BONUS CHAPTER

4

Solutions to "Practice Problems"

Bonus Chapter 1

1. $H_0 : \mu_1 = \mu_2 = \mu_3$ $\quad \bar{x}_1 = 22.12 \quad \bar{x}_2 = 19.67 \quad \bar{x}_3 = 18.94$

   H_1 : not all μ's are equal $\quad s_1^2 = 0.98 \quad s_2^2 = 1.45 \quad s_3^2 = 2.36$

   $N = 17 \quad n_1 = 6 \quad n_2 = 6 \quad n_3 = 5$

   $$SSW = \sum_{i=1}^{k}(n_i - 1)s_i^2 = (6-1)(0.98) + (6-1)(1.45) + (5-1)(2.36) = 21.59$$

   $$\bar{\bar{x}} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{22.5+20.8+22.0+23.6+...+18.0+21.1+19.8+18.6}{17} = 20.32$$

   $$SSB = \sum_{i=1}^{k} n_i(\bar{x}_i - \bar{\bar{x}})^2 = 6(22.12-20.32)^2 + 6(19.67-20.32)^2 + 5(18.94-20.32)^2 = 31.50$$

   $$MSB = \frac{SSB}{k-1} = \frac{31.50}{3-1} = 15.75, \quad MSW = \frac{SSW}{N-k} = \frac{21.59}{17-3} = 1.54$$

   $$F = \frac{MSB}{MSW} = \frac{15.75}{1.54} = 10.23, \quad F_c = F_{\alpha, k-1, N-k} = F_{0.05, 2, 14} = 3.739$$

   Since $F > F_c$, we reject H_0 and conclude that there is a difference between the sample means.

2. For $\bar{x}_1$ and $\bar{x}_2$, $F_s = \dfrac{(\bar{x}_a - \bar{x}_b)^2}{\dfrac{SSW}{\sum_{i=1}^{k}(n_i - 1)}\left[\dfrac{1}{n_a} + \dfrac{1}{n_b}\right]} = \dfrac{(22.12 - 19.67)^2}{\dfrac{21.59}{5+5+4}\left[\dfrac{1}{6} + \dfrac{1}{6}\right]} = 11.70$

For $\bar{x}_1$ and $\bar{x}_3$, $F_s = \dfrac{(\bar{x}_a - \bar{x}_b)^2}{\dfrac{SSW}{\sum_{i=1}^{k}(n_i - 1)}\left[\dfrac{1}{n_a} + \dfrac{1}{n_b}\right]} = \dfrac{(22.12 - 18.94)^2}{\dfrac{21.59}{5+5+4}\left[\dfrac{1}{6} + \dfrac{1}{5}\right]} = 17.88$

For $\bar{x}_2$ and $\bar{x}_3$, $F_s = \dfrac{(\bar{x}_a - \bar{x}_b)^2}{\dfrac{SSW}{\sum_{i=1}^{k}(n_i - 1)}\left[\dfrac{1}{n_a} + \dfrac{1}{n_b}\right]} = \dfrac{(19.67 - 18.94)^2}{\dfrac{21.59}{5+5+4}\left[\dfrac{1}{6} + \dfrac{1}{5}\right]} = 0.94$

$F_{sc} = (k - 1)F_{\alpha, k-1, N-k} = (3 - 1)(3.739) = 7.478$

Sample Pair	F_S	F_{SC}	**Conclusion**
$\bar{x}_1$ and $\bar{x}_2$	11.70	7.478	Difference
$\bar{x}_1$ and $\bar{x}_3$	17.88	7.478	Difference
$\bar{x}_2$ and $\bar{x}_3$	0.94	7.478	No Difference

We conclude that there is a difference between gas mileage of Cars 1 and 2 and Cars 1 and 3.

3. $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ $\quad \bar{x}_1 = 38.33 \quad \bar{x}_2 = 28.29 \quad \bar{x}_3 = 28.0 \quad \bar{x}_4 = 31.43$

H_1 : not all μ's are equal $\quad s_1^2 = 115.47 \quad s_2^2 = 72.57 \quad s_3^2 = 86.8 \quad s_4^2 = 132.62$

$N = 26 \quad n_1 = 6 \quad n_2 = 7 \quad n_3 = 6 \quad n_4 = 7$

$SSW = (6 - 1)115.47 + (7 - 1)72.57 + (6 - 1)86.8 + (7 - 1)132.62 = 2242.49$

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{36+48+32+28+...+36+18+30+21}{26} = 31.38$$

$SSB = 6(38.33 - 31.38)^2 + 7(28.29 - 31.38)^2 + 6(28 - 31.38)^2 + 7(31.43 - 31.38)^2 = 425.22$

$MSB = \dfrac{SSB}{k-1} = \dfrac{425.22}{4-1} = 141.74$, $\quad MSW = \dfrac{SSW}{N-k} = \dfrac{2{,}242.49}{26-4} = 101.93$

$F = \dfrac{MSB}{MSW} = \dfrac{141.74}{101.93} = 1.391$, $F_c = F_{\alpha, k-1, N-k} = F_{.05, 3, 22} = 3.049$

Since $F < F_C$, we do not reject H_0 and conclude that there is no difference between the sample means.

4. $H_0 : \mu_1 = \mu_2 = \mu_3$, Pop 1 = Dad, $\bar{x}_1 = 92.5$, $n_1 = 4$

H_1 : not all μ's are equal, Pop 2 = Brian, $\bar{x}_2 = 83.5$, $n_2 = 4$

$N = 12$, $k = 3$, $b = 4$, Pop 3 = John, $\bar{x}_3 = 83.5$, $n_3 = 4$

H_0' : the block means all are equal

H_1' : the block means all are not equal

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{93+98+89+90+...+80+88+84+82}{12} = 86.5$$

x_{ij}	$\bar{\bar{x}}$	$(x_{ij} - \bar{\bar{x}})$	$(x_{ij} - \bar{\bar{x}})^2$
93	86.5	6.5	42.25
98	86.5	11.5	132.25
89	86.5	2.5	6.25
90	86.5	3.5	12.25
85	86.5	-1.5	2.25
87	86.5	0.5	0.25
82	86.5	-4.5	20.25
80	86.5	-6.5	42.25
80	86.5	-6.5	42.25
88	86.5	1.5	2.25
84	86.5	-2.5	6.25
82	86.5	-4.5	20.25

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{b} (x_{ij} - \bar{\bar{x}})^2 = 329$$

$$SSB = \sum_{i=1}^{k} n_i(\bar{x}_i - \bar{\bar{x}})^2 = 4(92.5 - 86.5)^2 + 4(83.5 - 86.5)^2 + 4(83.5 - 86.5)^2 = 216$$

$$SSBL = \sum_{j=1}^{b} k(\bar{x}_j - \bar{\bar{x}})^2$$ where $j = 1$ for Course 1, etc.

Block (course) averages: $\bar{x}_1 = 86$, $\bar{x}_2 = 91$, $\bar{x}_3 = 85$, $\bar{x}_4 = 84$

$SSBL = (3)(86 - 86.5)^2 + (3)(91 - 86.5)^2 + (3)(85 - 86.5)^2 + (3)(84 - 86.5)^2 = 87$

$SSW = SST - SSB - SSBL = 329 - 216 - 87 = 26$

$MSW = \frac{SSW}{(k-1)(b-1)} = \frac{26}{(2)(3)} = 4.33$, $MSBL = \frac{SSBL}{b-1} = \frac{87}{3} = 29$, $F' = \frac{MSBL}{MSW} = \frac{29}{4.33} = 6.70$

$v_1 = b - 1 = 3$, $v_2 = (k - 1)(b - 1) = (2)(3) = 6$, $F_{c'} = F_{.05, 3, 6} = 4.757$

Since $F' > F_{C'}$, we reject H_0’ and conclude that the blocking procedure was effective and proceed to test H_0.

$MSB = \frac{SSB}{k-1} = \frac{216}{2} = 108$, $F = \frac{MSB}{MSW} = \frac{108}{4.33} = 24.92$,

$v_1 = k - 1 = 2$, $v_2 = (k - 1)(b - 1) = (2)(3) = 6$, $F_c = F_{.05, 2, 6} = 5.143$

Since $F > F_C$, we reject H_0 and conclude that there is a difference between the golfer means.

Bonus Chapter 2

1. H_0 : The arrival process can be described by the expected distribution

   H_1 : The arrival process differs from the expected distribution

Day	Expected Percentage	Sample Size	Expected Frequency (*E*)	Observed Frequency (*O*)
Mon	10%	215	0.10(215) = 21.5	31
Tues	10%	215	0.10(215) = 21.5	18
Wed	15%	215	0.15(215) = 32.25	36
Thurs	15%	215	0.15(215) = 32.25	23
Fri	20%	215	0.20(215) = 43	47
Sat	30%	215	0.30(215) = 64.5	60
Total	100%		= 215	215

Day	O	E	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
Mon	31	21.50	9.50	90.25	4.20
Tues	18	21.50	-3.50	12.25	0.57
Wed	36	32.25	3.75	14.06	0.44
Thurs	23	32.25	-9.25	85.56	2.65
Fri	47	43.00	4.00	16.00	0.37
Sat	60	64.50	-4.50	20.25	0.31
Total					$\chi^2 = \sum \frac{(O-E)^2}{E} = 8.54$

For $\alpha = 0.05$ and $d.f. = k - 1 = 6 - 1 = 5$, $\chi_c^2 = 11.070$. Since $\chi_c^2 > \chi^2$, we do not reject H_0 and conclude that the arrival distribution is consistent with the expected distribution.

2. H_0 : The process can be described with the Poisson distribution using $\lambda = 3$.

   H_1 : The process differs from the Poisson distribution using $\lambda = 3$.

Number of Hits Per Minute	**Poisson Probabilities**		**Number of Hits**		**Expected Frequency**
0	0.0498	x	380	=	18.92
1	0.1494	x	380	=	56.77
2	0.2240	x	380	=	85.12
3	0.2240	x	380	=	85.12
4	0.1680	x	380	=	63.84
5	0.1008	x	380	=	38.30
6	0.0504	x	380	=	19.15
7 or more	0.0336	x	380	=	12.77
Total	1.0000				380.00

Hits per Min	O	E	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	22	18.92	3.08	9.46	0.50
1	51	56.77	-5.77	33.32	0.59
2	72	85.12	-13.12	172.13	2.02
3	92	85.12	6.88	47.33	0.56
4	60	63.84	-3.84	14.75	0.23
5	44	38.30	5.70	32.44	0.84
6	25	19.15	5.85	34.19	1.79
7 or more	14	12.77	1.23	1.52	0.12
Total					$\chi^2 = \sum \frac{(O-E)^2}{E} = 6.65$

For $\alpha = 0.01$ and $d.f. = k - 1 = 8 - 1 = 7$, $\chi_c^2 = 18.475$. Since $\chi_c^2 > \chi^2$, we do not reject H_0 and conclude that the process is consistent with the Poisson distribution using $\lambda = 3$.

3. H_0 : Grades are independent of reading time

   H_1 : Grades are dependent of reading time

   Sample expected frequency calculations:

   $E_{1,1} = \frac{(265)(95)}{500} = 50.35$ $\quad$ $E_{1,2} = \frac{(265)(128)}{500} = 67.84$

   $E_{1,3} = \frac{(265)(155)}{500} = 82.15$

Row	Column	O	E	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	1	36	50.35	-14.35	205.92	4.09
1	2	75	67.84	7.16	51.27	0.76
1	3	81	82.15	-1.15	1.32	0.02
1	4	63	49.82	13.18	173.71	3.49
1	5	10	14.84	-4.84	23.43	1.58
2	1	27	26.60	0.40	0.16	0.01
2	2	28	35.84	-7.84	61.47	1.72

Row	**Column**	O	E	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
2	3	50	43.40	6.60	43.56	1.00
2	4	25	26.32	-1.32	1.74	0.07
2	5	10	7.84	2.16	4.67	0.60
3	1	32	18.05	13.95	194.60	10.78
3	2	25	24.32	0.68	0.46	0.02
3	3	24	29.45	-5.45	29.70	1.01
3	4	6	17.86	-11.96	140.66	7.88
3	5	8	5.32	2.68	7.18	1.35
Total						$\chi^2 = \sum \frac{(O-E)^2}{E} = 34.38$

For $\alpha = 0.05$ and $d.f. = (r-1)(c-1) = (3-1)(5-1) = 8$, $\chi_c^2 = 15.507$. Since $\chi^2 > \chi_c^2$, we reject H_0 and conclude that there is a relationship between grades and the number of hours reading.

4. H_0 : The process can be described with the Binomial distribution using $p = 0.4$.

   H_1 : The process differs from the Binomial distribution using $p = 0.4$.

Number of Visits Per Day	**Binomial Probabilities**		**Sample Size**		**Expected Frequency**
0	0.0778	×	140	=	10.9
1	0.2592	×	140	=	36.3
2	0.3456	×	140	=	48.4
3	0.2304	×	140	=	32.3
4	0.0768	×	140	=	10.8
5	0.0102	×	140	=	1.4
Total	1.0000		140		

Because we must meet the chi-square requirement of having at least five observations in each of the expected frequency categories, we combined the final two categories in the following table (4 and 5 visits).

Number of Visits	O	E	$(O-E)$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	10	10.9	-0.9	0.8	0.07
1	41	36.3	4.7	22.2	0.61
2	60	48.4	11.6	134.9	2.79
3	20	32.3	-12.3	150.2	4.66
4 and 5	9	12.2	-3.2	10.24	.84
Total					$\chi^2 = \sum \frac{(O-E)^2}{E} = 8.99$

For $\alpha = 0.05$ and $d.f. = k - 1 = 5 - 1 = 4$, $\chi_c^2 = 9.488$. Since $\chi^2 > \chi_c^2$, we reject H_0 and conclude that the process differs from the Binomial distribution using $p = 0.4$.

Bonus Chapter 3

1. Payroll Wins

x	y	xy	x^2	y^2
171	103	17613	29241	10609
108	75	8100	11664	5625
119	92	10948	14161	8464
43	55	2365	1849	3025
58	56	3248	3364	3136
56	62	3472	3136	3844
62	84	5208	3844	7056
43	78	3354	1849	6084
57	73	4161	3249	5329
75	67	5025	5625	4489
$\sum x = 745$	$\sum y = 792$	$\sum xy = 63{,}494$	$\sum x^2 = 77{,}982$	$\sum y^2 = 57{,}661$

$\bar{x} = \frac{792}{10} = 79.2$, $\bar{y} = \frac{745}{10} = 74.5$,

$$r = \frac{n(\sum xy)-(\sum x)(\sum y)}{\sqrt{\left[n\left(\sum x^2\right)-(\sum x)^2\right]\left[n\left(\sum y^2\right)-(\sum y)^2\right]}}$$

$$r = \frac{10(63{,}494)-(792)(745)}{\sqrt{\left[10(77{,}982)-(792)^2\right]\left[10(57{,}661)-(745)^2\right]}} \frac{44{,}900}{\sqrt{(152{,}556)(21{,}585)}} = 0.782$$

$H_0 : \rho = 0, H_1 : \rho \neq 0$

$t = r\sqrt{\frac{n-2}{1-r^2}} = 0.782\sqrt{\frac{10-2}{1-(0.782)^2}} = 3.549$, $d.f. = n - 2 = 10 - 2 = 8$, $t_c = 2.306$

Since $t > t_c$, we reject H_0 and conclude the correlation coefficient is not equal to zero.

2a. $b = \frac{n(\sum xy)-(\sum x)(\sum y)}{n\left(\sum x^2\right)-(\sum x)^2} = \frac{10(63{,}494)-(792)(745)}{10(77{,}982)-(792)^2} = \frac{44{,}900}{152{,}556} = 0.294$

$a = \bar{y} - b\bar{x} = 74.5 - (0.294)(79.2) = 51.21$, $\hat{y}_i = 51.21 + 0.294x_i$

2b. $H_0 : \beta = 0, H_1 : \beta \neq 0$

$s_e = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}} = \sqrt{\frac{(57{,}661)-51.21(745)-(0.294)(63.494)}{10-2}} = 10.26$

$s_b = \frac{s_e}{\sqrt{\sum_{i=1}^{n} x_i^2 - n(\bar{x})^2}} = \frac{10.26}{\sqrt{77{,}982-10(79.2)^2}} = 0.0831$

$t = \frac{b-B_1}{s_b} = \frac{0.294-0}{0.0831} = 3.538$

$d.f. = n - 2 = 10 - 2 = 8$, $t_c = 2.306$

Since $t > t_c$, we reject H_0 and conclude there is a relationship between payroll and wins.

2c. $\hat{y}_i = 51.21 + 0.294(70) = 71.79$

2d. $CI = \hat{y} \pm t_c s_e \sqrt{\frac{1}{n} + \frac{(x_i - \bar{x})^2}{\left(\sum_{i=1}^{n} x_i^2\right) - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}}$, $d.f. = n - 2 = 10 - 2 = 8$, $t_c = 3.355$

$CI = 71.79 \pm (3.355)(10.26)\sqrt{\frac{1}{10} + \frac{(70-79.2)^2}{(77{,}982) - \frac{(792)^2}{10}}}$

$CI = 71.79 \pm (3.355)(10.26)(0.325) = 71.79 \pm 11.19$, $(60.60, 82.98)$

2e. $R^2 = (0.782)^2 = 0.612$ or 61.2 percent

3. GMAT GPA

X	y	xy	x^2
660	3.7	2442	435600
580	3.0	1740	336400
450	3.2	1440	202500
710	4.0	2840	504100
550	3.5	1925	302500
$\sum x = 2{,}950$	$\sum y = 17.4$	$\sum xy = 10{,}387$	$\sum x^2 = 1{,}781{,}100$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n\left(\sum x^2\right) - (\sum x)^2} = \frac{5(10{,}387) - (2{,}950)(17.4)}{5(1{,}781{,}100) - (2{,}950)^2} = \frac{605}{203{,}000} = 0.003$$

$$a = \overline{y} - b\overline{x} = \left(\frac{17.4}{5}\right) - (0.003)\left(\frac{2{,}950}{5}\right) = 1.71$$

$$\hat{y}_i = 1.71 + 0.003\, x_i = 1.71 + (0.003)(600) = 3.51$$