Total No. of Questions : 9]

SEAT No. :

PD-4024

[Total No. of Pages : 4

[6401]-1901
F.E
ENGINEERING MATHEMATICS - I
(2019 Pattern)(Semester - I) (107001)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt Q.1, Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Figures to the right indicate full marks.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Assume suitable data, if necessary.*
5) *Use of electronic pocket calculator is allowed.*

Q1) **Write the correct option for the following MCQ's.**

i) If $u=\frac{1}{x^2+y^2}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$...... **[2]**

a) 0 b) $-2u$

c) $2u$ d) none

ii) If $x=uv, y=\frac{u}{v}$ then $\frac{\partial(x,y)}{\partial(u,v)}=$...... **[2]**

a) $-\frac{2u}{v}$ b) uv

c) $\frac{v}{2u}$ d) $-\frac{v}{2u}$

iii) Rank of a matrix $A=\begin{bmatrix}2 & 2 & 2\\ 0 & -2 & 0\\ 2 & 2 & 2\end{bmatrix}$ is .... **[2]**

a) 0 b) 1

c) 2 d) 3

iv) For the matrix $A=\begin{bmatrix}1 & 0 & 0\\ -1 & 2 & 0\\ 1 & 0 & 3\end{bmatrix}$ the eigenvalues of A^2 are ..... **[2]**

a) 1,2,3 b) 1,4,9

c) –1, –2, –3 d) –1, –4, –9

P.T.O.

v) If $z = xy$ then $\frac{\partial^2 z}{\partial x \partial y} = ...$ **[1]**

a) x b) y

c) 1 d) 0

vi) If $A^{-1} = A^{T}$ then matrix A is ________ **[1]**

a) Singular b) Non-singular

c) Orthogonal d) None

Q2) a) If $u = \tan^{-1}\left(\frac{x}{y}\right)$, then show that $u_{xy} = u_{yx}$. . **[5]**

b) If $u = ln(x) + ln(y)$, find the value of $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} + x u_x + y u_y$. **[5]**

c) If $u = f\left(e^{y-z}, e^{z-x}, e^{x-y}\right)$, find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$. **[5]**

OR

Q3) a) If $ux + vy = 0$ and $\frac{u}{x} + \frac{v}{y} = 1$, then show that $\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{x^2 + y^2}{y^2 - x^2}$ **[5]**

b) If $f(x, y) = x^2 + \left(x^2 + y^2\right)[ln(x) - ln(y)]$, then find the value of $x f_x + y f_y$ **[5]**

c) If $x = u + v + w$, $y = uv + vw + wu$, $z = uvw$ and F is a function of x,y,z show that $uF_u + vF_v + wF_w = xF_x + 2y\,F_y + 3z\,F_z$. **[5]**

Q4) a) If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(r,\theta)}{\partial(x,y)}$ **[5]**

b) Examine for functional dependence : **[5]**

$u = \frac{x-y}{x+y}$, $v = \frac{x+y}{x}$. If functionally dependent find the relation between them.

c) Discuss maxima and minima of $f(x,y) = x^2 + y^2 + 6x + 12$. **[5]**

OR

Q5) a) Verify $JJ' = 1$ for the transformation $x = uv,\ y = \dfrac{u}{v}$. **[5]**

b) In calculating the volume of a right circular cone, errors of 2% and 1% are made in measuring the height and radius of base respectively. Find the error in the calculated volume. **[5]**

c) By using Langrange's method, divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. **[5]**

Q6) a) Show that the system **[5]**

$3x + 4y + 5z = \alpha$

$4x + 5y + 6z = \beta$

$5x + 6y + 7z = \gamma$

is consistent only when α, β, γ are in arithmetic progression.

b) Examine for linear dependence or independence, if dependent find the relation between them. $X_1 = (1, 1, 1, 3)$, $X_2 = (1,2,3,4)$, $X_3 = (2,3,4,7)$ **[5]**

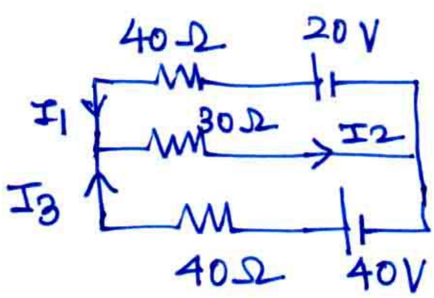
c) If $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & a \\ \frac{2}{3} & \frac{1}{3} & b \\ \frac{2}{3} & \frac{-2}{3} & c \end{bmatrix}$ is orthogonal. Find a,b,c. **[5]**

OR

Q7) a) Examine the consistency of the linear system of the following equations. If consistent solve it $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$. **[5]**

b) Examine for linear dependence or independence of the following vectors. It dependent, Find the relation among vectors (1,1,1) (1,2,3) (2,3,8). **[5]**

c) Determine the currents in the network given in figure below **[5]**



Q8) a) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$. **[5]**

b) Verify the Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ and use it to find A^{-1}. **[5]**

c) Find the modal matrix p such that $p^{-1}AP$ is diagonal matrix for $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **[5]**

OR

Q9) a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **[5]**

b) Reduce the following quadratic form to the "sum of the squares form" $2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz$. **[5]**

c) Verify the Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -3 \end{bmatrix}$ Hence find A^{-1}. **[5]**

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