Total No. of Questions : 9]

SEAT No. :

PC-1683

[Total No. of Pages : 4

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F.E.

ENGINEERING MATHEMATICS - II

(2019 Pattern) (Semester - I/II) (107008)

Time : 2½ Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Q.1 is compulsory.***
2) ***Attempt Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Figures to the right indicate full marks.***
5) ***Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.***
6) ***Assume suitable data, if necessary.***

Q1) Write the correct option for the following multiple choice questions.

i) The value of integral $\int_0^\infty \sqrt{x}\, e^{-x^3}\, dx$ by using substitution $x^3 = t$ is _____. **[2]**

a) $\frac{\sqrt{\pi}}{6}$ b) $\frac{\sqrt{\pi}}{2}$

c) $3\sqrt{\pi}$ d) $\frac{\sqrt{\pi}}{3}$

ii) The region of absence for the curve represented by the equation $y^2(2a - x) = x^3$ is _________. **[2]**

a) $x > 0$ and $x < 2a$ b) $x < 0$ and $x > 2a$

c) $x < 0$ and $x < 2a$ d) $x > 0$ and $x > 2a$

iii) Find the centre and radius of a sphere $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$. **[2]**

a) (2, –3, 1) and 5 b) (–2, 3, –1) and 11

c) (4, –6, –2) and 6 d) (2, 3, –1) and 5

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iv) The value of the integral $\int_0^1 \int_0^y dx\, dy$ is _______. **[2]**

a) $\frac{1}{4}$ b) $\frac{1}{3}x$

c) $\frac{1}{2}$ d) $\frac{1}{2}y$

v) Using polar transformations $x = r\cos\theta$, $y = r\sin\theta$ the cartesian double integral $\iint f(x, y)\, dx\, dy$ is transformed to **[1]**

a) $\iint_R f(r,\theta)\, dr d\theta$ b) $\iint_R f(r,\theta) \frac{1}{2} dr d\theta$

c) $\iint_R f(r,\theta)\, 2 dr d\theta$ d) $\iint_R f(r,\theta)\, r\, dr d\theta$

vi) The number of loops in the rose curve $r = a\cos 2\theta$ are ______ **[1]**

a) 4 b) 2

c) 8 d) = 3

Q2) a) If $I_n = \int_0^{\pi/2} x^n \cos x\, dx$ then prove that $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)\, I_{n-2}$. **[5]**

b) Evaluate $\int_0^{\infty} \frac{x^2}{2^x} dx$. **[5]**

c) Evaluate $\frac{d}{dx} erf\left(\sqrt{x}\right)$. **[5]**

OR

Q3) a) If $I_n = \int_0^{\pi/4} \sec^n x\, dx$ then prove that $I_n \frac{\left(\sqrt{2}\right)^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$. **[5]**

b) Evaluate $\int_0^3 x^4 (3-x)^5\, dx$. **[5]**

c) Prove that $\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log\left(\frac{a^2+1}{2}\right)$ **[5]**

Q4) a) Trace the curve $y^2(a-x) = x^2(a+x)$, $(a > 0)$. **[5]**

b) Trace the curve $r = a\,(1 - \sin\theta)$, $(a > 0)$. **[5]**

c) Find the arc length of the astroid $x = a\cos^3 t$, $y = a\sin^3 t$, $(a > 0)$. **[5]**

OR

Q5) a) Trace the curve $y^2 = (x-1)(x-2)(x-3)$. **[5]**

b) Trace the curve $r = a\cos 5\theta$, $(a > 0)$. **[5]**

c) Trace the curve $x = t^2,\ y = t - \dfrac{t^3}{3}$ **[5]**

Q6) a) Show that the plane $2x - y - 2z = 4$ is tangential to the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ and find the point of contact. **[5]**

b) Find the equation of right circular cone whose vertex is at (0, 0, 0), semivertical angle $\pi/4$ and axis along the line $\dfrac{x}{-2} = \dfrac{y}{1} = \dfrac{z}{-2}$. **[5]**

c) Find the equation of right circular cylinder having its radius '4' and equation of the axis is $\dfrac{x+1}{1} = \dfrac{y+1}{-1} = \dfrac{z+1}{1}$. **[5]**

OR

Q7) a) Show that the two spheres $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$ and $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other externally and find the point of contact. **[5]**

b) Find the equation of right circular cone whose vertex is (1, 1, 1), axis is the line $\dfrac{x-1}{1} = \dfrac{y-1}{2} = \dfrac{z-1}{3}$ and semivertical angle $\pi/4$. **[5]**

c) Find the equation of right circular cylinder of radius 03 units and axis is the line $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z-2}{3}$. **[5]**

Q8) a) Evaluate $\iint_R xy\,dxdy$ over the region R bounded by the parabolas $y^2 = x$ and $x^2 = y$. **[5]**

b) Find the area of one loop of the rose curve $r = a\cos 2\theta$. **[5]**

c) Find the x-coordinate of centre of gravity of the area enclosed by the parabola $y^2 = 4x$ and the line $y = 2x$. **[5]**

OR

Q9) a) Evaluate the integration by changing the order : **[5]**

$$\int_0^1 \int_y^1 e^{x^2}\,dx\,dy$$

b) Find the volume of the tetrahedron bounded by the co-ordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **[5]**

c) Prove that the moment of inertia of the area between the curves $y^2 = ax$ and $x^2 = ay$ about x-axis is $\frac{9}{35}Ma^2$.

(Given that the density $\rho = \frac{3M}{a^2}$ where M is the mass) **[5]**

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