Total No. of Questions : 9]

SEAT No. :

PB3586

[Total No. of Pages : 5

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F.E.
ENGINEERING MATHEMATICS-I
(2019 Credit Pattern) (Semester -I/II) (107001)

Time :2½Hours] *[Max. Marks :70*

Instructions to the candidates:

1) *Q.1 is Compulsory.*
2) *Answer Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
3) *Figures to the right indicate full marks.*
4) *Assume suitable data, if necessary.*
5) *Neat diagrams must be drawn wherever necessary.*
6) *Use of electronic pocket calculator is allowed.*

Q1) Write the correct option for the following MCQs. **[10]**

a) If $u = x^3 + y^3$ then $\frac{\partial^2 u}{\partial x \partial y} = \cdots ?$ **[2]**

i) 3 ii) -3

iii) 2 iv) 0

b) If $x = uv, y = \frac{u}{v}$ the $\frac{\partial(x,y)}{\partial(u,v)} = \cdots ?$ **[2]**

i) $\frac{-2u}{v}$ ii) uv

iii) $\frac{v}{2u}$ iv) $\frac{-v}{2u}$

c) Rank of matrixA=$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is ....? **[2]**

i) 0 ii) 1

iii) 2 iv) 3

P.T.O.

d) Using Cayley Hamilton theorem A^{-1} for the matrix A=$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ is given by; **[2]**

i) $\frac{1}{5}$(A+4I) ii) $\frac{1}{4}$(A+5I)

iii) $\frac{1}{4}$(A$-$5I) iv) $\frac{1}{5}$(A$-$4I)

e) If A^{-1}=A$'$ then matrix A is ....? **[1]**

i) Orthogonal ii) Singular

iii) Non-Singular iv) None of above

f) If $u = x^3 + 4y - 3x$, $\frac{\partial u}{\partial x}$=....? **[1]**

i) 4 ii) $3x^2 - 3$

iii) $3x^2 + 4y$ iv) $3x^2 + 1$

Q2) a) If $u = x^y + y^x$,find $\frac{\partial^2 u}{\partial x \partial y}$ **[5]**

b) If $u = \log\left(\frac{x^3 + y^3}{x^2 + y^2}\right)$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ **[5]**

c) If $u = f(y - z, z - x, x - y)$, Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **[5]**

OR

Q3) a) If $x^2 = au + bv$ and $y^2 = au - bv$, prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2}$ **[5]**

b) If $u = \sin^{-1}\left(\frac{y}{x}\right) + \sqrt{x^2 + y^2}$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ **[5]**

c) If $x = \frac{\cos\theta}{u}, y = \frac{\sin\theta}{u}$ and $z = f(x, y)$, then show that $u \frac{\partial z}{\partial u} - \frac{\partial z}{\partial \theta} = (y - x)\frac{\partial z}{\partial x} - (y + x)\frac{\partial z}{\partial y}$ **[5]**

Q4) a) If $x = uv$ and $y = \dfrac{u+v}{u-v}$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$ **[5]**

b) Examine for functional dependence:

$u = \dfrac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$. If dependent find the relation between them. **[5]**

c) Discuss maxima and minima of $f(x,y) = x^3 + y^3 - 3axy \quad a > 0$. **[5]**

OR

Q5) a) Prove that $JJ' = 1$ for the transformation $x = u\cos v, y = u\sin v$ **[5]**

b) Find the percentage error in computing the parallel resistance r of two resistances r_1 and r_2 from the formula $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$ where r_1 and r_2 are both in error by +2% each. **[5]**

c) Find maximum value of $u = x^2 y^3 z^4$ such that $2x + 3y + 4z = a$ by langrange's method. **[5]**

Q6) a) Find for what values of k, the set of equations **[5]**

$$2x-3y+6z-5t=3$$
$$y-4z+t=1$$
$$4x-5y+8z-9t=k$$

has i) No solution

ii) An intinite number of solutions.

b) Examine for linear dependence of vectors **[5]**

$(1,-1,1)$, $(2,1,1)$ and $(3,0,2)$

c) Show that $A = \dfrac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal. **[5]**

OR

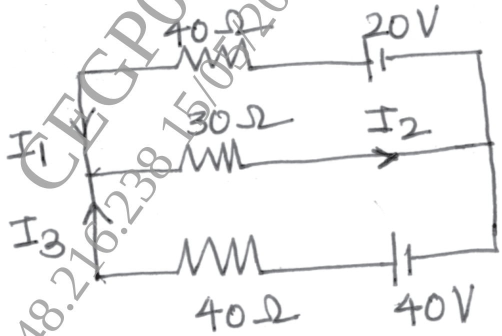
Q7) a) Examine for consistency the following set of equations and obtain the solution if consistent. **[5]**

$$2x-y-z=2$$
$$x+2y+z=2$$
$$4x-7y-5z=2$$

b) Examine for linear dependence of vectors **[5]**

(1,2,4),(2,−1,3),(0,1,2).

c) Determine the currents in the network given in figure below. **[5]**



Q8) a) Find the eigen values and eigen vectors of the following matrix. **[5]**

$$A=\begin{bmatrix}1 & 1 & 1\\0 & 2 & 1\\0 & 0 & 3\end{bmatrix}.$$

b) Verify Cayley - Hamilton theorem for $A=\begin{bmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{bmatrix}$ and use it to

Find A^{-1} **[5]**

c) Find the modal matrix P which transform the matrix

$A=\begin{bmatrix}1 & 1 & -2\\-1 & 2 & 1\\0 & 1 & -1\end{bmatrix}$ to the diagonal form. **[5]**

OR

Q9) a) Find the eigen values and eigen vectors of the following matrix

$$A=\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}.$$ **[5]**

b) Verify cayley Hamilton theorem for $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$. Hence find A^{-1}. **[5]**

c) Reduce the following quadratic form to the Sum of the squares form.
$3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$. **[5]**