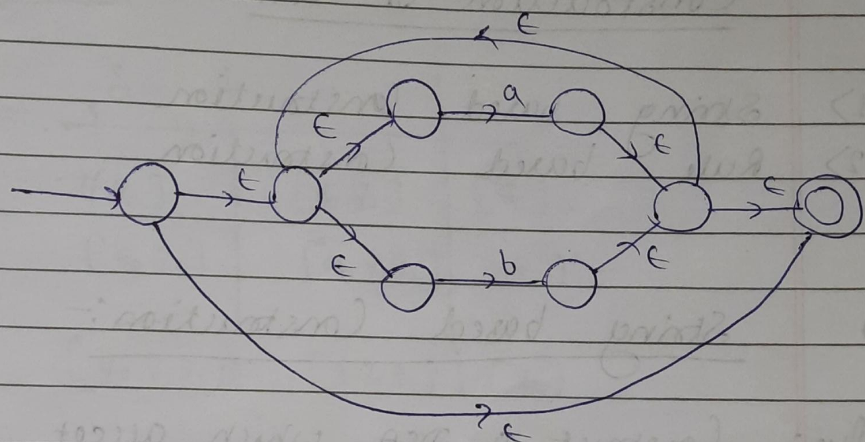
ab
$(a+b)^* = \epsilon, a, aa \ldots b, bb \ldots, aba, abba, abab$

VF Automata for (a+b)*

a+b :-



(a+b)* :



Questions On Regular expression :

Que:- Construct RE which accepts all even a & odd b where a followed by b.

Sol :- $(aa)^* \, b \, (bb)^*$

Que:- A RE which accepts atmost 2 a's & any no. of b

$b^* \, (\epsilon + a + aa) \, b^*$