Total No. of Questions : 9]

SEAT No. :

PB-3594

[Total No. of Pages : 4

[6260]-9

F.E. (All Branches)

ENGINEERING MATHEMATICS - II

(2019 Pattern) (Semester - I/II) (107008)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Q.No.1 is compulsory.*
2) *Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q,8 or Q.No.9.*
3) *Neat diagram must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice equations.

a) $\int_0^\infty e^{-x} x^{1/2}\, dx$ is equal to **[2]**

i) $\frac{1}{2}$ ii) $\frac{\sqrt{\pi}}{2}$

iii) $\frac{\pi}{2}$ iv) $\sqrt{\pi}$

b) $\int_0^{\pi/2} \cos^4 0$ do is equal to **[2]**

i) $\frac{3\pi}{16}$ ii) $\frac{3\pi}{4}$

iii) 0 iv) $\frac{3\pi}{8}$

c) The region of absence for the curve represented by the equation $y^2(2a - x) = x^3$ is **[2]**

i) $x > 0, x < 2a$ ii) $x < 0, x > 2a$

iii) $x < 0, x < 2a$ iv) $x > 0, x > 2a$

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d) The centre and radius of sphere $x^2 + y^2 + z^2 - 2z - 3 = 0$ is **[2]**

i) (0, 0, 1) and 2 ii) (0, 0, 0) and 3

iii) (1, 0, 0) and 2 iv) (0, 0, –2) and 3

e) The value of $\int_0^1 \int_0^x dx\, dy$ is **[1]**

i) $\frac{1}{3}$ ii) $\frac{1}{3}x$

iii) $\frac{1}{2}$ iv) $\frac{1}{3}y$

f) $\int_0^1 x^{3/2}(1-x)^{5/2} dx$ is equal to **[1]**

i) $\beta(5/2, 7/2)$ ii) $\beta(-1/2, 1/2)$

iii) $\frac{1}{2}\beta(5/2, 3/2)$ iv) $\beta(1/2, 5/2)$

Q2) a) If $\text{In} = \int_{\pi/4}^{\pi/2} \cot^n \theta \, d\theta$, then prove that $\text{In} = \frac{1}{n-1} - \text{In} - 2$. **[5]**

b) Evaluate $\int_0^\infty x^g e^{-2x^2} dx$. **[5]**

c) Show that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}[erf(b) - erf(a)]$. **[5]**

OR

Q3) a) If $\text{In} = \int_0^{\pi/2} x^n \cos x \, dx,$ then show that $\text{In} = \left(\frac{\pi}{2}\right)^n - n(n-1)\,\text{In} - 2$. **[5]**

b) Show that $\int_0^1 x^{m-1}(1-x^2)^{n-1} dx = \frac{1}{2}\beta\left(\frac{m}{2} 1n\right)$. **[5]**

c) Evaluate $\int_0^1 \frac{x^a - x^b}{\log x} dx \quad 0 < a < 1, 0 < b < 1$. **[5]**

Q4) a) Trace the curve $y^2(a - x) = x^3$. **[5]**

b) Trace the curve $r = \frac{a}{2}(1 + \cos\theta)$. **[5]**

c) Find the length of the upper arc of one loop of lemniscate $r^2 = a^2 \cos 2\theta$. **[5]**

OR

Q5) a) Trace the curve $xy^2 = a^2(a - x)$. **[5]**

b) Trace the curve $r = a \sin 2\theta$. **[5]**

c) Trace the curve $x = a(t - \sin t), y = a(1 - \cos t)$. **[5]**

Q6) a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. **[5]**

b) Find the equation of right circular cone whose vertex is at (1, 2, –3), semi-vertical angle $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ & axis is the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+1}{-1}$. **[5]**

c) Find the equation of right circular cylinder of radius 5 and axis is $\frac{x-2}{2} = \frac{y-3}{1} = \frac{z+1}{1}$. **[5]**

OR

Q7) a) Prove that the two spheres $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$ and $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other and find the point of contact. **[5]**

b) Find the equation of right circular cone which passes through the point (1, 1, 2) and has the axis as the line $6x = -3y = 4z$ and vertex at origin. **[5]**

c) Find the equation of right circular cylinder of radius 2, whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, 1, 2. **[5]**

Q8) a) Evaluate $\iint_R x^2 y^2 dydx$ over positive quadrant of $x^2 + y^2 = 1$. **[5]**

b) Find the area between the curve $y = x^2 - 2x - 8$ and x – axis. **[5]**

c) Find the position of the centroid of the area bounded by the curve $y^2(2a - x) = x^3$ and its asymptote. **[5]**

OR

Q9) a) Change the order of integration $\int_0^2 \int_0^{2-x} y\, dy\, dx$ and then solve it. **[5]**

b) Evaluate $\iiint_V \frac{dxdydz}{\sqrt{1 - x^2 - y^2 - z^2}}$ taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$ in the positive octant. **[5]**

c) Find the moment of inertia (M.I.) of the area enclosed by $r = a(1 - \cos\theta)$ about the line $\theta = \frac{\pi}{2}$. **[5]**

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