

TCS NQT / DIGITAL / NINJA

APTITUDE SOLVED QUESTIONS

PART 2 **TOPICS COVERED: (Part 2 Focus)**

-  **Number System**
-  **Linear and Quadratic Equations**
-  **Probability**
-  **Time, Speed and Distance**
-  **Permutations and Combinations**

**With Detailed Solutions & Shortcut Tricks****START SOLVING QUESTIONS!**

Number System

22
11
91

Divisibility rules:-

For 2 :- last digit should be even.

For 3 :- Sum of digits should be 3, 6, 9.

For 4 :- last two digits should be divisible by 4, or end with "00".

For 5 :- last digit should be 0 or 5.

For 6 :- Divisible by both "2" and "3".

For 7 :- multiply last digit by 2 and then subtract the result from the rest of the digits. If the final result is divisible by 7, then the no. is divisible by 7.

For 8 :- last three digits should be divisible by 8 or end with "000".

For 9 :- sum of digits should be 9.

For 10 :- should end with "0".

For 11 :- Sum of odd position digits - Sum of even position digits, should be 0 or multiple of 11.

For 12 :- divisible by both 4 and 3.

Que: If the no. 243P51 is divisible by 9, then value of the digit marked as P would be.

- (a) 3 (b) 2 (c) 2 (d) 4

$$\text{Sum} \Rightarrow 2 + 4 + 3 + P + 5 + 1 \Rightarrow 15 + P$$

$$\text{Hence} \Rightarrow 15 + 3 = 18 \text{ divisible by } 9$$

$$\text{Ans} = 3$$

Que: If 41895AB is divisible by 11, find the value of A.

$$4 + 8 + 5 + 8 - (1 + 9 + A) = 11$$

$$25 - 10 - A = 11$$

$$A = 25 - 10 - 11$$

$$A = 15 - 11$$

$$A = 4 \text{ Ans}$$

Que: Find A and B if the number A8563145B is divisible by 88.

$$88 \rightarrow 8 \text{ and } 11$$

45B should be divisible by 8

$$\begin{array}{r} 8 \overline{) 45B} \quad (5 \\ 40 \\ \hline 5B \\ 1 \\ \hline 0 \end{array}$$

B should be 6

because only 56 is a multiple of 8.

Now, A85631456 should be divisible by 11

$$(A + 5 + 3 + 4 + 6) - (8 + 6 + 1 + 5) = 11 \text{ or } 0$$

$$A + 18 - 20 = 11 \text{ or } 0$$

$$A - 2 = 11 \text{ or } 0$$

$$A = 11 + 2 = 13 \text{ Ans. or}$$

$$A = 0 + 2 = 2 \text{ Ans.}$$

HCF and LCM.

Ques: The LCM and HCF of $\frac{3}{5}$, $\frac{4}{7}$, $\frac{2}{9}$ are

$$\text{HCF} \left(\frac{3}{5}, \frac{4}{7}, \frac{2}{9} \right)$$

$$\text{LCM} \left(\frac{3}{5}, \frac{4}{7}, \frac{2}{9} \right)$$

→ HCF of Numerator
LCM of Denominator

→ LCM of Numerator
HCF of Denominator

$$\rightarrow \frac{\text{HCF}(3, 4, 2)}{\text{LCM}(5, 7, 9)} = \frac{1}{315}$$

$$\rightarrow \frac{\text{LCM}(3, 4, 2)}{\text{HCF}(5, 7, 9)} = \frac{12}{1}$$

$$\text{HCF} = \frac{1}{315}, \text{LCM} = 12 \quad \text{Ans}$$

Ques: 5 bells are heard at intervals of 3, 6, 8, 10, and 12 minutes respectively. If all the bells toll together exactly at 2:23 PM, they will again be heard together at?

$$\text{LCM}(3, 6, 8, 10, 12) \Rightarrow 120 \text{ minutes} \Rightarrow 2 \text{ hours}$$

Ans all the bells are heard together at 4:23 PM.

Ques: Find the greatest number that will divide 12, 36, and 54 exactly.

$$\text{HCF}(12, 36, 54) \rightarrow 6$$

Ques 1 Find the latest number that is exactly divisible by 6, 8, and 12.

$$\text{LCM}(6, 8, 12)$$

Ans:- 24

Ques: Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

$$\text{HCF}(43, 91, 183) \rightarrow$$

$a, b, c \rightarrow$ same remainder

$$\text{HCF of } (a-b, b-c, c-a)$$

$$\text{HCF of } (48, 92, 140) \rightarrow 4, \text{ Ans.}$$

Ques: Find the least number when divided by 36, 24 and 16 leaves a remainder of 11 in each case.

$$\text{LCM}(36, 24, 16) + \text{remainder}$$

$$\text{LCM}(36, 24, 16) + 11$$

$$\begin{array}{c|c} 4 & 36, 24, 16 \end{array}$$

$$\begin{array}{c|c} 3 & 9, 6, 4 \end{array}$$

$$\begin{array}{c|c} 2 & 3, 2, 4 \end{array}$$

$$\begin{array}{c|c} 1 & 1, 1, 2 \end{array}$$

$$\Rightarrow 4 \times 3 \times 2 \times 3 \times 1 \times 2 \Rightarrow 144$$

$$\text{Ans} \Rightarrow 144 + 11 = 155$$

Q: Find the greatest number that will divide 27, 38 and 47 leaving remainders of 3, 2 and 5 respectively.

$$\text{HCF}(a-x, b-y, c-z)$$

$$\text{HCF}(27-3, 38-2, 47-5)$$

$$\text{HCF}(24, 36, 42) \rightarrow 6$$

$$\text{Ans} \Rightarrow 6$$

Ques: Find the least number which when divided by 12 leaves a remainder of 10, when divided by 24 leaves a remainder of 22 and when divided by 32 leaves a remainder of 30.

No.	km.		
12	- 10	= 2	
24	- 22	= 2	Here $K = 2$
32	- 30	= 2	

$$\text{LCM}(12, 24, 32) - K$$

$$96 - 2 = 94 \text{ Ans.}$$

Remainder Theorem

Ex: 1) $4^{70} / 9 \rightarrow \frac{4}{9} \rightarrow R(4), \frac{4^2}{9} \rightarrow R(7), \frac{4^3}{9} \rightarrow R(1)$

$\frac{4^4}{9} \rightarrow R(4), \frac{4^5}{9} \rightarrow R(7), \frac{4^6}{9} \rightarrow R(1)$

Repetition at every

3 intervals.

Then:

$\frac{4^{70} \times 4}{9} = R(1) \times \frac{4}{9} \rightarrow \frac{4}{9} \rightarrow R(4) \text{ Ans.}$

2) $5^{72} / 7 \rightarrow \frac{5}{7} \rightarrow R(5), \frac{5^2}{7} \rightarrow R(4), \frac{5^3}{7} \rightarrow R(6)$

$\frac{5^4}{7} \rightarrow R(2), \frac{5^5}{7} \rightarrow R(3), \frac{5^6}{7} \rightarrow R(1), \frac{5^7}{7} \rightarrow R(5)$

no. of intervals $\rightarrow 6$

72 is divisible by 6 means at last

~~8~~

remainder of cycle

$R(1) \text{ Ans.}$

Que: Find the remainder when 3^{75} is divisible by 5?

$\frac{3^{75}}{5} \rightarrow \frac{3}{5} \rightarrow R(3), \frac{3^2}{5} \rightarrow R(4), \frac{3^3}{5} \rightarrow R(2), \frac{3^4}{5} \rightarrow R(1)$

Interval = 4

$\frac{3^{75}}{5} = \frac{3^{72} \times 3}{5} = \frac{R(1) \times 3}{R(3)} = \text{Ans is } R(2).$

Ques Find the remainder when 7^{203} is divided by 47

$\frac{7^{203}}{47}$ Now, $7 \rightarrow R(3), 7^2 \rightarrow R(1)$

$\frac{203}{2} \rightarrow R(2)$ means Ans is $R(3)$

Ques: Find the remainder when 13^{41} is divided by 7

$\frac{13^{41}}{7}$ Now, $13 \rightarrow R(6), 13^2 \rightarrow R(1)$

$\frac{41}{2} \rightarrow$ Ans is $\rightarrow R(6)$

$\frac{5^2}{7} \rightarrow R(5)$

ask

by 5?

(1)

BODMAS - Simplification of Complex Equation

Ques: $4 - 3.6 \div 4 + 0.2 \times 0.5 = ?$

$$4 - \frac{3.6}{4} + 0.2 \times 0.5$$

$$4 - 0.9 + 0.10$$

$$4 - 0.9 + 0.10 = 3.2$$

Ans

Ques: $108 \div 36 \text{ of } \frac{1}{3} + \frac{2}{5} \times \frac{3}{4}$

$$108 \div 36 \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4}$$

$$\frac{108}{36} + \frac{2 \times 3}{5 \times 4} = 3 + \frac{6}{20}$$

Ans

$$3 + \frac{3}{10} = \frac{33}{10}$$

Ques: $(2+2+2 \div 2) / (5+5+5 \div 5) =$

$$(2+2+1) / (5+5+1)$$

Ans

Ques: $\{(1232-22)(124+126)\} / (12 \times 15 - 15) =$

$$\{(1210)(250)\} / (180 - 15)$$

$$\{1210 \times 250\} / 165$$

Ans

Q: $1 + 1 \div \{1 + 1 \div (1 + 1/2)\}$

$$1 + 1 \div \{1 + 1 \div (3/2)\}$$

$$2 \div \{2 \div 3/2\}$$

$$1 + 1 \div \{1 + \frac{2}{3}\}$$

$$1 + 1 \div \{5/3\}$$

$$1 + \frac{3}{5} = \frac{8}{5} \text{ Ans}$$

Ques: The value of $1 \div \{1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}\}$ is

$$1 \div \{1 + 1 \div \{1 + 1 \div (3/2)\}\}$$

$$1 \div \{1 + 1 \div \{1 + \frac{2}{3}\}\}$$

$$1 \div (1 + \frac{3}{5})$$

Ans

(Powers)
Exponents and Surds

Ques- The value of $(11111)^2$ is

$$1^2 = 1$$

$$11^2 = 121$$

$$111^2 = 12321$$

$$1111^2 = 1234321$$

$$11111^2 = 123456789021 \text{ Ans}$$

Q: $\sqrt{8 + \sqrt{57 + \sqrt{38 + \sqrt{108 + \sqrt{169}}}}} = ?$

$$\sqrt{169} = 13$$

$$\sqrt{121} = 11$$

$$\sqrt{38 + 11} = \sqrt{49} = 7$$

$$\sqrt{57 + 7} = \sqrt{64} = 8$$

$$\sqrt{8 + 8} = \sqrt{16} = 4 \text{ Ans}$$

Q $\sqrt{189} - \sqrt{625} \div \sqrt{25}$ is equal to

$$17 - 25 \div 5$$

$$17 - 5 = 12 \text{ Ans}$$

Q Find the square root of 15376

$$\sqrt{15376} \rightarrow \text{ending with 6}$$

means last digit should be

4 or 6

$$\begin{array}{r} 12 \\ 12 \\ \hline 119 \\ 132 \end{array}$$

Now ignoring last two digit

153 is closest to 12^2 and nearest small to 13^2

$$\begin{array}{r} 12^2 \\ 144 \end{array} \quad \begin{array}{r} 13^2 \\ 169 \end{array}$$

take least value 12

$$124^2 \text{ or } 126^2$$

Now for

$$125^2 \Rightarrow 15625 > 15376$$

So, the answer is 124^2

Find the cube root of 386989

1. last digit of no

2. Ignore last 3 digit

last digit = 3 means cube root must be of last digit 7

neglect last 3 digit

$$3869$$

$$15^3 \rightarrow 3375$$

$$16^3 \rightarrow 4096$$

Ans is 15

Ans is 157

Simplification using Algebraic Identities

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a^2 - b^2) = (a-b)(a+b)$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{If } a+b+c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc$$

Ques:- The value of $\frac{[(3.2)^3 - 0.008]}{[(3.2)^2 + 0.64 + 0.04]}$ is

$$= \frac{(3.2)^3 - 0.008}{(3.2)^2 + 0.64 + 0.04} = \frac{(3.2)^3 - (0.2)^3}{(3.2)^2 + 3.2 \times 0.2 + (0.2)^2}$$

$$\frac{(3.2 - 0.2) [(3.2)^2 + 3.2 \times 0.2 + (0.2)^2]}{(3.2)^2 + 3.2 \times 0.2 + (0.2)^2}$$

$$= 3.2 - 0.2$$

$$= 3.0 \text{ Ans}$$

$$Q. \frac{0.9 \times 0.9 \times 0.9 + 0.2 \times 0.2 \times 0.2 + 0.3 \times 0.3 \times 0.3 - 3 \times 0.9 \times 0.2 \times 0.3}{0.9 \times 0.9 + 0.2 \times 0.2 + 0.3 \times 0.3 - 0.9 \times 0.2 - 0.2 \times 0.3 - 0.3 \times 0.9}$$

$$= \frac{(0.9)^3 + (0.2)^3 + (0.3)^3 - 3(0.9)(0.2)(0.3)}{(0.9)^2 + (0.2)^2 + (0.3)^2 - (0.9)(0.2) - (0.2)(0.3) - (0.3)(0.9)}$$

$$= \frac{(0.9 + 0.2 + 0.3) [(0.9)^2 + (0.2)^2 + (0.3)^2 - (0.9)(0.2) - (0.2)(0.3) - (0.3)(0.9)]}{(0.9)^2 + (0.2)^2 + (0.3)^2 - (0.9)(0.2) - (0.2)(0.3) - (0.3)(0.9)}$$

$$= 0.9 + 0.2 + 0.3 = 1.4 \text{ Ans}$$

Q. $(112^2 + 115^2 + 117^2 - 112 \times 115 - 115 \times 117 - 117 \times 112)$ is equal to

$$a^2 + b^2 + c^2 - ab - bc - ca$$

No direct formula

mul and divide by 2

$$\frac{1}{2} [2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$$

$$\frac{1}{2} [a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca]$$

$$\frac{1}{2} [a^2 + b^2 - 2ab + a^2 + b^2 - 2ca + b^2 + c^2 - 2bc]$$

$$\frac{1}{2} [(a-b)^2 + (a-c)^2 + (b-c)^2]$$

$$\frac{1}{2} [(112-115)^2 + (112-117)^2 + (115-117)^2]$$

$$= \frac{1}{2} [9 + 25 + 4] = \frac{34}{2} = 17 \text{ Ans}$$

Q $\frac{[133 \times 133 + 133 \times 133 + 133 \times 133]}{[133 \times 133 \times 133 - 133 \times 133]}$ = ?

$\Rightarrow \frac{[(133)^2 + 133 \times 133 + (133)^2]}{(133^3 - 133^2)}$

$\Rightarrow \frac{[(133)^2 + 133 \times 133 + (133)^2]}{(133 - 133) [(133)^2 + 133 \times 133 + (133)^2]}$

= $\frac{1}{4}$ Ans

Q Solve : $\frac{[(999 + 588)^2 - (999 - 588)^2]}{(999 \times 588)}$

$\frac{(\cancel{999})^2 + (\cancel{588})^2 + 2(999)(588) - (\cancel{999})^2 - (\cancel{588})^2 + 2(999)(588)}{(999 \times 588)}$

+ $\frac{4(999)(588)}{(999 \times 588)} = 4$ Ans

Decimal simplification

Q If $547.527 / 0.0082 = x$, then find the value of $547527 / 82$ is.

$x = \frac{547.527 \times 10000}{0.0082 \times 1000}$

$x = \frac{547527 \times 100}{82}$

$\frac{547527}{82} = \frac{x}{100}$ Ans

Q $(0.9 \times 0.9 \times 0.9 + 0.1 \times 0.1 \times 0.1)$ is approximately equal to.

$\Rightarrow [(0.9)^3 + (0.1)^3]$

= $[(0.9) + (0.1)] [(0.9)^2 - (0.9)(0.1) + (0.1)^2]$

= $[1.0] [0.81 - 0.09 + 0.01]$

= $0.81 - 0.09$

= $0.72 + 0.01$

= 0.73 Ans

$$\frac{7.5 + 3 - 1.5}{47 + 8 - 36} \Rightarrow \frac{9}{19} \text{ Ans}$$

Q The simplified value of $\frac{(0.0519 - 0.002) \times 0.4 + 0.56 \times 0.07}{0.04 \times 0.25}$

$$\Rightarrow \frac{(0.0519 - 0.0020) \times 0.4 + 0.56 \times 0.07}{0.04 \times 0.25}$$

$$\Rightarrow \frac{(0.0519) \times 0.4 + 0.56 \times 0.07}{0.04 \times 0.25}$$

$$(0.0519) \times 0.4 + 0.4 \times \frac{1.4}{10} \times 0.07$$

$$0.4 \frac{(0.0519 + 1.4 \times 0.07)}{0.04 \times 0.25}$$

$$0.4 \frac{(0.0519 + 0.098)}{0.04 \times 0.25}$$

$$\frac{0.4 \times 0.1499}{0.04 \times 0.25}$$

$$= \frac{4 \times 1499}{4 \times 25} \times \frac{10000}{100000}$$

$$= \frac{1499}{250} = 5.996$$

Decimal to fraction

Convert the following into vulgar fraction.

$$(A) 0.25 \rightarrow \frac{25}{100} = \frac{1}{4}$$

$$(B) 4.004 \rightarrow \frac{4004}{1000} = \frac{1001}{250}$$

$$(C) 0.0056 \rightarrow \frac{56}{10000} = \frac{14}{2500} = \frac{7}{1250}$$

Q Convert $87.434343 \dots$ into a vulgar fraction (rational number).

$$x = 87.434343 \dots$$

$$100x = 8743.434343 \dots$$

$$100x - x = 8743.434343 \dots - 87.434343 \dots$$

$$99x = 8743 - 87$$

$$99x = 8656$$

$$x = \frac{8656}{99}$$

Q $1.0232323 \dots$ as a vulgar fraction

$$\frac{10.232323 \times 10}{10} \Rightarrow \frac{(1023 - 10) \times 1}{99 \times 10}$$

$$\frac{1013}{990} \text{ Ans}$$

Linear & Quadratic Equation.

62
- 13
49

Linear Equation.

Solve for x.

A) $6x - 77 + 3x = 4 + 9 - x$ (D) $3(x + 10y + 4) + 8 = 5(x + 6y)$

$$10x = 13 + 27$$

$$10x = 40$$

$$x = 4 \text{ Ans}$$

$$3x + 30y + 12 + 8 = 5x + 30y$$

$$5x - 30x = 20$$

$$2x = 20$$

$$x = 10 \text{ Ans}$$

B) $19(x + 5) + 17 = 19(-x + 5) - 21$

$$19x + 195 + 17 = -19x + 195 - 21$$

$$38x = -78$$

$$x = -1 \text{ Ans}$$

(C) $x + \frac{x}{2} + \frac{x}{3} + 5 = 27$

$$\frac{11x}{6} + 5 = 27$$

$$11x = 22$$

$$x =$$

$$x = \frac{22 \times 6}{11} = 12 \text{ Ans}$$

$$x =$$

90
- 42
48

Solve the following

A) $5x + 5 = 13$; $x + 5y = 17$

$$(5x + 5 = 13) \times 5 \Rightarrow 25x + 55 = 65$$

$$(x + 5y = 17) \Rightarrow x + 5y = 17$$

$$24x = 48$$

$$x = 2$$

$$y = 3 \text{ Ans}$$

B) $4x - 5 = 13$; $2x + 3y = 31$

$$4x - 5 = 13 \Rightarrow 4x - 5 = 13$$

$$(2x + 3y = 31) \times 2 \Rightarrow 4x + 6y = 62$$

$$7y = 49$$

$$y = 7 \rightarrow x = 5 \text{ Ans}$$

Q If 6 pens and 3 pencils together costs 90 rs. while 5 pens and 6 pencils cost Rs 76, what will be the cost of each pen.

$$6x + 3y = 90$$

$$(6x + 3y = 90) \times 5 \Rightarrow 30x + 35y = 450$$

$$(5x + 6y = 76) \times 6 \Rightarrow 30x + 36y = 456$$

$$y = 6$$

$$x = 8$$

Concept of unique solution

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned} \rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Concept of Infinite and no soln.

Infinite soln:-

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned} \rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

No. soln:-

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned} \rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Quadratic eqn:-

$$ax^2 + bx + c = 0$$

Factorization method.

Ex 1:- $x^2 + 10x + 24 = 0$
 $a.c = 1 \times 24 = 24$ $\left\{ \begin{array}{l} 6 \\ 4 \end{array} \right\} + = b = 10$

$$\begin{aligned} x^2 + 6x + 4x + 24 &= 0 \\ x(x+6) + 4(x+6) &= 0 \Rightarrow (x+6)(x+4) = 0 \end{aligned}$$

$$x^2 + 5x + 6 = 0$$

$$a.c = 1 \times 6 = 6 \left\{ \begin{array}{l} 2 \\ 3 \end{array} \right\} + = 5$$

$$\begin{aligned} x^2 + 5x + 6 &= 0 \\ x(x+6) + 1(x+6) &= 0 \\ (x+6)(x+1) &= 0 \\ x &= -6, -1 \end{aligned}$$

$$2x^2 + 3x - 9 = 0$$

$$2 \times -9 = -18 \left\{ \begin{array}{l} -6 \\ 3 \end{array} \right\} +$$

$$\begin{aligned} 2x^2 + 6x - 3x - 9 &= 0 \\ 2x(x+3) - 3(x+3) &= 0 \\ (2x-3)(x+3) &= 0 \\ x &= \frac{3}{2}, -3 \text{ Ans} \end{aligned}$$

Quadratic formula:-

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

right angled

A triangle has sides of x , $x+1$, and 5.
 If the longest side is 5. Then x is

$$\begin{aligned} H &= 5 \\ H^2 &= P^2 + Q^2 \\ 5^2 &= x^2 + (x+1)^2 \\ 5^2 &= x^2 + x^2 + 1 + 2x \\ 0 &= 2x^2 + 2x - 24 \end{aligned}$$

$$\begin{aligned} x^2 + x - 12 &= 0 \\ 1 \times -12 = -12 \left\{ \begin{array}{l} -4 \\ 3 \end{array} \right\} + \\ x^2 + 4x - 3x - 12 &= 0 \\ x(x+4) - 3(x+4) &= 0 \end{aligned}$$

$$x = 3, -4 \quad [-4 \text{ (neglected)}]$$

1.5
2.8

Q Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

$$\begin{array}{lcl} \text{Rohan} = x & \rightarrow & x+3 \Rightarrow x+3 \\ \text{Rohan's mother} = 26+x & \rightarrow & 26+x+3 \Rightarrow 29+x \end{array}$$

$$(x+3)(29+x) = 360$$

$$29x + x^2 + 29 \times 3 + 3x = 360$$

$$x^2 + 32x + 87 = 360$$

$$x^2 + 32x$$

nature of roots

$$ax^2 + bx + c = 0$$

$$b^2 - 4ac \rightarrow 0$$

i) Two distinct real roots = $b^2 - 4ac > 0$

ii) Two equal ^{real} roots = $b^2 - 4ac = 0$

iii) No real roots = $b^2 - 4ac < 0$

Q find the nature of the roots.

$$i) 2x^2 - 3x + 5 = 0$$

$$\Rightarrow b^2 - 4ac = (3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31$$

Here $b^2 - 4ac < 0 \Rightarrow$ no real roots

$$ii) 3x^2 - 4\sqrt{3}x + 4 = 0$$

$$\Rightarrow b^2 - 4ac = (4\sqrt{3})^2 - 4 \times 3 \times 4 = 48 - 48 = 0$$

Two equal real roots

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{4\sqrt{3}}{6}$$

$$= \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}$$

$$\text{Q1b } 2x^2 - 6x + 3 = 0$$

$$\begin{aligned} b^2 - 4ac &= 36 - 4 \times 2 \times 3 = \\ &= 36 - 24 \\ &= 12 > 0. \end{aligned}$$

Two distinct real roots

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{+6 \pm \sqrt{12}}{4}$$

$$\Rightarrow \frac{3}{2} + \frac{\sqrt{12}}{4}, \frac{3}{2} - \frac{\sqrt{12}}{4}$$

$${}^6C_3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$$

Probability

Q A committee of 3 members is to be selected out of 3 men and 2 women. What is the probability that the committee has at least one woman?

$P(\text{at least 1 woman})$

Favorable outcomes: 2 men + 1 woman or 1 woman + 2 men

$$\begin{aligned} \Rightarrow \frac{{}^2C_1 \times {}^3C_2 + {}^2C_2 \times {}^3C_1}{{}^5C_3} &= \frac{2 \times 3 + 1 \times 3}{10} = \frac{6}{10} = \frac{3}{5} \text{ Ans} \end{aligned}$$

Q If A speaks the truth 80% of the time B speaks the truth 60% of the times. What is the probability that they both will speak truth?

$P(A \& B) \Rightarrow \text{Truth}$

$$A = 80\% = \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$$

$$B = 60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

$$P(A \& B = \text{Truth}) = \frac{4}{5} \times \frac{3}{5} = \frac{12}{25} = 0.48$$

Coins

Q When two coins are tossed, find the probability of getting at least one head.

outcomes $\rightarrow \{HH, HT, TH, TT\}$

$$P(\text{at least one head}) = \frac{3}{4}$$

Q When three coins are tossed find the probability of getting.

$$\text{Total outcomes} = 2^3 = 8$$

$$= \{HHH, HHT, HTH, THH, THT, TTH, TTT, HTT\}$$

(i) AT least 2 heads.

$$= \frac{4}{8} = \frac{1}{2}$$

(ii) AT least 1 head and 1 tail.

$$= \frac{6}{8} = \frac{3}{4}$$

(iii) AT most 2 Heads.

$$= \frac{7}{8}$$

(iv) No Heads

$$= \frac{1}{8}$$

(v) AT least 1 head

$$= \frac{7}{8}$$

Dice

Q When 2 Dice are rolled, how many ways are possible to get a sum as 3 or 5 or 7?

2 Dice rolls: Total outcomes = 36

$$\text{For sum 3} = \{(1, 2), (2, 1)\}$$

$$\text{For sum 5} = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$$

$$\text{For sum 7} = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

$$P(3 \text{ or } 5 \text{ or } 7) = \frac{12}{36} = \frac{2}{6} = \frac{1}{3}$$

There are 12 possible outcomes or ways.

Q When 3 dice are rolled, what is the probability of getting the sum as 8?

For 2 Dice

$$\text{Sum of 8} = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

$$= P(\text{sum of 8}) = \frac{5}{36}$$

For 3 Dice

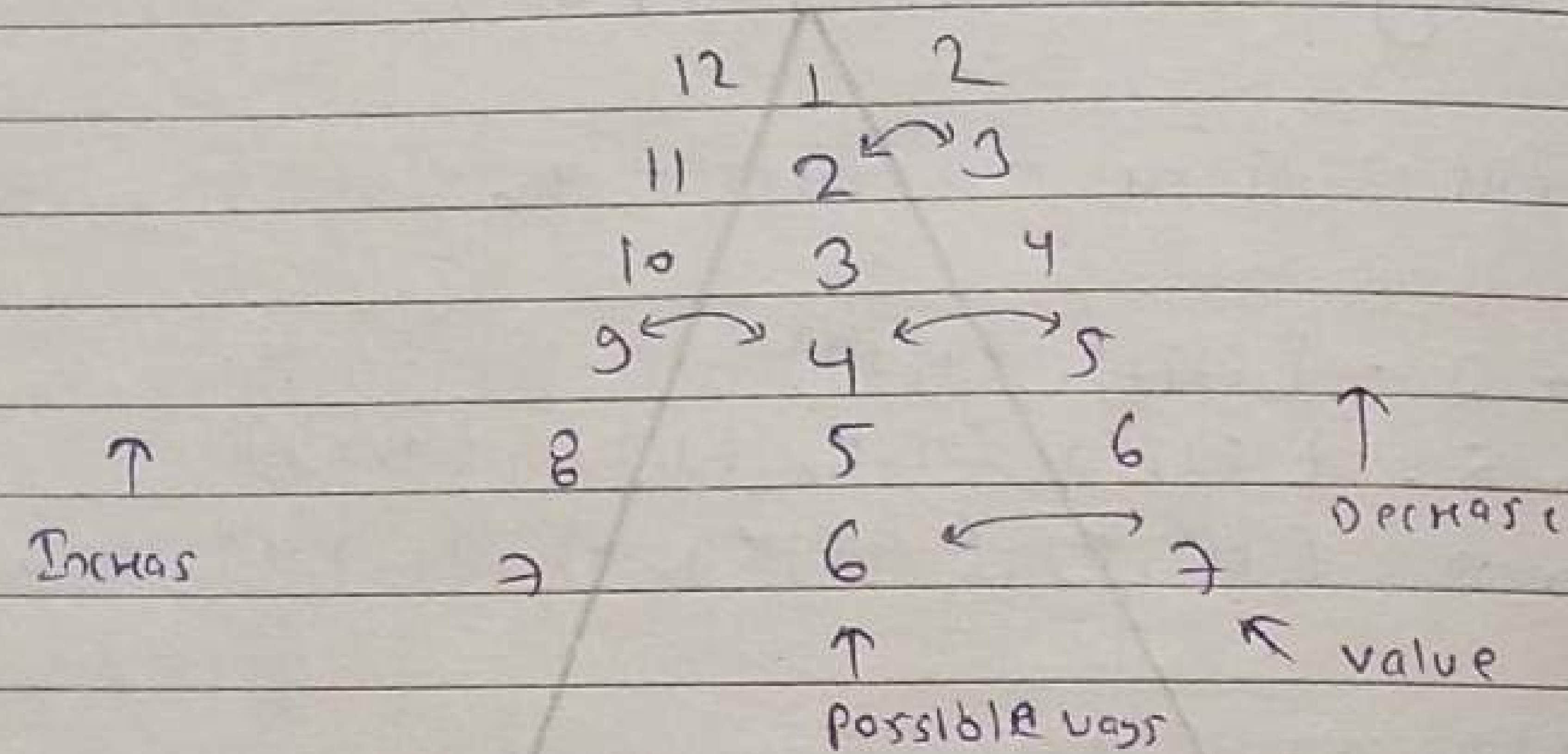
$$\text{Sum of 8} = (1, 1, 6), (1, 2, 5), (1, 3, 4), (2, 2, 4), (2, 4, 2)$$

$$= \frac{3!}{2!1!} + \frac{3!}{2!1!} + \frac{3!}{1!1!1!} + \frac{3!}{2!1!} + \frac{3!}{2!1!} = 3 + 3 + 6 + 3 + 3 = 18$$

$$= \frac{18}{216} = \frac{1}{12}$$

For 2 dice rolled

Trick :- Pyramid Trick



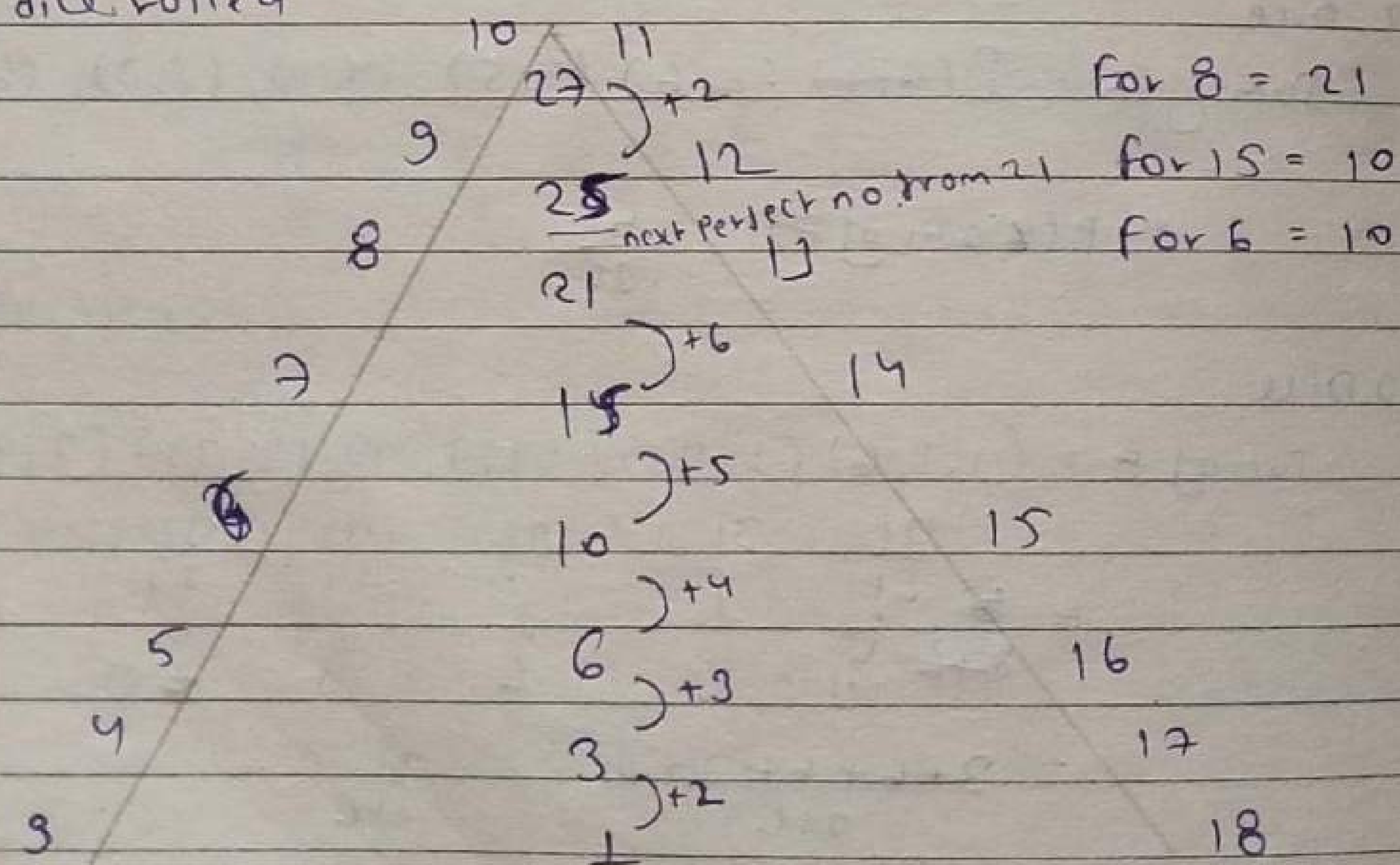
For 3 → 2

For 5 → 4, For 12 → 1

For 7 → 6

For 9 → 4

For 3 dice rolled



Bag & Ball

A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

2 Red, 3 Green, 2 Blue

$P(2 \text{ balls, no blue}) =$

one from red & one from green + Both Red + Both Green.

$$\Rightarrow \frac{{}^2C_1 \times {}^3C_1 + {}^2C_2 + {}^3C_2}{{}^7C_2}$$

$$\Rightarrow \frac{2 \times 3 + 1 + 3}{7 \times 6 / 2 \times 1} = \frac{10}{21} \text{ Ans}$$

Ques. Two bags are there, in the first bag there are 5 red & 3 blue where as in the second bag there are 6 red and 2 blue. Find the probability of drawing 2 red and 1 blue colored balls?

$P(2R \& 1B)$

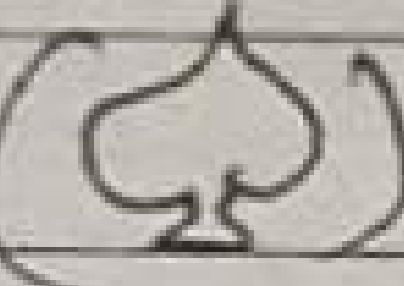
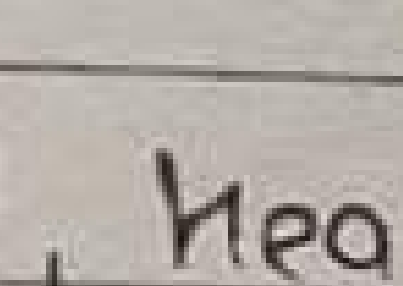
Either Bag 1 or Bag 2.

$$= \frac{1}{2} \left[\frac{{}^5C_2 \times {}^9C_1}{{}^8C_3} \right] + \frac{1}{2} \left[\frac{{}^6C_2 \times {}^2C_1}{{}^8C_3} \right]$$

$$= \frac{1}{2} \left[\frac{30}{56} \right] + \frac{1}{2} \left[\frac{90}{56} \right]$$

$$= \frac{30}{56} \times \frac{2}{2} = \frac{60}{112}$$

Cards

Spade () , club () , Diamond () , Heart ()

Total = 52 cards

Red
(26 cards)

Black
(26 cards)

Clubs (13)

Spades (13)

Diamond (13)

Hearts (13)

Ace, 2, 3, 4, 5, 6,
7, 8, 9, 10,
Jack, Queen,
King.

Ace, 2, 3, 4, 5,
6, 7, 8, 9, 10,
Jack, Queen,
King

Ace, 2, 3, 4, 5,
6, 7, 8, 9, 10,
Jack, Queen,
King

Ace, 2, 3, 4, 5,
6, 7, 8, 9, 10,
Jack, Queen,
King.

Q. A card is drawn from a deck of cards. Find the probability of getting.

i) A spade card. $= \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

ii) 2 Red cards $= \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26 \times 25}{52 \times 51} = \frac{25}{102}$

iii) A King. $= \frac{4}{52} = \frac{1}{13}$

iv) 3rd ace cards $= \frac{0}{52} = 0$

v) 2-face cards $= \frac{{}^{12}C_2}{{}^{52}C_2} = \frac{12 \times 11}{52 \times 51} = \frac{11}{221}$

vi) A-Red face card $= \frac{{}^6C_1}{{}^{52}C_1} = \frac{6}{52}$

vii) Queen of hearts $= \frac{1}{52}$

viii) Neither the heart nor the king.

no. of hearts = 13

no. of kings = 4 including 1 heart

Total = 16

$$\Rightarrow \frac{52 - 16}{52} = \frac{36}{52}$$

xix) Neither an Ace nor a King.

$$\text{no. of Ace} = 4$$

$$\text{no. of Kings} = 4$$

$$\text{Total to be selected} = 52 - 8 = 44$$

$$P(\text{Neither Ace nor King}) = \frac{44}{52} = \frac{11}{13}$$

(X) Probab: 2 cards but not black cards.

$$\Rightarrow \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26 \times 25}{52 \times 51} = \frac{25}{102}$$

Time, Speed & Distance

$$\text{Distance :- } \text{Km} \times 1000 = 1000 \text{ m}$$

$$\text{Time :- } 1 \text{ hr} = 60 \text{ min} = 3600 \text{ sec}$$

$$1 \text{ min} = 60 \text{ sec}$$

$$\text{Speed :- } \frac{\text{Km}}{\text{hr}} = \frac{1000 \text{ m}}{3600 \text{ sec}} = \frac{5 \text{ m}}{18 \text{ sec}}$$

Ques: Walking at $(S/3)^{\text{th}}$ of your usual speed, you will reach the market 16 minutes late. What is the usual time taken by you, to reach the market?

Thick:

distance same so.

usual = unusual.

$$\text{speed}_1 \times \text{time}_1 = \text{speed}_2 \times \text{time}_2$$

$$\frac{S}{3} \times (t + \frac{760}{60}) = S \times t$$

$$\frac{S(t + \frac{760}{60})}{3} = S \times t$$

$$\frac{S}{3} + \frac{S \times \frac{760}{60}}{3} = t$$

$$t - \frac{S}{3} = \frac{S \times \frac{760}{60}}{3}$$

$$2t = \frac{S \times 760}{3}$$

$$t = 40 \text{ minutes}$$

$$t = \frac{S \times 8}{3} = \frac{S \times 980}{3}$$

$$t = 1900 \text{ sec}$$

$$t = \frac{1900}{3} = 633.33 \text{ sec} = 10 \text{ min } 33.33 \text{ sec}$$

$$\frac{S}{3} \rightarrow t + 16 \text{ min}$$

$$\frac{S}{3} \rightarrow \frac{7}{5} t$$

$$t + 16 = \frac{7}{5} t$$

$$5t - 7t = -5 \times 16$$

$$2t = 80$$

$$t = 40 \text{ min.}$$

Ques:- Rohit drives from his home at a speed of 30 km/hr and reaches his bank 20 minutes late. Then the next day he increases his speed by 15 km/hr but still he is late by 8 minutes. How far is his bank from his home?

$$t_1 = t + 20 \text{ min}, \quad t_2 = t + 8 \text{ min.}$$

subtract both eqn

$$t_1 - t_2 = t + 20 - t - 8$$

$$t_1 - t_2 = 12 \text{ mins.}$$

$$\left(t = \frac{D}{S} \right)$$

$$\frac{D}{30} - \frac{D}{45} = 12$$

$$\frac{30 - 20}{90} = 12$$

$$\frac{D}{90} = \frac{12}{60}$$

$$D = \frac{12 \times 90}{60}$$

$$D = 18 \text{ km}$$

Q A farmer travelled a distance of 61 km in 5 hours. He travelled partly on foot @ 4 km/hr and partly on bicycle @ 9 km/hr. The distance travelled on foot is:-

$$d_f = x, \quad d_b = 61 - x$$

$$t_f = t, \quad t_b = 5 - t$$

$$t_f + t_b = 5 \text{ hr}$$

$$t = \frac{\text{Distance}}{\text{Speed}}$$

$$\frac{x}{4} + \frac{61-x}{9} = 5$$

$$\frac{9x + 244 - 4x}{36} = 5$$

$$5x + 244 = 180$$

$$5x = 180 - 244$$

$$5x = 80$$

$$x = 16 \text{ km}$$

Ques shilpa travelling at the speed of 100 kmph reaches her destination in 80 min. If she travels at the speed of 125 kmph, in how many minutes will she reach her destination?

$$100 \times 80 \times 60 = 125 \times t$$

$$t = \frac{100 \times 80 \times 60}{125} = 3840 \text{ min}$$

$$t = \frac{3840}{60} = 64 \text{ min}$$

$$t = 64 \text{ min}$$

Ques:- Excluding stoppages, the speed of a bus is 54 km/h and including stoppages, it is 45 km/h. For how many minutes does the bus stop per hour.

$$54 \text{ km/hr} = 1 \text{ hr (no stoppage)}$$

$$45 \text{ km/hr} = 1 \text{ hr (stoppages)}$$

$$\text{Distance covered by stoppages} = 54 - 45 \\ = 9 \text{ km.}$$

$$T = \frac{D}{S} = \frac{9}{54} = \frac{1}{6} \text{ hr.}$$

$$T = \frac{1}{6} \times 60 \text{ min} = 10 \text{ min.}$$

Avg Average speed.

Ques:- If you travel 210 km from Town A to Town B at a speed of 30 kmph and from there to Town C at a speed of 20 kmph for another 140 km, then what is your average speed during the entire journey?

$$\text{Avg speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

$$T_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{210}{30} = 7 \text{ hr}$$

$$T_2 = \frac{\text{Distance}}{\text{Speed}} = \frac{140}{20} = 7 \text{ hr}$$

$$\text{Avg speed} = \frac{210 + 140}{7 + 7} = \frac{350}{14} = 25 \text{ kmph}$$

Ques:- Arjun covers a distance by cycle at 18 km/h. He then goes to the starting point in a bus at a speed of 45 km/h. Find the average speed for the journey.

$$\text{Distance same} \Rightarrow \text{Avg speed} = \frac{2 \times S_1 \times S_2}{S_1 + S_2}$$

$$\text{Time same} \Rightarrow \text{Avg speed} = \frac{2 \times S_1 \times S_2}{S_1 + S_2} \times \frac{S_1 + S_2}{2}$$

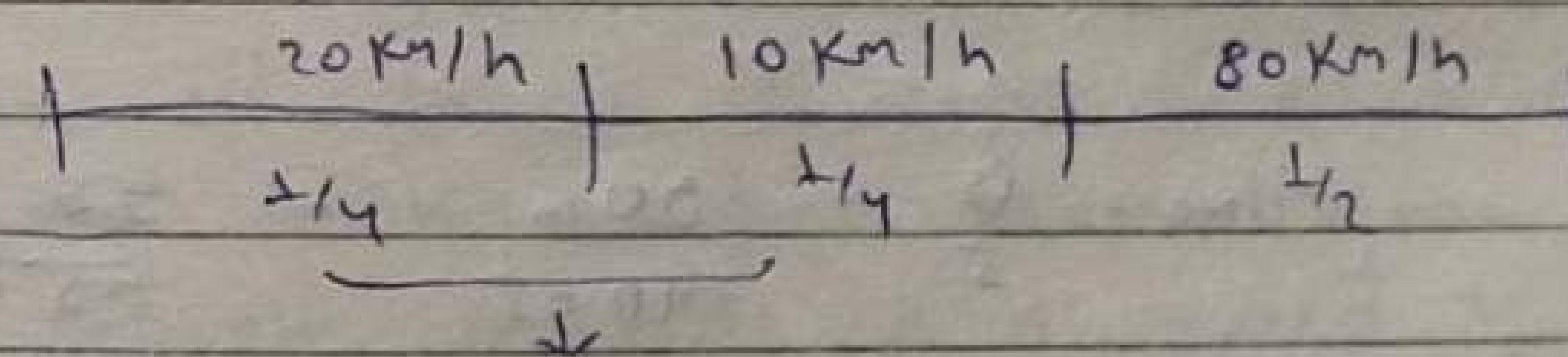
Here, Distance is same:

$$\text{Avg speed} = \frac{2 \times 18 \times 45}{45 + 18} = \frac{1620}{63} = 25.71 \text{ km/hr}$$

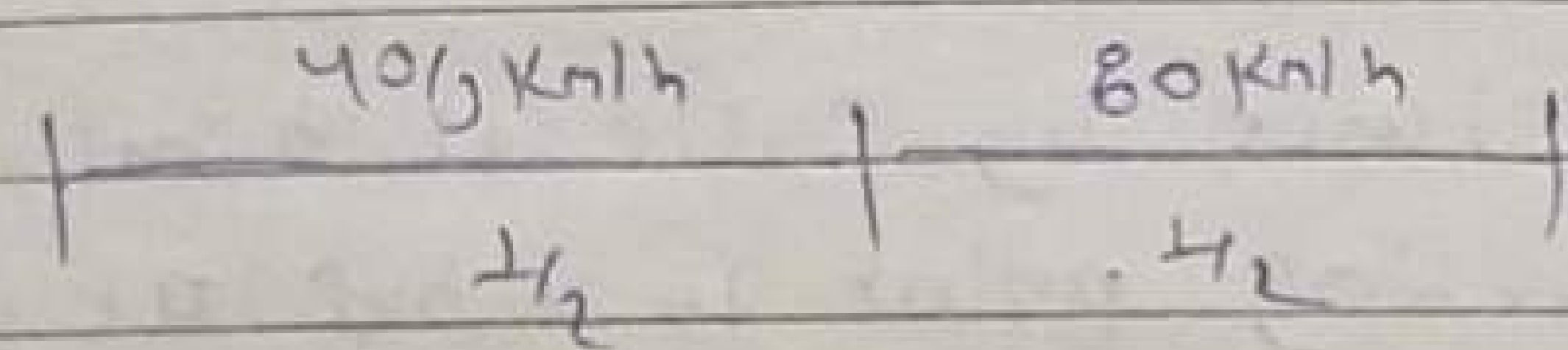
$$= \frac{2 \times 18 \times 45}{63} = 25.71 \text{ km/hr}$$

$$= 180 = 25.71 \text{ km/hr}$$

Ques:- Krishna covers one-fourth of the total distance at 20 km/hr, one-fourth at 10 km/hr and rest of his journey at 80 km/h. Find Krishna's average speed for the whole distance?



$$\text{Avg speed} = \frac{2 \times 20 \times 10}{20 + 10} = \frac{2 \times 200}{30} = \frac{400}{30} = 13.33 \text{ km/hr}$$



$$\text{Avg speed} = \frac{2 \times \frac{40 \times 80}{2}}{40 + 80} = \frac{2 \times 40 \times 80}{40 + 240} = \frac{6400}{280} = \frac{160}{7} = 22.85 \text{ km/h}$$

Relative speed.

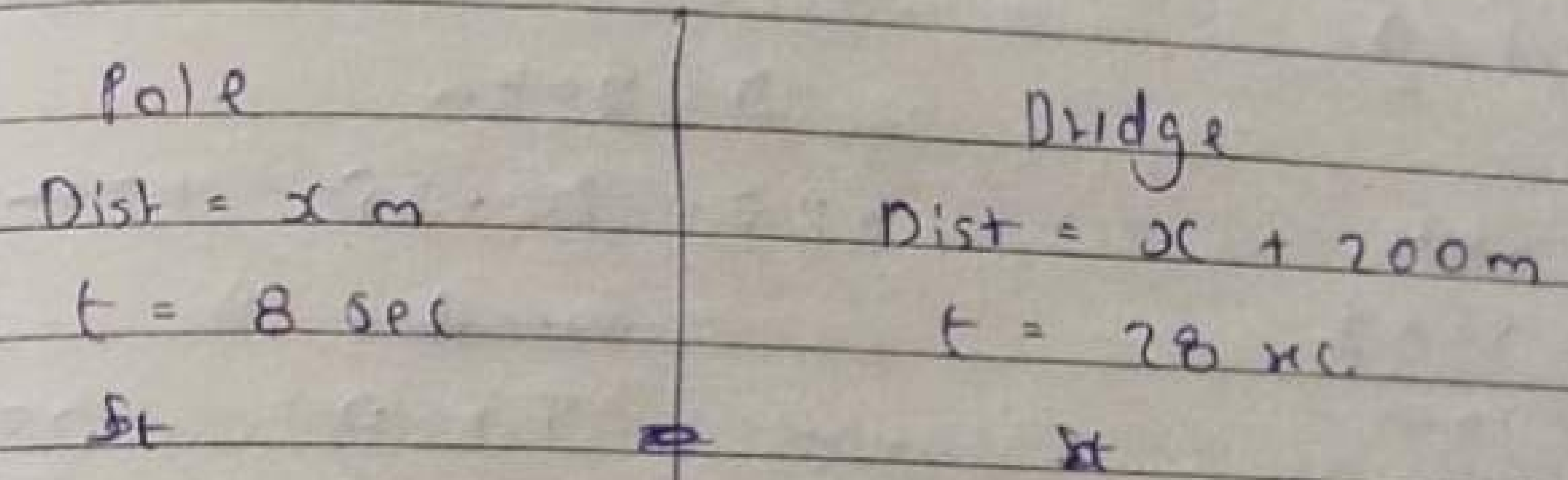
Que:- A train 90m long is travelling at a speed of 40 km/h. A man is running at a speed of 5 km/h in a direction opposite to the motion of the train. What would be the time taken by the train to cross the man?

Train
 $L = 90 \text{ m}$
 $S = 40 \text{ km/h}$

man
 $S = 5 \text{ km/h}$

$$\text{time} = \frac{D}{S} = \frac{90 \text{ m}}{40 + 5} = \frac{90}{45} = \frac{90 \times 1000}{45 \times 1000} = \frac{90000}{45000} = \frac{36}{5} = 7.2 \text{ sec}$$

Que: A train passes a stationary pole in 8 sec. The train also passes a 200m long bridge in 28 seconds. What is the length and the speed of the train?



Speed is equal.

$$S_t = S_r$$

$$\frac{x}{8} = \frac{x + 200}{28}$$

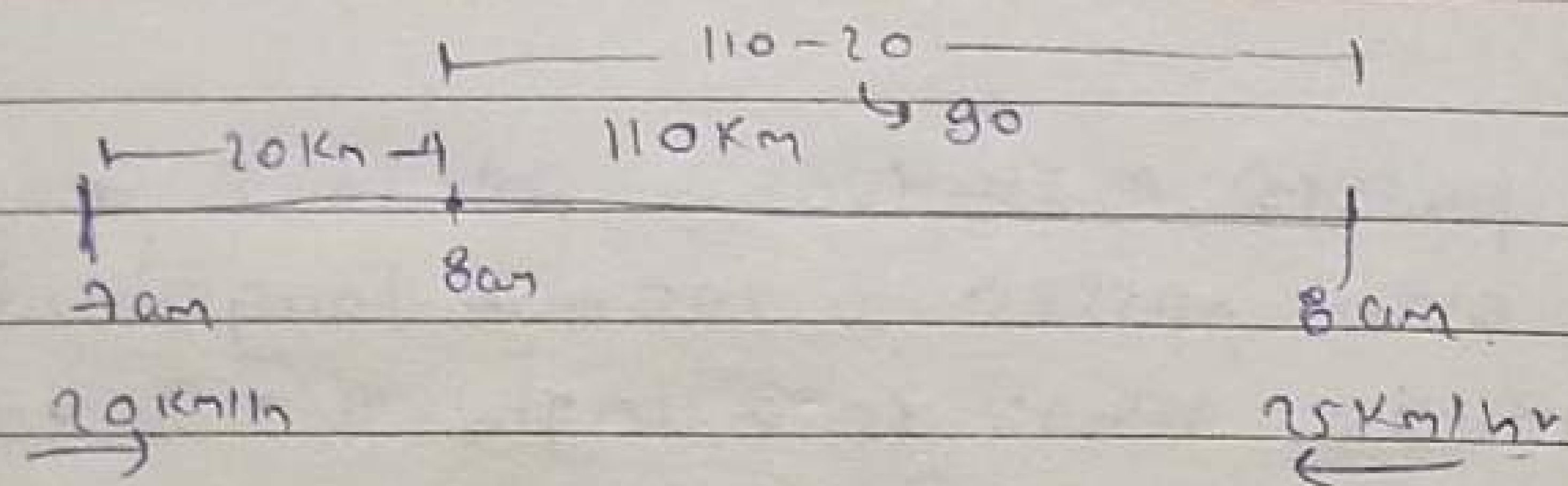
$$28x = 8x + 1600$$

$$20x = 1600$$

$$x = \frac{1600}{20} = 80 \text{ m}$$

$$\text{Speed of train} = \frac{x}{8} = \frac{80}{8} = 10 \text{ m/s}$$

Que: Two stations are A and B are 110 km apart on a straight line. One train starts from A at 7 am and travels towards B at a speed of 20 km/h. Another train starts from B at 8 am and travels towards A at speed of 25 km/h. At what time will they meet?



$$\text{Speed} = \frac{D}{T}$$

$$D = S \times t$$

$$D = 20 \times 2 = 40 \text{ km}$$

$$D = 90 \text{ km}$$

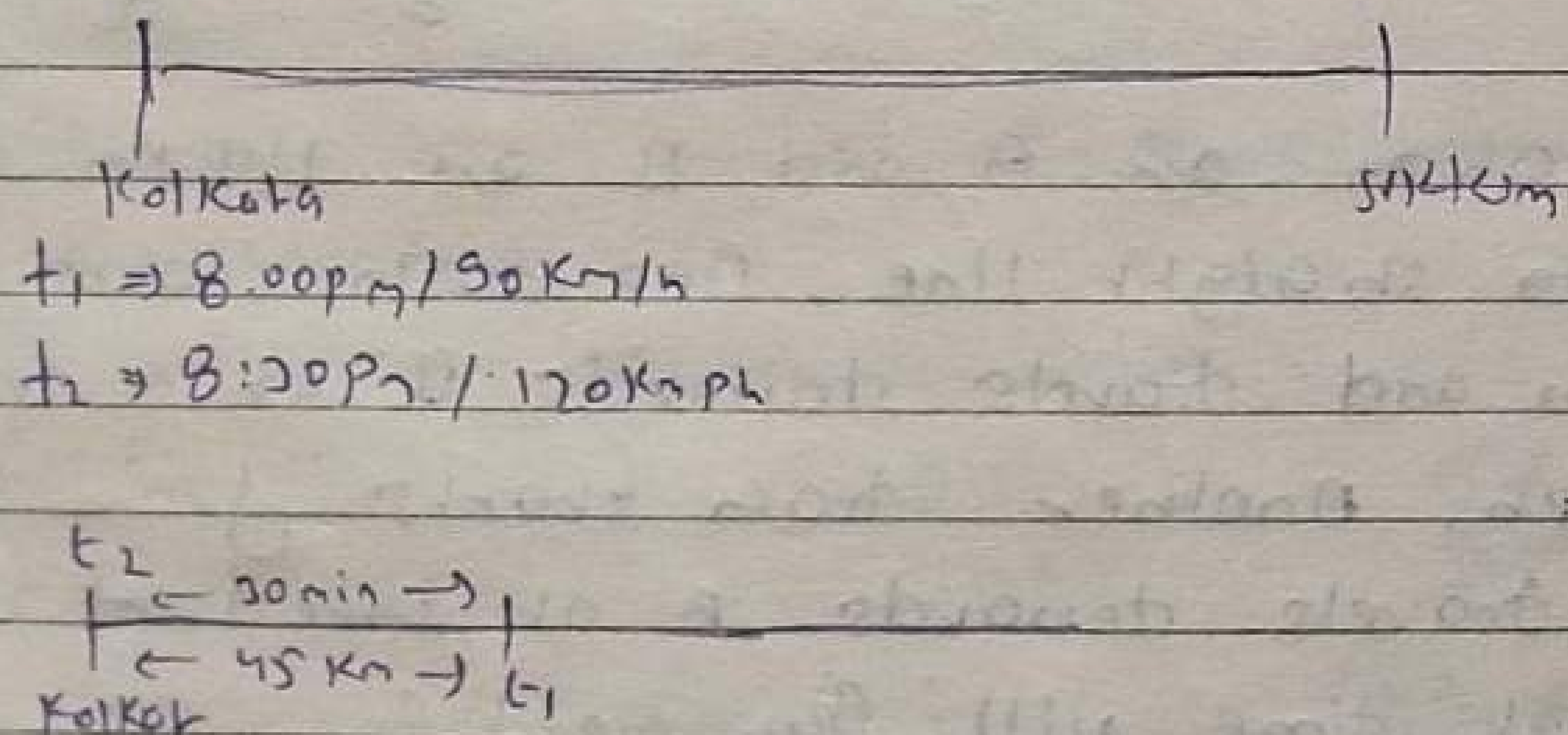
$$RS = 20 + 25 = 45 \text{ km/hr}$$

$$\text{Speed} = \frac{D}{T}$$

$$T = \frac{D}{S} = \frac{90}{45} = 2 \text{ hr}$$

$$Pos = 10 \text{ a.m.}$$

Que:- Two trains A and B leave Kolkata for Sikkim at 8:00 pm and 8:30 pm respectively and run at 90 km/h and 120 km/h respectively. At what distance from Kolkata, will the two trains meet?



$$S = 90 \text{ kmph}$$

$$t = 30 \text{ min} = \frac{1}{2} \text{ hr}$$

$$D = 90 \times \frac{1}{2} = 45 \text{ km}$$

$$D = 45 \text{ km}$$

$$RS = 120 - 90 = 30 \text{ km/h}$$

$$T = \frac{D}{S} = \frac{45}{30} = \frac{3}{2} = 1.5 \text{ hr}$$

So, The Distance is

$$D = \text{Speed} \times \text{time} = 120 \times \frac{3}{2} = 180 \text{ km}$$

Que:- A train overtakes two boys, who are running at 8 km/hr and 16 km/hr in the same direction as the train. The train completely passes them in 26 seconds and 40 seconds respectively. What is the length of the train?

$$\text{length of the train} = x, \text{ speed of train} = y$$

Boy (1)

$$D = x$$

$$t = 26 \text{ sec}$$

$$RS = y - 8$$

$$x = 26 \times (y - 8) \times \frac{5}{18}$$

$$26 \times (y - 8) \times \frac{5}{18} = 40 \times (y - 16) \times \frac{5}{18}$$

$$26y - 26 \times 8 = 40y - 40 \times 16$$

Boy (2)

$$D = x$$

$$t = 40 \text{ sec}$$

$$RS = y - 16$$

$$x = 40 \times (y - 16) \times \frac{5}{18}$$

$$40y - 36y = 40 \times 16 - 36 \times 8$$

$$4y = 640 - 288$$

$$4y = 352$$

$$y = 88$$

Then, length of the train is.

$$= 26 \times (y - 8) \times \frac{5}{18}$$

$$= 26 \times (88 - 8) \times \frac{5}{18}$$

$$= 26 \times 80 \times \frac{5}{18}$$

$$= 800 \text{ m Ans}$$

Effective speed.

Upstream: Against the flow [speed_{obj} - speed_{stream}]

Downstream: Along the flow. [speed_{obj} + speed_{stream}]

Q A Boat can travel at a speed of 12 Km/h in still water. If the speed of the stream is 4 Km/h, find the time taken by the boat to travel 68 Km downstream.

Effective speed = Speed (boat) + Speed (stream).

$$\text{E Speed} = 12 + 4 = 17$$

$$\text{Time Taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{68}{17} = 4 \text{ hours.}$$

Q A motorboat travels 40 Km Downstream in 5 hours and 30 Km upstream in 6 hours. Find the speed of the stream (in Km/h).

DS

$$D = 40 \text{ Km}$$

$$T = 5 \text{ h}$$

$$ES = 8 \text{ Km/h}$$

US

$$D = 30 \text{ Km}$$

$$T = 6 \text{ h}$$

$$ES = 5 \text{ Km/h}$$

Effective speed in DS and US.

$$ES = S_0 + S_s$$

$$8 = S_0 + S_s$$

$$ES = S_0 - S_s$$

$$5 = S_0 - S_s$$

$$8 = S_0 + S_s = 8$$

$$-S_0 + S_s = -5$$

$$2S_s = 3$$

$$S_s = \frac{3}{2} = 1.5 \text{ Km/h}$$

Que:- A man can row a boat at a speed of 6 Km/h in still water. If it takes him twice as long to row up as to row down the river, the rate of current in the stream would be.

$$T_u = 2T_d$$

$$\text{means } \frac{D}{S_u} = 2 \times \frac{D}{S_d}$$

$$S_d = 2S_u$$

$$S_0 + S_r = 2(S_0 - S_r)$$

$$6 + S_r = 2 \times 6 - 2S_r$$

$$3S_r = 6$$

$$S_r = 2$$

Q: A boat sails 15 km of a river towards upstream in 5 hours. How long will it take to cover the same distance downstream, if the speed of current is one-fourth the speed of the boat in still river water?

UP

$$D = 15 \text{ km}$$

$$t = 5 \text{ hr}$$

$$E(S_u) = 3 \text{ km/h}$$

DS

$$D = 15 \text{ km}$$

$$S_0 = S_0 + S_r$$

$$S_0 = 4 + 1 = 5$$

$$S_r = \frac{1}{4} S_0$$

$$S_0 = S_0 - S_r$$

$$= \cancel{S_0} - \frac{1}{4} S_0$$

$$3S_0 = \frac{3}{4} S_0$$

$$S_0 = 4$$

$$\text{Time taken} = \frac{15}{5} = 3 \text{ hr}$$

Q: Ratio b/w speed of boat in still water to speed of stream is 5:2. If 224 km is travelled by downstream in 4 hours. Find the difference between speed of boat in still water and speed of stream?

$$S_0 = 5x$$

$$S_r = 2x$$

Dis.

$$D = 224 \text{ km}$$

$$t = 4 \text{ hr}$$

$$S_0 = \frac{224}{4} = 56 \text{ kmph}$$

$$S_0 = 56 \text{ kmph}$$

$$S_0 + S_r = 56$$

$$5x + 2x = 56$$

$$x = \frac{56}{7} = 8$$

Difference

$$\rightarrow 5x - 2x = 3x$$

$$\Rightarrow 3 \times 8 = 24 \text{ Ans}$$

Q: A boat goes 260 km upstream and returns to the same point in 25 hrs. If the speed of current is 3 km/h what is speed of boat?

$$t_u + t_d = 25$$

$$\frac{D}{S_0} + \frac{D}{S_u} = 25$$

$$\frac{260}{(x+3)} + \frac{260}{(x-3)} = 25$$

$$D = 260$$

$$S_u = x - 3$$

$$S_0 = x + 3$$

Concept of linear race

Ques: In a race of 200m, A can beat B by 21m and C by 18m.
In a race of 350m C will beat B by?

A	B	C	
200 m	179 m	182 m	
	?	350 m	Cross multiply

$$x = \frac{179 \times 350}{182} = 328 \text{ m}$$

Ques: In a race of 500m, the ratio of the speeds of two contestants A and B is 3:4. A has a head start of 140m. In A will win by?

Distance should cover by A = $500 - 140$
= 360m

A = 360 m

B = 500 m

A	→	B
3 m	→	4 m
360 m	→	?

$$x = \frac{360 \times 4}{3} = 480 \text{ m}$$

B cover 480 m

A will win by 20 m.

Q In a 200m race, A gives B a head start of 25m and wins by 10sec. If A gives B a head start of 45m, the race ends in a dead heat, what will be the time taken by A to complete the race?

A	B
200 m	175 m
t sec	t + 10 sec
200 m	155 m
t sec	t sec

B → 20m in 10 sec

So, 155m = $\frac{155 \times 10}{20} \text{ sec}$

t = 37.5 sec

Concept of Circular race

Q: Two persons start running simultaneously around a circular track of length 200m from the same point at speeds of 15 km/hr and 25 km/hr. When will they meet for the first time anywhere on the track if they are moving in opposite directions?

$T = \frac{P}{S}$

$P = 15 + 25 = 40 \text{ km/hr} = \frac{40 \times 5}{18} \text{ m/sec} = \frac{100}{9} \text{ m/sec}$

$$T = \frac{200m}{\frac{100}{5}} = \frac{200 \times 5}{100} = 10 \text{ sec}$$

Ques: Three friends A, B and C run around a circular of length 120 m at speed of 5 m/sec, 7 m/sec and 15 m/sec, starting simultaneously from the same point and in the same direction. How often will the three of them meet?

Time for A & B $\rightarrow D = 120m$ $T = \frac{120}{2} = 60 \text{ sec}$
 $R.S = 7 - 5 = 2 \text{ m/s}$

Time for B & C $\rightarrow D = 120m$ $T = \frac{120}{8} = 15 \text{ sec}$
 $R.S = 15 - 7 = 8 \text{ m/s}$

Time for C & A $\rightarrow D = 120m$ $T = \frac{120}{10} = 12 \text{ sec}$
 $R.S = 15 - 5 = 10 \text{ m/s}$

(60 sec, 15 sec, 12 sec)
 LCM $\rightarrow 60 \text{ sec}$

Permutation and Combination

Concept of Principle of Counting:-

or $\rightarrow +$ and $\rightarrow \times$.

Q. A shopping mall has 2 distinct glass doors and 2 distinct metal doors for entry and has 5 distinct glass doors and a wooden door for exit.

Entry: G_1, G_2, G_3, M_1, M_2
 Exit: $G_1, G_2, G_3, G_4, G_5, W_1$

(i) In how many ways can you enter the mall?
 $\rightarrow 5 \text{ ways}$

(ii) In how many way can you leave the mall?
 $\rightarrow 6 \text{ ways}$

(iii) In how many way the entry and the exit can happen?
 $\rightarrow 5 \times 6 = 30 \text{ ways}$

Q. If there are 2 trains from A to B, 5 trains from B to C, 6 trains from C to D and 2 trains from D to A. In how many ways can a person travel from A to D (via B to C) and return back to A again?

$$A-B = 3 \text{ trains}$$

$$A-C = 5 \text{ trains}$$

$$C-D = 6 \text{ trains}$$

$$D-A = 2 \text{ trains}$$

$$A-D$$

$$A-B \& B-C \& C-D \& D-A$$

$$3 \times 5 \times 6 \times 2 = 180 \text{ ways}$$

Q. Each of 2 friends has 20 stamps and 10 post cards. we all call it an exchange fair if they exchange a card for a card (or) stamp for a stamp. how many ways are there to carry out one fair exchange between these two friends?

A

$$20S \& 10P$$

B

$$20S \& 10P$$

(card to card (or) stamp to stamp

$$(10 \times 10) + (20 \times 20) = 500 \text{ ways}$$

Q. The total no. of ways of answering 5 objective type questions each question having 4 choices is?

obj 1

obj 2

obj 3

obj 4

obj 5

$$(1+1+1+1) \times (1+1+1+1) \times (1+1+1+1) \times (1+1+1+1) \times (1+1+1+1)$$

$$4 \times 4 \times 4 \times 4 \times 4$$

$$1024 \text{ Ans}$$

Concept of Combination.

$${}^nC_r = \frac{n!}{r!(n-r)!} = n = \text{total elements}$$

$$= r = \text{favourable elements}$$

$${}^nC_n = 1$$

$${}^nC_1 = n$$

$${}^nC_{(n-1)} = n$$

$${}^nC_r = {}^nC_{n-r}$$

Q. A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways this can be done when at least 2 women are included?
6M & 4W

$$2W \& 3M \text{ or } 3W \& 2M \text{ or } 4W \& 1M$$

$$2 \times 3 + 3 \times 2 + 4 \times 1$$

$$6 + 6 + 4 = 16$$

$${}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$\frac{4 \times 3}{1 \times 2} \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} + \frac{4 \times 3 \times 2}{1 \times 2 \times 3} \times \frac{6 \times 5}{1 \times 2} + 1 \times 6$$

$$6 \times 20 + 4 \times 15 + 6$$

$$120 + 60 + 6$$

$$186 \text{ Ans}$$

Q. A Conference is attended by 25 participants. If each participant shakes hand with every other participant what will be the resultant number of handshaking?

$$= {}^{25}C_2 = \frac{25 \times 24}{1 \times 2} = 300 \text{ Ans.}$$

Q. A Department contains 17 male and 15 female employee. In how many ways can a male and a female employee be chosen to represent the department?

$${}^{17}C_1 \times {}^{15}C_1 \\ = 17 \times 15 = 255$$

Q. Out of 11 points there are 4 collinear points. How many triangles can be formed by using the points as vertices?

$$\Rightarrow {}^{11}C_3 - {}^4C_3$$

$$\Rightarrow \frac{11 \times 10 \times 9}{1 \times 2 \times 3} - \frac{4 \times 3 \times 2}{1 \times 2 \times 3}$$

$$\Rightarrow 165 - 4 \\ \Rightarrow 161 \text{ Ans}$$

Q. How many parallelograms are formed by a set of 6 parallel lines (horizontal) intersecting another set of 4 parallel lines (vertical)?

$$\Rightarrow {}^6C_2 \text{ \& } {}^4C_2$$

$$\Rightarrow \frac{6 \times 5}{1 \times 2} \times \frac{4 \times 3}{1 \times 2}$$

$$\Rightarrow 15 \times 6 = 90 \text{ Ans}$$

Concept of Permutation.

$${}^nP_r = \frac{n!}{(n-r)!} \Rightarrow {}^{10}P_2 \Rightarrow 10 \times 9 = 90 \\ {}^5P_4 \Rightarrow 5 \times 4 \times 3 \times 2 = 120$$

$${}^nP_0 = n!$$

$${}^nP_1 = n$$

$${}^nP_{n-1} = n! = {}^nP_n$$

Q. How many 3 letter words can be formed by using different alphabets?

$${}^{26}P_3 = 26 \times 25 \times 24 = 15,600$$

Q. A out of 6 consonants and 5 vowels, how words of 3 consonants and 2 vowels can be formed.

6 cons & 5 vowels

3 cons & 2 vowels

Q Selection:- ${}^6C_3 \times {}^5C_2$

$$\text{Arrangement} : ({}^6C_3 \times {}^5C_2) \times {}^5P_5$$

$$= \left(\frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times \frac{5 \times 4}{1 \times 2} \right) \times 5!$$

$$= 20 \times 10 \times 120$$

$$= 2400$$

Q How many ways 5 letter words can be formed using the letters of the word "GREAT".

i) If repetition of alphabets are allowed.

$$5 \times 5 \times 5 \times 5 \times 5 \rightarrow 5^5 \rightarrow 3125$$

ii) If repetition of alphabets are not allowed.

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

iii) Such that all the vowels are together.

EA G R T

$$4! \times 2! = 24 \times 2 = 48 \text{ ans}$$

4) Such that all the vowels are not together.

Total Arrangements - vowel together

$$= 120 - 48 = 72$$

5) Such that all the constants are together

GRT E A

$$\rightarrow 3! \times 3! = 36$$

6) Such that all the consonants are not together.

Total arrangement - consonant together

$$120 - 36 = 84$$

7) Such that all the vowels are together and all the consonants are together

EA GRT

$$2! \times 2! \times 3!$$

$$\rightarrow 2 \times 2 \times 6 = 24$$

Alphabet and number related Problems:-

Q: In how many ways can be the letters of the word "STRANGE" be arranged so that the vowel may appear in odd places?

STRANGE

(a, e, i, o, u)

~~5!~~

$$5! \times 4P_2$$

$$120 \times 4 \times 3 \rightarrow 1440$$

Q: How many 5 digit no. can be formed on having the numbers 5, 1, 6, 7 and 4? (repetition not allowed)

~~5!~~

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

Q: How many 4 digit numbers can be formed on having the numbers 0, 1, 2, 3 and 4? (repetition not allowed)

$$4 \times 4 \times 3 \times 2 = 96$$

Q: How many 2 digit telephone numbers can be formed with 6 and 5 at the right and left extreme ends resp.

1: 5

6

2: 6

5

$$8 \times 7 \times 6 \times 5 \times 4 = 6720$$

Q: How many 6 digit even numbers can be formed from digits 1, 2, 3, 4, 5, 6, 7 so that the digit should not repeat and the second last digit is even?

even = 2, 4, 6

$$5 \times 4 \times 3 \times 2 \times 2 \times 3 = 720 \text{ ways}$$

Q: Find the number of 5 digit numbers that are formed by using digits 1, 2, 5, 6, and 7 (without repetition)

even = 2, 6

$$5! = 120$$

i) divisible by 2:

$$4 \times 3 \times 2 \times 1 \times 2$$

$$= 240 = 48$$

ii) divisible by 4:

$$\rightarrow 12, 16, 52, 56, 72, 76$$

$$3 \times 2 \times 1 \times 6 = 36 \text{ ways}$$

iii) Divisible by 5

(0 or 5)

$$4 \times 3 \times 2 \times 1 \times 1 \rightarrow \text{fixed} \quad (5)$$

$$= 24 \text{ ways}$$

Q What is the sum of all 4-digit numbers that can be formed using the digits 1, 2, 3, 4 where repetition of digits is not allowed?

Sum of ^{numbers} digit that are formed

$$= (\text{sum of digits}) \times (n-1)! \times 1111$$

$$= 10 \times 3! \times 1111$$

$$= 66660$$