Unit-1

Quantum Physics

Concept of wave –particle Duality:

The optical phenomenon such as interference, polarization and diffraction of light could be explained by the *wave theory of light.* But wave theory is unable to explain the Black body radiation, Photoelectric Effect, Compton Effect etc .This phenomenon could easily explained by the *quantum theory of physics*. This theory explains that the energy of light is concentrated in to small regions and it travels in the form of energy bundles known as quanta or photons. The energy of each photon is given by

$$E = h\upsilon$$

Hence light shows both wave as well as particle nature. This nature of light is known as duel nature of light and the property is known as wave particle duality.

DE – BROGLIE'S HYPOTHESIS

The concept of duel nature of light led Louis de – Broglie in 1924, to introduce the idea that matter also posse's dual nature like light. According to him:-

"There is a wave associated with every moving particle the wavelength of that wave depends upon the mass of the particle and velocity. These wave associated with the matter particle are known as ***'matter waves'*** *or* ***'de – broglie waves'****."*

$\lambda = \frac{h}{mv}$ or $\lambda = \frac{h}{p}$

where h= $6.62*10^{-34}$ Js

$p = mv = momentum\ , m = mass\ of\ the\ particle\ and\ v = velocity\ of\ the\ particle$

His concept was based on the following facts:-

- Matter and light, both are forms of energy and each of them can be transformed in to each other by energy mass relation of Einstein-

  $E=mc^2$ Where, c= speed of light ($3x10^7$m/s)

- Both are governed by the space time symmetries of the theory of relativity.

De – Broglie equation provides the connection between wavelength and momentum. His relation is-

Velocity of Debroglie wave OR phase velocity :

We know that

$c = \upsilon\lambda$ $\quad v_p = VELOCITY\ of\ de - Broglie\ wave$

$v_p = \upsilon\lambda$ ……..(1) $\quad we\ Know\ that$

$E = h\upsilon$ ….. (2) $\quad v_p = \frac{E}{h}\frac{h}{mv}$

and

$E = mc^2$ ….. (3) $\quad hence \quad v_p = \frac{mc^2}{h}\frac{h}{mv}$

$v_p = \frac{c^2}{v}$ …..(4)

$for\ light\ v_p = c$

FOR ANY OTHER PARTICLE

$v_p = \frac{c*c}{v} \quad v_p > c$

Properties of matter wave:

1. The wavelength of de Broglie wave associated with moving light particle is greater than the wavelength associated with heavy particle.
2. The wavelength of de Broglie wave associated with slow particle is greater than the wavelength associated with fast moving particle.
3. For a particle is at rest i.e. v= 0 the wavelength $\lambda = \infty$ that means wave become indeterminate. If $v = \infty$ then $\lambda = 0$ this indicate that the matter waves are generated only when particles are in motion.
4. The matter waves are generated by moving charged particle as well as by moving neutral particle.
5. Velocity of de Broglie wave is given by $v_p = \frac{c^2}{v}$ that means de Broglie waves have velocity greater than the speed of light.
6. de Broglie waves are not electromagnetic waves .
7. The wave and particle aspect of matter never appear simultaneously in the same experiment.

8. The velocity of matter wave is not constant like the radiations which move with constant velocity equal to the velocity of light. The velocity of matter wave depends upon the velocity of material particle

Different formulas to calculate de-Broglie wavelength

1. $\lambda = \frac{h}{mv} = \frac{h}{p}$ ….. (1)

where h= $6.62*10^{-34}$ Js

$p = mv = momentum$, $m = mass\ of\ the\ particle\ and\ v = velocity\ of\ the\ particle$

2. If Kinetic energy of the particle is given then using

$$E = \frac{1}{2} mv^2$$

$\lambda = \frac{h}{\sqrt{2mE}}$ ….. (2)

3. According to kinetic theory of gases , the average kinetic energy of the particle is given by

$$E = \frac{1}{2} mv^2 = \frac{3}{2} KT$$

K= Boltzmann constant = $1.38*10^{-23}$ J/K

T= Absolute temp.of the particle

$\lambda = \frac{h}{\sqrt{3mKT}}$ …..(3)

4. suppose an electron is accelerated through aa potential difference V volt .

Work done by electric field = gain in K.E.

eV = $\frac{1}{2} mv^2$

$$m^2v^2 = 2meV$$

$$mv = \sqrt{2meV}$$

$\lambda = \frac{h}{\sqrt{2meV}}$ …. (4)

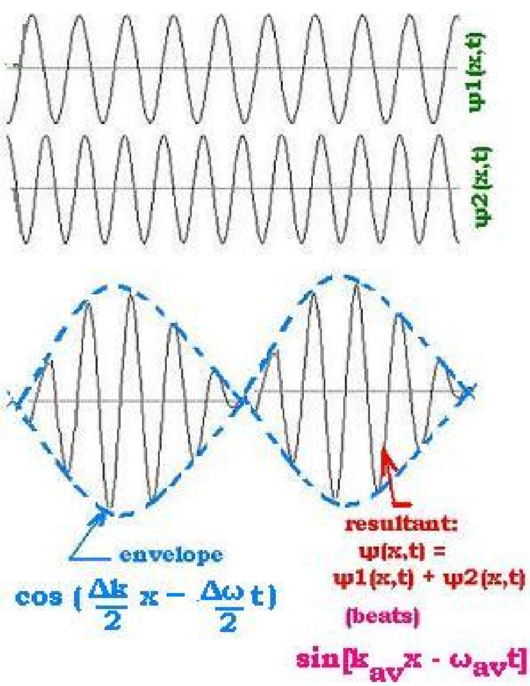
Concept of wave packet :

{We have studied:

1. *Velocity of matter wave is greater than the speed of light*
2. *According to Einstein's theory of relativity , no material particle can have speed greater than the speed of light c ,*
3. *Point 1 is contradiction with that*
4. *Secondly , if the speed of matter wave associated with moving particle is greater than the speed of particle, the particle left behind.*

Schrodinger assumed that a moving particle is equivalent to a wave packet (group of waves) instead of a single wave.}

Wave packet: A wave packet comprises a group of waves slightly differing in velocity and wavelength ,with phases and amplitude such that they interfere constructively over a small region of space where the particle can be located and outside this space they interfere destructively so that the amplitude reduces to zero .



Supperposition of two waves of almost the same frequency

Phase velocity : Velocity of component waves of a wave packet is called phase velocity it is denoted by V_p.

Group Velocity :

Velocity of the wave packet is known as group velocity it is denoted by Vg

It can be obtained due to superposition of waves travelling in a group.

Relation between group velocity and Phase velocity :

Let us consider two waves of nearly same amplitude and slightly differ in angular frequency

The wave can be represented as

$y_1 = a\sin(\omega_1 t - K_1 x)$ ...1

$y_2 = a\sin(\omega_2 t - K_2 x)$ 2

$Y = y_1 + y_2$

$Y = a\sin(\omega_1 t - K_1 x) + a\sin(\omega_2 t - K_2 x)$

$$Y = 2a\sin\frac{((\omega_1 t - K_1 x) + (\omega_2 t - K_2 x))}{2}\cos\frac{((\omega_1 t - K_1 x) - (\omega_2 t - K_2 x))}{2}$$

$$Y = 2a\sin\frac{((\omega_1 + \omega_2)t - (K_1 + K_2)x)}{2}\cos\frac{((\omega_1 - \omega_2)t - (K_1 - K_2)x)}{2}$$

$Y = 2a\sin(\omega t - kx).\cos\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x\right)$ ....(3)

where $= \frac{\omega_1 + \omega_2}{2}$, $k = \frac{k_1 + k_2}{2}$, $\partial\omega = \omega_1 - \omega_2$, $\partial k = k_1 - k_2$

$(\omega t - kx) = constant$

$(\omega dt - kdx) = 0$

$\frac{dx}{dt} = v_p = \frac{\omega}{k}$

$\omega = kv_p$ .... (4)

$\left(\frac{\partial\omega}{2}t - \frac{\partial k}{2}x\right) = 0$

$v_g = \frac{\partial\omega}{\partial k}$

$v_g = \frac{d\omega}{dk}$

From eq. (4)

$$v_g = \frac{d \ (kv_p)}{dk}$$

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$k = \frac{2\pi}{\lambda}$$

$$v_g = v_p + \frac{2\pi}{\lambda}\frac{dv_p}{d\left\{\frac{2\pi}{\lambda}\right\}}$$

On solving this we get

$$\boldsymbol{v_g = v_p - \lambda\frac{dv_p}{d\lambda}} \quad \textbf{................(5)}$$

This is the relationship between group velocity and Phase velocity.

Different cases

1. If $\frac{dv_p}{d\lambda} = 0$ i.e. the medium is such that in it phase velocity doesnot depend on the wave length, then $v_g = v_p$ i.e. the group velocity is equal to phase velocity . Such a medium is called non-dispersive medium.

2. If $\frac{dv_p}{d\lambda} = positive$ medium is such that in it phase velocity depends on the wave length, $v_g < v_p$ i.e. the group velocity is less than the phase velocity. Such a medium is called dispersive medium.

Relations between group velocity and particle velocity

1. **For Relativistic particle**

   Consider de-broglie waves associated with a particle of rest mass m_0 moving with velocity v . For this angular frequency ω and propagation constant k are given by

   We know that

$$\omega = 2\pi\vartheta \quad .. \quad (1)$$

$$\omega = 2\pi\frac{mc^2}{h}$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\omega = 2\pi \frac{m_0\, c^2}{h\sqrt{1-\frac{v^2}{c^2}}} \quad \ldots\ldots. (2)$$

$$k = \frac{2\pi}{\lambda} \ldots\ldots. \qquad (3)$$

$$k = \frac{2\pi m v}{h}$$

$$k = \frac{2\pi v}{h} \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \ldots. (4)$$

Phase velocity

$$v_p = \frac{\omega}{k}$$

Put the values from eq. (2) and (4)

Phase velocity $v_p = \frac{c^2}{v}$

Group velocity

$$v_g = \frac{d\omega}{dk} = \frac{d\omega}{dv} \Big/ \frac{dk}{dv} \qquad \ldots (5)$$

$$\frac{d\omega}{dv} = \frac{d}{dv}\left(2\pi \frac{m_0\, c^2}{h\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0\, c^2}{h} \frac{d}{dv}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0\, c^2}{h} \frac{d}{dv}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0}{h} \frac{v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \quad \ldots\ldots \quad (6)$$

$$\frac{dk}{dv} = \frac{d}{dv} \frac{2\pi v}{h} \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \left(1 - \frac{v^2}{c^2}\right)^{-1/2} + \frac{v^2}{c^2}.\left(1 - \frac{v^2}{c^2}\right)^{-3/2}$$

$$= \frac{2\pi m_0}{h}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}$$

$\frac{dk}{dv} = \frac{2\pi m_0}{h} . \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}$ ....(7)

Put the value of eq 6 and 7 in to (5)

$\boldsymbol{v_g = v}$ **.....(6)**

Or $\boldsymbol{v_g . v_p = c^2}$ **...... (7)**

2. Non Relativistic Free Particle

According to Debroglie Hypothesis

$$\lambda = \frac{h}{mv_g} \quad .... (1)$$

Total Energy $E = \frac{1}{2} m\, {v_g}^2$ (2)

$E = h\vartheta$

$$\vartheta = \frac{E}{h} = \frac{m\, {v_g}^2}{2h} \quad (3)$$

Phase velocity $v_p = \vartheta\lambda$ (4)

$$v_p = \frac{m\, {v_g}^2}{2h} \frac{h}{mv_g}$$

$$\boldsymbol{v_p = \frac{v_g}{2}} \quad \textbf{(5)}$$

This is the relation between phase velocity and group velocity for non relativistic free particle.