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Practical limitations of Talbot imaging with microlens arrays

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Abstract. The Talbot effect which also appears for the foci of microlens arrays can, in principle, be used for array illuminators. In particular, the multiplication of the foci in the fractional Talbot planes seems to be promising. Certainly, in practice there are three effects which limit the application of the fractional Talbot effect for microlens arrays. Theoretical, numerical and experimental results are given to demonstrate these limitations.

1. Introduction

The Talbot effect is a phenomenon which appears for all periodic structures and thus also for the foci of a microlens array [1]. In the fractional Talbot effect, for planes with a distance $z = M/N\, z_T$ from the focal plane (z_T is the Talbot length and M and N are non-negative integers), the number of foci per line or column is multiplied by a factor N for odd N and by a factor $N/2$ for even N [2] so that the effective period between the multiplied foci is reduced by the same factor (see figure 1). The multiplied foci can be used, in principle,

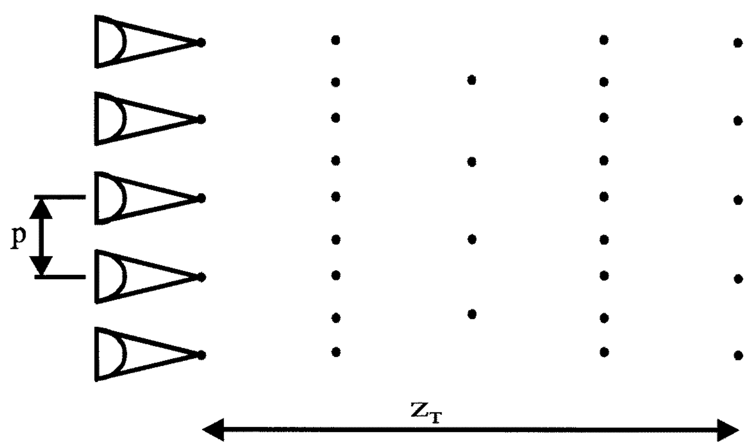


Figure 1. Illustration of the Talbot effect for the foci of an array of microlenses. In the $\frac{1}{4}$ and $\frac{3}{4}$ Talbot planes the number of foci is, for example, doubled whereas in the $\frac{1}{2}$ Talbot plane the number of foci is not increased but the foci are laterally shifted by half a period p. $z_T = 2p^2/\lambda$ is the Talbot length.

as array illuminators [3]. However, theoretical and experimental results show that there are several limitations on the practical use of this effect. In practice, there are three different effects which limit the application:

(i) The intensities of the multiplied foci in the fractional Talbot planes are in general different [4].

(ii) If large spatial frequencies are present, which is the case for lenses with high numerical apertures, the foci are blurred because the Talbot effect in the original meaning occurs only for small spatial frequencies [5].

(iii) Walk-off effects for arrays with a finite size are quite severe for lenses with high numerical apertures.

All of these effects will be discussed in the following in more detail, whereby for (i) an analytical method is used, for (i) and (ii) numerical simulations are explained and for (iii) a simple geometrical optical model is described.

2. Analytical expressions for the intensity of the foci in the fractional Talbot planes

An array of microlenses which is periodic in the two orthogonal directions x and y with period p is illuminated by a plane wave. It is assumed in this section that the array is of infinite extent or in practice that it is so large that walk-off effects can be neglected. Moreover, it is assumed in the derivation of the expressions that only small spatial frequencies occur, i.e. that the lenses only have small numerical apertures. The derivation of the expressions is described in an earlier publication [4] and only the equations which are essential for our considerations will be repeated.

The foci in the focal plane of the microlens array at $z = 0$ also form a periodic array. The complex wave amplitude $u(x, y, z = 0)$ of the foci can be represented by the convolution of the wave amplitude $A(x, y)$ of a single focus and the array generating function $g(x, y)$

$$u(x, y, z = 0) = A(x, y) \otimes g(x, y) \tag{1}$$

with

$$g(x, y) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \delta(x - kp)\delta(y - lp). \tag{2}$$

The symbol $\otimes$ is used to indicate a convolution between two functions. The amplitude function in a plane parallel to the focal plane at $z = z_0$ can be calculated with the method of the propagation of the angular spectrum of plane waves [6]:

$$u(x, y, z = z_0) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{u}(\nu_x, \nu_y) \exp\left[2\pi \mathrm{i} \frac{z_0}{\lambda} \sqrt{1 - \lambda^2 \left(\nu_x^2 + \nu_y^2\right)}\right] \times \exp\left[2\pi \mathrm{i} \left(\nu_x x + \nu_y y\right)\right] \mathrm{d}\nu_x \, \mathrm{d}\nu_y \tag{3}$$

where λ is the wavelength of the light, ν_x and ν_y are spatial frequencies and the function $\tilde{u}(\nu_x, \nu_y)$ is the Fourier transform of $u(x, y, z = 0)$:

$$\tilde{u}(\nu_x, \nu_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(x, y, z = 0) \exp\left[-2\pi \mathrm{i} \left(\nu_x x + \nu_y y\right)\right] \mathrm{d}x \, \mathrm{d}y. \tag{4}$$

Since we assume that $\tilde{u}$ has values considerably different from zero only for small spatial frequencies, i.e.

$$\nu_x^2 + \nu_y^2 \ll 1/\lambda^2 \tag{5}$$

and

$$z_0\lambda^3(\nu_x^2+\nu_y^2)^2 \ll 1 \tag{6}$$

the square root of equation (3) can be approximated by the first two terms of its Taylor series (parabolic approximation) and the wave amplitude in the fractional Talbot planes results in

$$u\left(x, y, z = \frac{M}{N}z_{\mathrm{T}}\right) = \exp\left[2\pi \mathrm{i}\frac{Mz_{\mathrm{T}}}{N\lambda}\right]\sum_{k=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty} F_{k,l,M,N}\; A\left(x - k\frac{p}{N}, y - l\frac{p}{N}\right) \tag{7}$$

with the complex factor

$$\begin{aligned} F_{k,l,M,N} &= \frac{1}{N^2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\exp\left[2\pi\mathrm{i}\frac{km + ln - M(m^2+n^2)}{N}\right] \\ &= \left(\frac{1}{N}\sum_{m=0}^{N-1}\exp\left[2\pi\mathrm{i}\frac{km - Mm^2}{N}\right]\right)\left(\frac{1}{N}\sum_{n=0}^{N-1}\exp\left[2\pi\mathrm{i}\frac{ln - Mn^2}{N}\right]\right) \\ &=: f_{k,M,N}\, f_{l,M,N}. \end{aligned} \tag{8}$$

The complex factor F is responsible for the fact that in the half Talbot plane the foci are not multiplied, but only shifted by half a period p because f, in this case, is zero for even numbers k or l. It should be mentioned that the theory up to here is not only valid if A is the wave amplitude of one focus of the microlens array, but also if it is an arbitrary function which is repeated after a period p in x and y directions and which is positioned in a plane at $z = 0$.

If we examine the influence of F and A on the intensity of the multiplied foci in the fractional Talbot planes we have to distinguish mainly between two cases:

(i) A is only different from zero in a small range compared to the reduced period p/N. Then, there will be no overlap between the multiplied functions A and the intensity of the peaks will be proportional to $|F|^2$, i.e. the multiplied foci will have equal intensities.

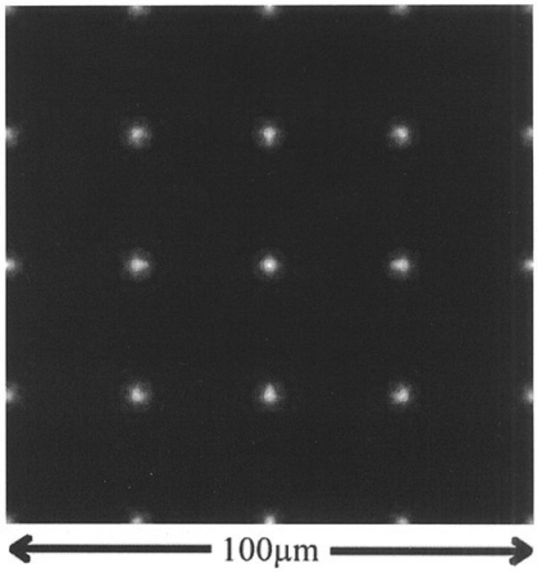
(ii) A is different from zero in a range comparable to p/N or larger than p/N. Then, the shifted functions $A(x - kp/N, y - lp/N)$ will overlap and the resulting wave amplitude is dependent on the complex factor F and A itself. This leads to the fact that the multiplied foci in the fractional Talbot planes will, in general, have different intensities. In practice, this effect occurs, for example, for a lens array with circular lenses and no stops between the lenses so that the light which passes through the space between the lenses interferes with the foci in the fractional Talbot planes. A numerical simulation which shows this effect will be given at the end of the next section.

3. Numerical simulations of the Talbot effect

If the assumption that only small spatial frequencies occur in the wave amplitude is no longer valid (e.g. for microlenses with high numerical apertures) the analytic expression (7) cannot be used. For this purpose we have implemented, in the latest version of our optical analysis and design software RAYTRACE 6.0, a numerical simulation tool which also uses the angular spectrum of plane waves and assumes only an infinite size of the microlens array. The method itself is described in our earlier publication [4].

With the help of the numerical simulation tool the result of the analytical calculations that the intensity of the multiplied foci in the fractional Talbot planes is, in general, different can be verified. Figure 2 shows the simulation of a refractive microlens array in the $\frac{1}{8}$ Talbot plane. The diameter of the planoconvex lenses is 80 μm, the period of the array is 100 μm,

(*a*)



(*b*)

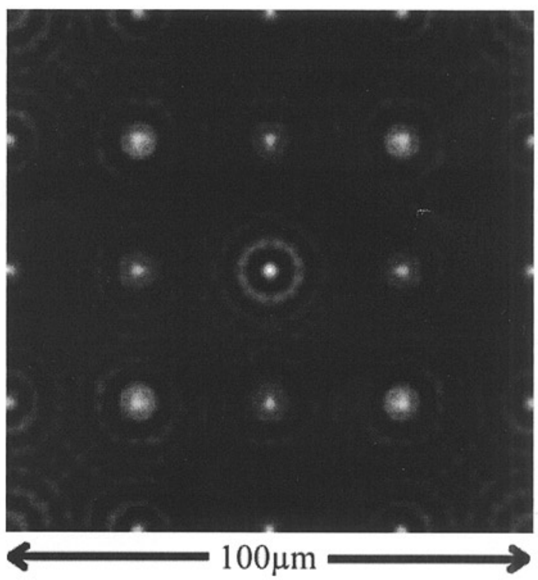


Figure 2. Simulation of the foci of a refractive microlens array in the $\frac{1}{8}$ Talbot plane. Shown is one period of the array with 4×4 multiplied foci. In (*a*) there are stops between the lenses whereas in (*b*) there are no stops between the lenses.

the focal length of the lenses is 250 μm and the wavelength is 633 nm. In figure 2(*a*) there are stops between the microlenses and it can be seen that there is only a small variation of the intensity of the multiplied foci. In figure 2(*b*) there are no stops between the lenses so that the light which passes through the space between the lenses interferes with the multiplied foci and a large variation of the intensity can be observed.

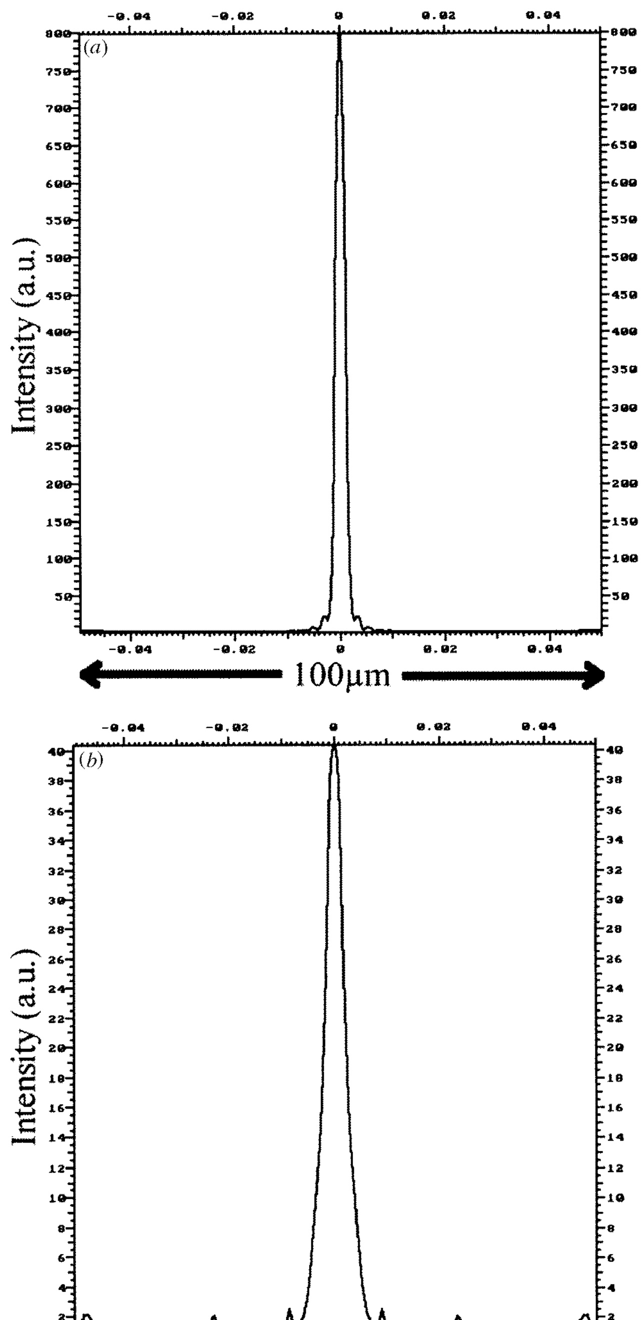


Figure 3. Simulation of the blurring of the foci in the 1 Talbot plane. (a) 0 Talbot plane (focal plane), (b) 1 Talbot plane.

Figure 3 shows a simulation of the intensity distribution in the 0 and 1 Talbot planes of the array without stops. The foci in the 1 Talbot plane are blurred. The reason for this is that equation (6) is not fulfilled and the higher-order terms in the Taylor series of the square root in equation (3) cannot be neglected. So, the conditions for the Talbot effect are violated to some degree and only a blurred self-image results in the 1 Talbot plane.

4. Simple model for walk-off effects

A simple geometrical optical model shall be used to describe the walk-off effects of microlens arrays. It is assumed in this model that the total light power in one period can be estimated with geometrical optics, whereas the distribution of light in one period is of course a multiple-beam interference effect which causes, for example, the formation of foci in the fractional Talbot planes.

After the focus, the light of one lens with numerical aperture NA forms a cone with the radius

$$x \approx z\,\mathrm{NA} \tag{9}$$

where z is the distance from the focus (see figure 4). So, the intensity I_{lens} of the light coming from one lens decreases as $1/z^2$:

$$I_{\mathrm{lens}} \propto \frac{1}{z^2}. \tag{10}$$

On the other hand, the lenses which supply light to a point at a distance z also lie in a circle with radius x so that their number N_{lens} increases proportionally with z^2 (see also figure 4):

$$N_{\mathrm{lens}} \propto z^2. \tag{11}$$

Since the total intensity at a point is proportional to $I_{\mathrm{lens}}N_{\mathrm{lens}}$ there is no intensity loss because of walk-off effects as long as the lateral distance of this point from the edges of the array is larger than x. Of course, there are walk-off effects for points which have a lateral distance smaller than x from the edges of the array. In this case the number of lenses which supply light to this point has to be calculated and to be divided by the number of lenses which lie in a circle of radius x to obtain the decrease of the intensity.

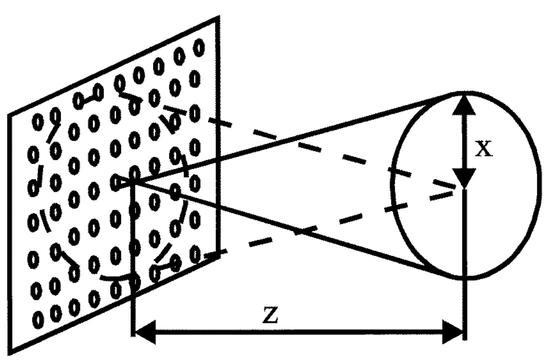


Figure 4. Illustration of walk-off effects. The intensity of light of one lens with numerical aperture NA at a distance z is decreased but the number of lenses which supply light to the foci in the fractional Talbot planes is increased.

5. Experimental results

Experiments were carried out with an array of planoconvex refractive microlenses made of photoresist without stops between the lenses. They were illuminated by a plane wave with the plane side of the lens towards the plane wave, which is the most common situation for microlenses because of their small focal lengths. The parameters of the array were:

(*a*) (*b*) (*c*) (*d*) (*e*)

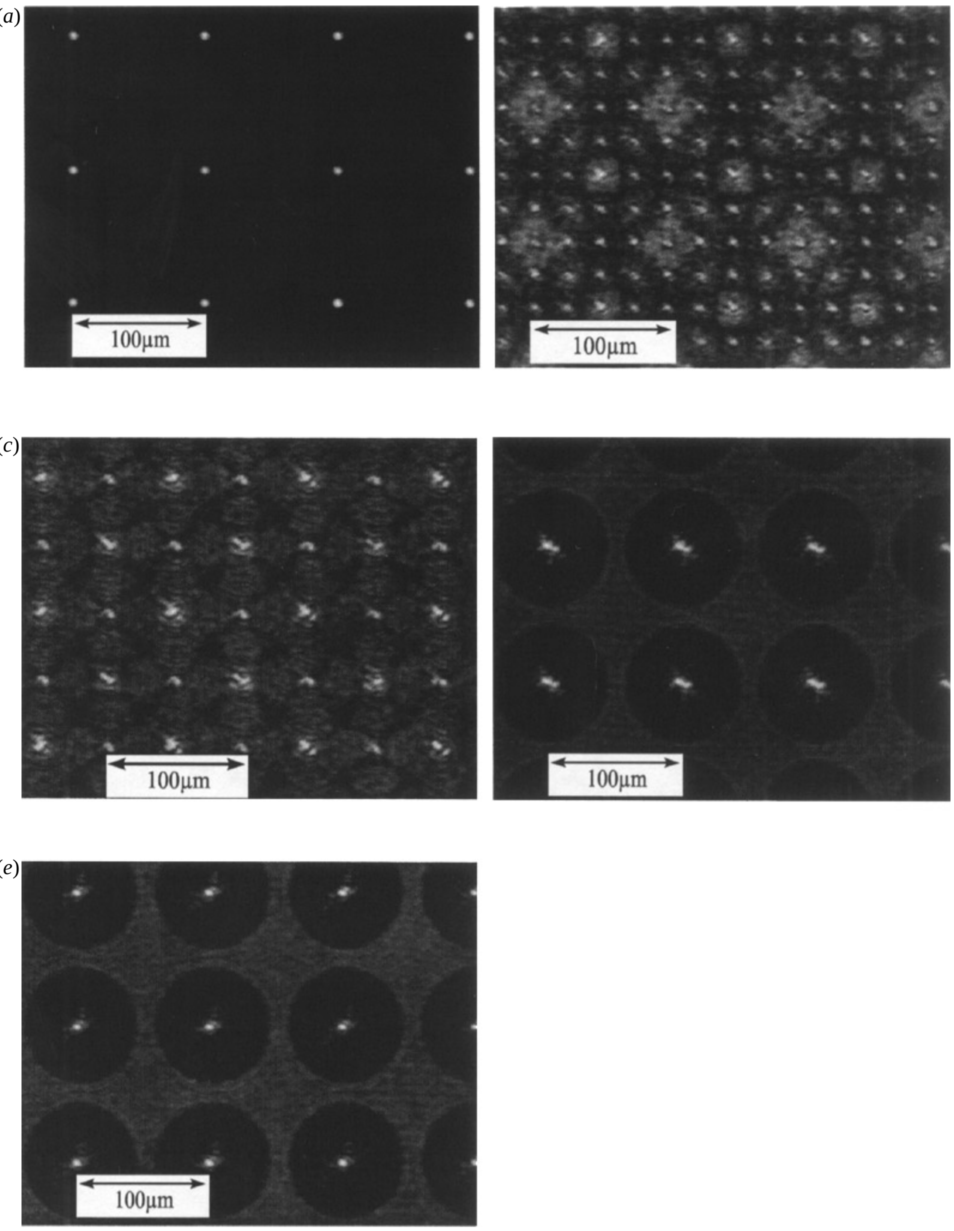


Figure 5. Foci in different (fractional) Talbot planes. (*a*) 0 Talbot plane, (*b*) $\frac{1}{8}$ Talbot plane, (*c*) $\frac{1}{4}$ Talbot plane, (*d*) $\frac{1}{2}$ Talbot plane, (*e*) 1 Talbot plane.

period of the microlens array $p = 100\ \mu$m; diameter of a single lens $d = 80\ \mu$m; focal length of each lens $f = 130\ \mu$m; wavelength of the illuminating plane wave $\lambda = 633$ nm. Figures 5(*a*)–(*e*) show the foci in different (fractional) Talbot planes. It can be seen that the multiplied foci in the $\frac{1}{8}$ and $\frac{1}{4}$ Talbot planes have different intensities and that the foci in the $\frac{1}{2}$ and 1 Talbot planes are blurred compared to the original foci in the 0 Talbot plane. In each picture the intensity distribution was adapted to the dynamic range of the CCD camera by using different absorption filters. By considering the amount of light in the space between the lenses, which is of course nearly constant in the different planes, it can also be seen that the maximum value of the intensity of the foci decreases in the $\frac{1}{2}$ and 1 Talbot plane. This is due to the same effect as shown in figure 3. Walk-off effects could be excluded in this experiment because the microlens array was large enough. A more detailed comparison between experiments and simulations was given in [4].

6. Summary

It has been shown that the fractional Talbot effect for microlens arrays can, in principle, be used for array illuminators. Nevertheless, in practice there are several effects which limit the application: (i) different intensities of the multiplied foci in the fractional Talbot planes; (ii) blurring of the foci for lenses with high numerical apertures; (iii) walk-off effects. These effects and theoretical and numerical simulation methods for their calculation have been presented and experimental results for their verification have been given.

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