X, 359 *Rules for the Direction of the Mind* (1618?–1628?)

1. The end of all studies should be to direct the mind toward the enunciation of solid and true judgments on all things presented to it.

It is the custom of people, whenever they notice any similarity between two things, to attribute to both of them, even in those respects in which they differ, whatever they have found to be true of either one. So they make a false comparison between the sciences, which all consist in the cognition of the mind, and the arts, which demand a certain belief and disposition of the body. They see that one person cannot learn all the arts at the same time, but that a person who practices
360 only one art more easily emerges an excellent artist. For the same hands cannot so easily be applied both to tilling the fields and to strumming on the lyre, or to several different occupations of this kind, as to only one of them. Hence they have held the same opinion of the sciences, and, distinguishing them from one another by the diversity of their objects, they thought it proper to pursue each one of them singly to the neglect of all the others. But in this they are plainly mistaken. Since all the sciences are nothing but human intelligence, which always remains one and the same, however different the subjects to which it is applied, and which receives no more alteration from those subjects than does the light of the sun from the variety of things it illumines, there is no need to impose any boundaries upon the mind; nor, indeed, does the knowledge of one truth, like the practice of a single art, keep us from the discovery of another, but rather assists us. Indeed, it amazes me how most people study with the greatest diligence the customs of humans, the properties of plants, the motions of the stars, the transformations of metals, and the objects of other such disciplines, while at the same time almost no one thinks about good sense, or about universal intelligence, although as a matter of fact all other things are to be valued, not for themselves, but because they contribute something to universal intelligence. Thus it is not without reason that we set down this first rule, since nothing takes us farther afield from the right road for seeking truth than the direction of our studies to particular ends rather than to this one general one. Nor am I speaking of perverse and reprehensible ends, like vain glory or the love of filthy lucre; it is clear that
361 pretended arguments and sophistries suited to vulgar minds open a much easier road to them than the solid knowledge of truth could do. But I am thinking even of decent and laudable ends, since we are often more subtly deceived by these—as, for example, if we pursue sciences useful either for the comforts of life, or for that pleasure which is found in the contemplation of truth, and which is almost the only happiness in this life that is pure and untroubled by pain. For these legitimate fruits of the sciences we can certainly expect to attain; but, if we think

about these things in the midst of our studies, they often make us omit much that is necessary to the knowledge of other things—whether because such material appears at first sight of little use, or of little interest. It must be recognized, however, that all the sciences are so related to one another that it is much easier to learn them all at one time than to separate one from the others. If therefore anyone wishes seriously to investigate the truth of things, he should not choose any single science; for they are all interconnected and reciprocally dependent. He should rather think only of increasing the natural light of reason, not in order to resolve this or that problem of the School, but in order that in every particular situation of his life his intellect may show his will what choice to make. Soon he will be amazed to find that he has made much greater progress than those who study particular things, and that he has attained not only what others desire, but also higher things which they could not expect to reach.

2. We should concern ourselves only with those objects of which our minds appear to be adequate in gaining their certain and indubitable knowledge. *362*

All science is certain and evident knowledge. He who doubts of many things is not more learned than he who has never thought about them. Indeed, the former seems even more ignorant than the latter, if he has conceived a false opinion of any of them. So it is better not to study at all than to occupy oneself with objects so difficult that, in our inability to distinguish true from false, we are forced to admit doubtful things for certain; for in these matters there is not so much hope of increasing our learning as there is danger of diminishing it. And so through this proposition we reject all knowledge that is only probable, and we declare that only those things ought to be believed which are perfectly known and of which there can be no doubt. Scholars may perhaps have convinced themselves that there is little knowledge of this kind, because they have neglected to reflect on it as being too easy and open to anyone at all—a vice common to the human race. But I warn them that such knowledge is much more plentiful than they think and sufficient to demonstrate with certainty innumerable propositions on which they have been able until now to argue only with probability. And because they have thought it unworthy of a scholar to admit ignorance of anything, they have been accustomed to adorn their false arguments so well that they come to persuade themselves, and so they have ended up trumpeting them as true. *363*

Indeed if we observe this rule well there will be very few things we may suitably undertake to learn. For there is scarcely one question in the sciences on which clever people have not often disagreed. But each time the judgments of two people diverge on a single point, it is certain that at least one of them is wrong and not even one of them, it seems, has scientific knowledge. For if the argument of the one was certain and evident, he would be able to expound it to the other in such a way as finally to convince his intellect also. Therefore we see that in all such probable opinions we cannot acquire perfect science, for we may not without temerity hope for more than others have achieved. Consequently, if

our reckoning is correct, there remain of all the sciences already discovered only arithmetic and geometry to which the observation of this rule reduces us.

Nevertheless we do not therefore condemn that manner of philosophizing so suitable for jousting that others have already invented, that is, the scholastics' weapons of probable syllogisms. They do indeed train the minds of children and stimulate them by a certain emulation. It is much better to mold them with opin-
364 ions of this kind, uncertain though they seem when disputed among the erudite, than to leave them free to themselves. For perhaps without a guide they might cast themselves into some abyss; but while they follow in their masters' footsteps, they may indeed deviate somewhat from the truth, yet they will certainly take a road that is more secure at least in this sense, that it has already been tried by those who are more prudent; and we ourselves rejoice that we were once trained in the schools in this way. But since we are now freed of that obligation that bound us to the words of our masters, and since as adults we withdraw our hand from under the rod, if we wish seriously to set ourselves rules with the help of which we may ascend to the height of human knowledge, we must surely admit among the first the one that warns us not to abuse our leisure, as do many who neglect everything simple and are occupied only with arduous matters. They certainly make the subtlest conjectures on such subjects and devise very probable arguments. But after many labors they finally notice too late that they have only increased their doubts, without having learned any science.

But now, since we just said that of all the disciplines known by others, only arithmetic and geometry are free from every taint of falsity and uncertainty, we should examine more carefully the reason why this is so. And for this purpose we
365 must observe that we can arrive at knowledge of things by two paths, namely by experience or by deduction. We must observe, further, that while experiences of things are often deceptive, deduction or a pure inference of one thing from another, though it may be passed over if it is not noticed, can never be erroneously executed by an intellect even minimally rational. And I find of little use for this purpose those bonds by which the dialecticians seek to rule human reason, although I do not deny that they are most suitable for other uses. For all the error to which people are subject (people, I say, not beasts), results, never from faulty inference, but only from the fact that experiments insufficiently understood are admitted or that judgments are asserted rashly and without basis.

From this the explanation is evident why arithmetic and geometry are much more certain than other disciplines. The reason is that they alone are concerned with an object so pure and simple that they suppose absolutely nothing which experience has rendered uncertain, but they consist entirely in consequences rationally deduced. They are therefore the easiest and clearest of all the sciences, and have the kind of object we require, since in them it appears that human nature scarcely ever errs, except through inattention. Nevertheless, we ought not to wonder if many apply their minds more readily to other arts or to philosophy. For this happens because everyone feels free to guess with more confidence in
366 an obscure than in an evident subject matter, and because it is much easier to make conjectures on any random question than to arrive at truth itself in a single one, however simple.

From all this one must conclude, not, indeed, that one must learn nothing but arithmetic and geometry, but only that those who seek the right road of truth should not occupy themselves with any object concerning which they cannot possess a certainty equal to that of the demonstrations of arithmetic and geometry.

3. Concerning the objects presented to us we should investigate, not what others have thought nor what we ourselves conjecture, but what we can intuit clearly and evidently or deduce with certainty, since scientific knowledge is acquired by no other means.

The books of the ancients should be read, since it is a tremendous advantage for us to be able to use the labors of so many persons: as much to learn what has been correctly discovered in the past as to be counseled what more remains to be thought out in all the disciplines. On the other hand, there is great danger that perhaps some traces of the errors acquired by too attentive a reading of those authors may remain with us, however unwilling we may be and however much we guard against them. For writers are in fact so inclined that whenever, through thoughtless credulity, they have slipped into a judgment on some controversial subject, they always try by the subtlest arguments to draw us along in the same direction. Whenever, on the contrary, they have happily discovered something certain and evident, they never display it except in a wrapping of various detours, *367*
either because they fear that the dignity of their discovery might be diminished by the simplicity of the argument, or else because they begrudge us the obvious truth.

Yet even if they were all guileless and open, and never imposed upon us any doubtful opinions as true, but expounded every subject in good faith, we should still be perpetually uncertain which of them ought to be believed, since scarcely anything has been pronounced by someone whose contrary has not been asserted by another. And it would not help to count votes, so that we might follow the opinion held by the greater number of authorities. For when it is a case of a difficult question, it is more likely that the truth should have been discovered by few than by many. But even if all of them agreed, their doctrine would still be inadequate. For instance, we shall not turn out to be mathematicians, even though we keep in mind all the demonstrations of others, unless we are equipped intellectually for the solution of any kind of problem. Nor shall we turn out to be philosophers if we have read all the arguments of Plato and Aristotle but are unable to form a solid judgment on a given question. In fact we seem in this fashion to have learned not sciences but histories.

Further, we should be warned never at any time to admit any conjectures whatsoever as an admixture to our judgments on the truth of things. This counsel is of no small importance. For the chief reason why nothing is found in the vulgar philosophy so evident and certain as to be incapable of controversial treatment is this: scholars, not content with knowing what is clear and certain, first hazarded *368*
further affirmations about obscure and unknown matters which they arrived at only by probable conjectures; and then gradually attaching to such matters a complete faith, and mixing them indiscriminately with what is true and evident,

they have finally grown unable to draw any conclusion that does not appear to depend on some such proposition, and so is not uncertain.

But in order that we may not fall into the same error, let us here enumerate all the acts of our intellect through which we can arrive at knowledge of things without any fear of error. We admit only two: namely, intuition and deduction.[3]

By *intuition* I understand neither the fleeting testimony of the senses nor the deceptive judgment of the imagination with its false constructions, but a conception of a pure and attentive mind, so easy and so distinct, that no doubt at all remains about what we understand. Or, what comes to the same thing, intuition is the indubitable conception of a pure and attentive mind arising from the light of reason alone; it is more certain even than deduction, because it is simpler, even though, as we noted above, people cannot err in deduction either. Thus everyone can intuit with his mind that he exists, that he is thinking, that a triangle is bounded by only three lines, a sphere by a single surface, and the like. Such things are much more numerous than most people think, because they disdain to turn their minds toward matters so easy.

369 But so that some may not be disturbed by the term intuition in this new sense, or still others by my being forced to depart in the same way from common meanings in the following pages, I add here the general warning: I do not in the least consider the way in which particular terms have been used in the schools recently, since it would have been very difficult to use the same words and inwardly to have such different thoughts. I consider only what each word means in Latin, so that when proper words are lacking I may transform whatever terms appear to me most suitable to fit my meaning.

This evidence and certainty of intuition is required, however, not only for single statements, but also for discursive reasoning of every kind. Thus, for example, given this conclusion: 2 and 2 amount to the same as 3 and 1, one must see by intuition not only that 2 and 2 make 4, and that 3 and 1 also make 4, but also that the third proposition is a necessary inference from the other two.

Thus there may now be some doubt as to why we should have added here another mode of knowledge besides intuition, that is, one proceeding by deduction, by which we understand all that is necessarily inferred from other things that are certainly known. But this procedure was necessary, since many things are known with certainty which nevertheless are not themselves evident, simply because they are deduced from true and known principles by the continuous and uninterrupted movement of a mind which clearly intuits each step. Thus we know that the last link of a long chain is connected with the first, even though we *370* do not take in with a single glance of the eyes all the intermediate links on which the connection depends—provided only that we run through them successively and remember that from first to last each one was attached to the one next to it. Therefore we distinguish here intuition from certain deduction by the fact that some movement or succession is conceived in the latter but not in the former.

3. The manuscript has *inductio*, but either *deductio* was intended or Descartes did not carefully differentiate between induction and deduction.

Moreover, evidence is not necessarily present for deduction, as it is for intuition, but deduction rather acquires its certainty, in a sense, from memory. From all this we may conclude that those propositions which follow immediately from first principles are known according to the way we look at it, now by intuition, now by deduction, but that the first principles themselves are known only by intuition, and the remote conclusions, in contrast, only by deduction.

These then are the two most certain paths to scientific knowledge. No others should be admitted by the mind, but all the rest rejected as suspect and liable to error. This does not, however, prevent our believing those matters which are divinely revealed to be more certain than all knowledge. For faith in these, although it concerns obscure matters, is not an act of intellect but of will, and if they have a basis in the intellect, they can and ought to be, more than all other things, discovered by one or the other of the two ways already mentioned, as we may perhaps indicate at greater length.

4. Method is necessary for the investigation of truth. *371*

Mortals are possessed by such blind curiosity that they often lead their minds through unknown paths, without any ground for hope, but simply venturing on the chance that what they seek might lie that way: as if a person is burning with so stupid a desire to find a treasure that he constantly roams about the streets to see if by chance he might find some article lost by a traveler. It is in this manner that almost all the chemists, most geometers, and not a few philosophers work. To be sure, I do not deny that they sometimes stray so fortunately as to find something true; still I do not therefore hold them more efficient, but only more fortunate. And it is much better never to think of investigating the truth of anything at all, than to do it without method. For it is very certain that through such disorderly studies and obscure meditations the natural light is obscured and our minds blinded. Thus all those who accustom themselves to walking in the dark weaken the acuteness of their eyes so much that afterward they cannot bear the light of the day. This is also confirmed by experience; for how often do we not see those who have never devoted themselves to letters judging much more solidly and clearly of the things that come their way than do those who have spent all their time in the schools? By method, then, I understand certain and simple rules such that if a person follows them exactly, he will never suppose anything false to be true, and, spending no useless mental effort, but gradually and steadily increasing his knowledge, will arrive at true knowledge of all those things to which his powers are adequate. *372*

Two things should be noted here: never to suppose true what is false, and to arrive at knowledge of all things. For if we are ignorant of some one of all the things that we can know, that happens only because we have never discovered any way that would lead us to such knowledge, or because we have slipped into the opposite error. But if the method explains correctly how the intuition of the mind is to be used, and how deductions are to be made, so that we may arrive at knowledge of all things, nothing more seems to me to be required to make it complete, since we have already said that there can be no scientific knowledge

except through an intuition of the mind or through a deduction. Nor indeed must the method extend to showing how these operations themselves are to be conducted, since they are the first and simplest of all—so much so that, unless our intellect already knew how to use them, it could understand none of the precepts of the method itself, however simple. As to the other operations of the mind, moreover, which dialectic struggles to direct with the aid of these prior ones, they
373 are useless here—or rather they may be counted as obstructions, since nothing can be added to the pure light of reason without in some way obscuring it.

Since therefore the usefulness of this method is so great that it seems more harmful than useful to devote oneself to the study of the sciences without it, I am readily convinced that, doubtless with the sole guide of nature, the greatest minds have formerly perceived it in some fashion. For the human mind possesses an I-know-not-what that is divine, in which the first seeds of useful thoughts are scattered, so that often, though neglected and suffocated by perverse studies, they bear spontaneous fruit. We have experience of this in the simplest of the sciences, arithmetic and geometry; for we have sufficient evidence that the ancient geometers used a certain analysis, which they extended to the resolution of all problems, even though they begrudged it to posterity. And now there also exists a kind of arithmetic, called algebra, which does with numbers what the ancients did with figures. And these two are nothing but spontaneous fruits born of the innate principles of this method. Nor do I wonder that, with regard to the extremely simple objects of these arts, these fruits have developed more happily than in others, where greater obstacles usually stifle them. Even there, however, if only they are cultivated with the greatest care, they can without doubt arrive at full maturity.

This, then, is what I have principally undertaken to do in this treatise. Indeed, I should not make much of these rules, if they were adapted only to the solution of the vain problems with which logicians and geometers are accustomed to play at their leisure; for in that case I should think I had succeeded only in playing
374 with trifles perhaps more subtly than others had done. True, I shall often speak here of figures and of numbers (since one cannot expect from any other discipline examples so evident or so certain), yet whoever considers my meaning attentively will easily perceive that there is nothing of which I am thinking here less than of vulgar mathematics; but that I am expounding another discipline, of which these are the outer husks rather than the parts. This science should in fact contain the first rudiments of human reason, and should need only to extend itself in order to elicit truths on any subject whatsoever; and to speak freely, I am convinced that it is more powerful than all the other knowledge that mankind has taught us, because it is the source of all the rest. But I said outer husks, not because I want to wrap up this doctrine and hide it to keep the crowd away, but rather in order to decorate and ornament it, so that it may be more suitable for the human mind.

When I first applied my mind to the mathematical disciplines I began by reading most of those things that mathematical authors usually teach, and I paid most
375 attention to arithmetic and geometry, since they were said to be simplest and at the same time paths to the others. But in neither case did I at that time lay my

hand on authors who fully satisfied me. I did indeed read in their works several statements about numbers which after making calculations I found to be true; and even with regard to figures, they set, so to speak, many things before my eyes, and inferred them from certain consequences. But they did not seem to my mind to exhibit satisfactorily why these matters stood thus, and how they had been discovered. So it did not surprise me that after tasting these arts, most persons of talent and knowledge at once set them aside as puerile and vain, or on the contrary are deterred at the very start from learning them because they appear so difficult and intricate. For indeed nothing is more futile than to occupy oneself with bare numbers and imaginary figures, in such a way as to appear willing to rest content with knowledge of such trifles; nor is anything more futile than so to attach oneself to those superficial demonstrations, which are more frequently discovered by chance than by art and have more to do with the eyes and the imagination than with the intellect, that one becomes in a sense unaccustomed to the use of reason. At the same time nothing is more complicated than to dispose in this manner of new difficulties hidden by the confusion of numbers. But then when I went on to think that those who first discovered philosophy long ago were unwilling to admit to the study of wisdom anyone untrained in mathematics, as if this discipline seemed to them the easiest and most necessary of all in training *376*
minds and preparing them to understand other and higher sciences, I strongly suspected that they knew some mathematics very different from the vulgar mathematics of our age. Not that I think they knew it very perfectly, for their mad celebrations and thanksgivings for trifling discoveries indicate clearly how little advanced they were. Nor do certain of their machines that are celebrated by historians move me from my opinion; for although they were doubtless very simple, they could be praised to a degree of fame befitting miracles by the ignorant and astonished crowd. But I am convinced that the first seeds of truth, sown by nature in the human mind, but which we stifle in ourselves by reading and hearing every day so many errors of every kind, had such force in that crude and simple antiquity that, by the same light of the mind that made them see they ought to prefer virtue to pleasure and the good to the useful, although they were ignorant why it should be so, people had true ideas of philosophy and mathematics, although they had not yet been able to acquire perfectly these sciences themselves. In fact it seems to me that traces of that true mathematics are still visible in Pappus and Diophantus,[4] who, though not of the first age, still lived many centuries before our time. But this I believe was later suppressed, with a sort of evil cunning, by these authors themselves. For, as many artisans have done for their inventions, they feared perhaps that being very easy and simple their method might lose its price if given to the crowd. In order that we should admire them they preferred to give us instead of their discoveries a few sterile verities, subtly deduced, as the fruits of their art, rather than to teach the art itself, which would *377*
clearly dispel the admiration. Finally there were some very ingenious persons who tried in this century to revive this art. For that art which is called by the barbarous name of algebra seems to be nothing else, provided only one could

4. Third- and fourth-century A.D. Alexandrian (Greek) mathematicians.

disentangle it from the multitudinous numerals and inexplicable figures with which it is encumbered, so that it might no longer lack that clarity and that supreme facility which ought, as we have said, to be present in true mathematics. When these thoughts had led me from the particular study of arithmetic and geometry to a general study of mathematics, I inquired first of all precisely what everyone means by this word, and why not only those two sciences of which we have already spoken, but also music, optics, mechanics, and several others are called parts of mathematics. For it is not enough in this case to consider the etymology of the word; since, as the term *mathesis* signifies simply science, the other sciences would have no less right than geometry itself to be called mathematics. Moreover, we see no one who, if he has so much as set foot in a school, fails to distinguish easily among those subject matters that are presented to him what belongs to mathematics and what belongs to other disciplines. And if one reflects on this matter more attentively, one finally observes
378 that all and only those subjects in which order and measurement are investigated are referred to mathematics, no matter whether such measure is sought in numbers, in figures, in stars, in sounds, or in some other object. One concludes, therefore, that there must be some general science explaining all that can be investigated concerning order and measure, without application to a particular material; and that this science is called, not by a strange name, but by a name already ancient and received by usage, *universal mathematics*, because it includes all that material by virtue of which other sciences are called parts of mathematics. How much it excels in usefulness and facility the sciences that depend on it is clear from the fact that it extends to all the objects which they treat and to many others; and that all the difficulties it involves are found also in the other sciences, accompanied in addition by many other difficulties, which arise from their particular objects, and which it for its part does not possess. But now, since everyone knows its name and knows what it deals with, even without applying it, how does it happen that most people try to learn the other sciences that depend on it, while no one takes the trouble to study it in itself? I should certainly be amazed at this, if I did not know that it is considered by everyone to be very simple, and if I had not observed long ago that the human mind, leaving aside what it thinks easy of attainment, hurries on to new and loftier things.

But I, conscious of my weakness, have decided constantly to observe in the
379 investigation of truth an order such that, always beginning with the simplest and easiest matters, I never proceed to others before it seems to me that nothing remains to be desired in the first. That is why I have cultivated up to now this universal mathematics to the best of my abilities; hence I believe that when I go on, as I hope to do soon, to deal in turn with higher sciences, my efforts will not be premature. But before I take this step I shall try to unite and to set in order all that I have found worthy of notice in my earlier studies—both in order to find them without trouble in this book, if need be, at a time when with increasing age my memory will fail, and in order to be able to carry a freer mind to other things, having discharged my memory of them.

5. All method consists in the order and disposition of those things toward which the eye of the mind must be directed if we are to discover any truth. And we follow this method exactly if we reduce involved and obscure propositions step by step to simpler ones, and then attempt to ascend by the same steps from the intuition of all those that are entirely simple to the cognition of all the others.

In this alone lies the sum total of human endeavor, and this must be followed no less carefully by one who would arrive at a knowledge of things than the *380* thread of Theseus by him who would penetrate the labyrinth. But many people either do not reflect on what this precept teaches, or are completely ignorant of it, or suppose they do not need it. Hence they often examine the most difficult questions with so little order that they seem to me to behave as if they were trying to get from the bottom to the top of a building with one jump, either taking no account of the stairs intended for this use, or failing to notice them. That is what all the astrologers do, who, without knowing the nature of the heaven and without even having observed its motions adequately, hope to be able to indicate its effects. That is what many do, who study mechanics apart from physics, and rashly manufacture new instruments for the production of motions. That is also what those philosophers do, who neglect experience but think that truth will spring from their own brains, like Minerva from the head of Jupiter.

Indeed it is evident that all these err with respect to the present rule. But since the order required here is so obscure and intricate that not everyone can make out what it is, they can scarcely take enough care to avoid error, unless they diligently observe what is expounded in the following proposition.

6. To distinguish the simplest things from those which are complex, and to follow them out in order, it is necessary, in every sequence of things in which we have directly deduced certain truths from others, to observe what constituent has the greatest simplicity, and in what way all the others are more or less or equally removed from it. *381*

Although this proposition appears to teach nothing new, it contains nevertheless the chief secret of this art, and there is no more useful proposition in all this treatise; for it counsels that all things can be arranged in certain sequences. Not indeed, that they can be so arranged insofar as they are referred to some genus of being, as the philosophers have divided them into their categories, but insofar as certain ones can be known through others. Thus, each time any difficulty occurs, we can see immediately whether it will be profitable to run through certain other matters first, and which ones, and in what order.

In order that this may be done correctly, however, it must first be noted that all things, to the degree to which they can be useful to our project (when we do not consider their natures in isolation but compare them with one another, in order that certain ones may be known through others), may be said to be either absolute or relative.

I call absolute everything that contains within itself the pure and simple nature in question: as all that is considered independent, cause, simple, universal, equal,
382 similar, straight, or the like; and I call this the simplest and easiest of all, so that we may use it for resolving questions.

The relative, on the other hand, is what participates in the same nature, or at least in something of it, in accordance with which it can be referred to the absolute, and deduced from it through some sequence, but which, in addition, involves in its conception other things I call relations. Such is all that is called dependent, effect, compounded, particular, many, unequal, dissimilar, oblique, etc. These relatives are removed from absolutes in proportion to the number of mutually subordinate relations they contain. And it is the necessity of distinguishing such relations that the present rule teaches. It also teaches the need of observing the pattern of interconnections between them and their natural order in such a fashion that we can proceed from the last of them to the most absolute, passing through all the rest.

And the secret of the whole art consists in this: that we notice carefully in all things what is most absolute in them. For some things are more absolute than others from one point of view, but more relative from another. Thus the universal is indeed more absolute than the particular, since it has a simpler nature, but at the same time one can say it is more relative since it depends on individuals for its existence. Again, there are sometimes things that are really more absolute than others, even though they are never the most absolute of all. Thus, if we consider individuals, the species is an absolute, and if we consider the genus, it is a relative; among measurable things, extension is an absolute, but among exten-
383 sions length is an absolute, etc. In the same way, finally, in order to make it clearer that we are here considering the sequences of things as objects of knowledge and not the nature of each one of them, we have purposely counted cause and equal among the absolutes, although their nature is really relative—for among the philosophers cause and effect are in fact correlatives. But here, if we are in fact inquiring into the nature of the effect, we must first know the cause, and not the reverse. Equals likewise correspond with one another; but we know unequals only by comparison with equals, and not the reverse, etc.

It should be noted, secondly, that there are only a few pure and simple natures, which we may intuit in themselves, independently of all others, whether in trials by experience, or by the light implanted in us. Moreover we declare that these must be painstakingly observed; for it is these we call the simplest in every sequence. All others, in contrast, can be perceived only insofar as they are deduced from these, either immediately or proximately, or through the mediation of two or three or more separate conclusions. And the number of these conclusions must also be noted, so that we may know if they are removed from the first and simplest propositions by a smaller or greater number of steps. And such is everywhere the nexus of consequences, from which arise those sequences of objects of investigation, to which every question is to be reduced in order that it
384 may be examined by a sure method. But because it is not easy to review them all, and since, besides, they do not need so much to be retained by the memory as distinguished by some insight of the mind, we must seek for something which

will form the mind so as to let it perceive these sequences whenever it needs to do so. For this purpose, I can say from experience, nothing is more effective than to reflect with some sagacity on the very smallest of those things we have already perceived.

Finally, it should be noted, in the third place, that we ought not to begin an inquiry with the investigation of difficult matters. Rather, before we set out to attack any definite questions, we must first collect indiscriminately all the truths that spontaneously present themselves, then gradually see if others can be deduced from them, and from these last yet others, and so on. That done, we must reflect attentively on the truths we have discovered, and consider carefully why we have been able to find some sooner and more easily than others, and which ones they are. This we do so that we may also be able to judge, when we begin some definite question, to what other inquiries we could profitably apply ourselves first. For example, if it occurred to me that 6 is the double of 3, I should look further for the double of 6, that is to say, 12; then I should look, if I liked, for the double of that, that is to say 24, and the double of that, that is 48, etc.; thus I should conclude, as it is easy to do, that there is the same proportion between 3 and 6 as between 6 and 12, and the same between 12 and 24, etc., and that consequently the numbers 3, 6, 12, 24, 48, etc., form a continuous proportion. Although all these things are so clear as to appear almost childish, I understand, on attentive reflection, in what way all questions are involved which can be *385* posed about proportions or the relations of things, and in what order they should be investigated: and this alone embraces the whole of the science of pure mathematics.

For first I observe, to begin with, that it is more difficult to find the double of 6 than the double of 3; and similarly, in every case, once we have found the proportion that exists between any two numbers, we can find other magnitudes in indefinite number having the same proportion to one another. And the nature of the difficulty does not change if we look for three or four or more, for the reason that we have to find each one separately and without taking account of the others. Further, I observe that although, given the magnitudes 6 and 3, you easily find the third in continuous proportion, namely, 12, it is nevertheless not so easy given the two extremes, that is, 3 and 12, to be able to find the mean proportional, that is, 6. If we look into the matter, we find that this is clearly a different kind of difficulty from the preceding, since, to find the mean proportional, we must attend at the same time to the two extremes and to the proportion that exists between them, so that something new is produced by their division. This is something very different from what is required, given two magnitudes, to find the third in some proportion. I go even further and ask whether, given the magnitudes 3 and 24, it would be as easy to find one of the two mean proportionals, namely, 6 and 12. Here we have yet another kind of problem, more involved than *386* previous ones, since here we have to attend, not to one or two, but to three things at the same time in order find a fourth. Let us go even further than this, and see whether, given only 3 and 48, it would be more difficult to find one of the three mean proportionals, namely, 6, 12, and 24. At first sight it does seem so. But then it occurs to us at once that this difficulty can be divided and diminished. Plainly,

we first look for one main proportional between 3 and 48, namely, 12, and then we look for another mean proportional between 3 and 12, that is, 6, and another between 12 and 48, namely, 24. And in this way the difficulty is reduced to the second kind already discussed.

From all the above I observe, further, how the knowledge of one and the same thing can be pursued by different paths, which differ from one another considerably in difficulty and obscurity. For example, if we are to find the four proportionals, 3, 6, 12, 24, given any consecutive pairs, that is, 3 and 6, 6 and 12, or 12 and 24, in order to find the rest, that will be very easy to do. In that case we shall say that the proportion to be found is being examined directly. If, however, two are given alternately, namely, 3 and 12 or 6 and 24, and the others are to be found, then we shall say that the difficulty is examined indirectly to the first degree. If, finally, we suppose the two extremes, that is, 3 and 24, so that we are looking for
387 the two intermediates, 6 and 12, then this will be examined indirectly to the second degree. And I could go further and deduce many other things from this one example. But these are sufficient to let the reader observe what I mean when I say that some progression is deduced directly or indirectly, and to enable him to understand how it is that from some very simple things that are known first, many others also, in many disciplines, can be deduced by those who reflect attentively and inquire with sagacity.

7. In order to attain complete scientific knowledge, it is necessary to run through, one by one, in a movement of thought which is continuous and nowhere interrupted, all those matters which bear upon our undertaking; they must also be included in a sufficient and ordered enumeration.

The observation of what is propounded here is necessary for the admission among certain truths of those which, as we have said above, are not immediately deduced from first principles known through themselves. Sometimes, in fact, this deduction is made by a chain of consequences so long that, when we get to the end, we do not easily remember the whole path that has led us to this point; and that is why we said that it is necessary to aid the weakness of the memory by a continuous movement of thought. Thus if I have found out by separate operations, for example, what relation there is between the magnitudes A and B, next between B and C, and then between C and D, and finally between D and E, I do
388 not therefore see what relation there is between A and E, nor can I understand it with accuracy from the facts I have already learned, unless I remember them all. To remedy this, I should run over them several times with a continuous movement of the imagination that gives an intuition of every single one and at the same time passes to others, until I had learned to pass from the first to the last so rapidly that next to no part was left to memory, but I seemed to intuit the whole thing at once. For by this means, while it helps the memory, the sluggishness of the mind is corrected, and its capacity in a certain sense extended.

We add, moreover, that the movement must nowhere be interrupted. For often those who wish to deduce something too quickly and from distant principles do

not run through the whole chain of intermediate propositions with sufficient care to prevent their rashly overlooking many points. But surely, wherever even the smallest point is omitted, the chain is immediately broken, and the whole certainty of the conclusion falls.

We say here, further, that enumeration is required for the complete attainment of scientific knowledge. To be sure, other precepts are of assistance in the solution of many questions; but only the aid of enumeration can bring it about that, to whatever question we may apply our minds, we would always make a true and certain judgment, and that therefore nothing at all would escape us, but we would appear to know something about everything.

This enumeration, then, or induction, is an inventory of everything that bears on any given question—an inventory so painstaking and accurate that we conclude from it with certainty and evidence that nothing has mistakenly been omitted by us. Thus every time we have used it, if the thing we are looking for escapes us, we are *389* at least wiser in this respect: that we perceive with certainty that it can be found by no way known to us; and if perchance, as often happens, we have succeeded in reviewing all the ways to it open to men, we may boldly affirm that knowledge of it lies entirely beyond the reach of human intelligence.

It should be noted, further, that by sufficient enumeration or induction, we understand only the means by which truth is more certainly inferred than by any other kind of proof except simple intuition. As often as a cognition cannot be reduced to intuition, since we have thrown off all syllogistic fetters, there remains to us only this one way on which we should fasten all our faith. For whatever single propositions we have deduced immediately from others are already reduced to a true intuition if the inference was evident. If, however, we infer some one thing from many and disconnected facts, the capacity of our intellect is often insufficient to embrace them all in a single intuition, in which case the certitude of the present operation should suffice. In the same way we are unable to distinguish with a single glance of the eyes all the links of a very long chain; yet if we see the connection of each one to the next, that is enough to let us say that we have seen how the last is connected with the first.

I have said that this operation ought to be sufficient, because it can often be defective and in consequence liable to error. For sometimes, even though we review by enumeration a great number of things that are really evident, if never- *390* theless we omit even the smallest point, the chain is broken, and the whole certainty of the conclusion falls. Sometimes, moreover, we embrace the whole with certainty in an enumeration, but we do not distinguish the single points from one another, and so know the whole only confusedly.

Besides, this enumeration should sometimes be complete, sometimes distinct, but sometimes neither is necessary; and that is why it has been stated only that it ought to be sufficient. For if I want to prove by enumeration how many genera of things are corporeal, or fall in some way under sense, I shall not declare that there are so many and no more, until I know for certain that I have included them all in my enumeration, and have distinguished each from the others. But if I wish to show by the same means that the rational soul is not corporeal, it will not be necessary for the enumeration to be complete, but it will be sufficient if I include

all bodies at once in certain classes, in such a way as to demonstrate that the rational soul can be referred to none of them. And finally if I wish to show by enumeration that the area of the circle is greater than the area of other figures whose perimeter is equal, it is not necessary to review all the figures, but it is sufficient to demonstrate this of some particular figures, in order by induction to reach this same conclusion concerning all the others.

I have also added that enumeration should be ordered, not only because there is no better remedy for the defects already listed than to examine everything with
391 order, but also because it often happens that, if it were necessary to examine separately every one of the things that bear on a given question, no human life would suffice for it, either because these things are too numerous, or because the same things would keep cropping up for renewed consideration. But if we dispose of all things in the best order, they will (for the most part) be (as far as possible) reduced to definite classes. It will then be enough to examine carefully either a single one of them, or something from each, or some rather than others; or at least we shall not review the same thing twice to no purpose. This procedure is so helpful that often because of a well-established order one traverses in a short time and with little effort a great many things which at first sight looked immense.

The order of things to be enumerated, however, can often vary, and it depends on the choice of each person. So, to grasp it more accurately, we must recall what was said in the fifth proposition. In the more trivial inventions of men, there are many things whose method of discovery consists entirely in disposing of things in this orderly way. Thus if you wish to construct a perfect anagram by transposing the letters of a name, there is no need to pass from the easy to the difficult, nor to distinguish absolute from relative. Here there is no place for these things; but it will be sufficient to adopt an order for transposing the letters under examination, such that one never comes twice to the same one, and that their number, for instance, is distributed in fixed classes so that where is the best hope of finding what is sought may immediately appear. In this fashion the work will often not take too long, but be mere child's play.

392 On the other hand, these three last propositions are not to be separated, because for the most part we must think of them at the same time, and because all concur equally in the perfection of our method. It did not make much difference which was given first; and we have explained them here briefly, because we have practically nothing left to do in the rest of the treatise, except to show in particular what we have considered here in general.

8. If in the series of things to be examined anything presents itself which our intellect is unable to intuit sufficiently well, we must stop there and should not examine what follows, but abstain from superfluous labor.

The three preceding rules prescribe order and explain it; this one shows when it is absolutely necessary and when it is only useful. Thus whatever constitutes a complete step in that series by which we must pass from relatives to some

absolute or the reverse, must necessarily be examined before anything that follows it. If, however, as often happens, many things belong to the same step, it is indeed always useful to run through them all in order, but in this case we are not forced to observe order so strictly nor so rigidly. Often, although we do not know all these things clearly, but only a small number of them or just one, it is still possible to pass beyond them. *393*

This rule follows necessarily from the reasons given for the second rule. However it must not be supposed that it contains nothing new for the advancement of science, even though it appears only to keep us from the discussion of certain things and to propound no truth. As for beginners, indeed, it teaches them only not to waste their time, in almost the same way as the second rule. But to those who have perfectly learned the seven preceding rules, it shows how in any science whatsoever they can satisfy themselves so as to desire nothing further. For whoever has observed the preceding rules exactly in the solution of any difficulty and has nevertheless received from this rule the order to halt, will then know with certainty that he cannot by any device discover the knowledge he is seeking—and that not by the fault of his mind, but because the nature of the difficulty itself or the condition of humanity prevents him. This knowledge is science no less than is what exhibits the nature of the thing itself; and he would not appear of sound mind who should extend his curiosity further.

Let us illustrate this by one or two examples. If someone who studies only mathematics looks for that line which in dioptrics is called anaclastic, and in which parallel lines are refracted in such fashion that all of them, after the refraction, meet in a single point, he will easily observe, according to rules five and six, that the determination of this line depends on the proportion of the angles of refraction to the angles of incidence. But as he will not be capable of investigating this matter, since it does not belong to mathematics but to physics, he will have to stop immediately. And it would be of no use to him if he wished to hear from the philosophers or draw from experience the knowledge of this truth; for he would be sinning against the third rule. Besides, this proposition is still composite and relative; but it is only in things that are perfectly simple and absolute that experience can be considered certain, as we shall show in the proper place. Moreover, it would be useless for him to postulate, between angles of this kind, some proportion he suspected to be truest of all; for then he would no longer be looking for the anaclastic line, but only for the line that should be a logical consequence of his supposition. *394*

On the other hand, if someone who does not study mathematics alone, but who tries, according to the first rule, to look for the truth on any subject that presents itself, should fall into the same difficulty, he will go farther and discover that this proportion between the angles of incidence and the angles of refraction depends on the variation of these same angles in virtue of the difference of the media; that this variation in turn depends on the manner in which the ray penetrates into the transparent body; that knowledge of the property of penetrating into a body presupposes equally that the nature of illumination is known; and that finally to understand illumination one must know what a natural power is in general—and this is the last and most absolute term in this whole sequence. Then when he has *395*

perceived this clearly by intuition, he will repeat the same steps, according to the fifth rule; and if in the second step he cannot at once recognize the nature of illumination, he will enumerate all the other natural powers, in accordance with the seventh rule, in order that, thanks to the knowledge of some one of them, he may understand it also, at least by analogy (of which I will speak later). This done, he will investigate the manner in which the ray penetrates the whole transparent body; and in this way he will run through the rest in order, until he has arrived at the anaclastic line itself. Although up to now this has been vainly attempted by many people, I see nothing to keep someone who makes perfect use of our method from evident knowledge of this line.

But let us give the most noble example of all. If a person proposes to himself the problem of examining all the truths for the knowledge of which human reason suffices—a task which should be undertaken at least once in his life, it seems to me, by anyone who is in all seriousness eager to attain excellence of mind—he will certainly discover by the rules given above that nothing can be known before the intellect, since the knowledge of all other things depends on this, and not the reverse. Then, when he has examined everything that follows immediately after the knowledge of the pure intellect, he will enumerate, among other things, all the other instruments of knowledge we possess besides the intellect;
396 and these are only two: namely, imagination and the senses. He will then devote all his care to distinguishing and examining these three modes of knowledge; and seeing that strictly speaking truth or falsity can exist only in the intellect, but that they often take their source from the other two as well, he will carefully attend to everything by which he can be deceived so that he may be on guard against it. And he will enumerate exactly all the paths to truth that are open to humans so that he may follow the sure one—for there are not so many that he cannot discover them all easily through a sufficient enumeration. And, what will seem marvelous and incredible to the inexperienced, as soon as he has distinguished, for each object, those cognitions which only fill and embellish the memory from those in virtue of which one may truly be said to be more learned, a distinction which it is also easy to make, [. . . .][5] he will feel that there is absolutely nothing of which he is ignorant through a defect of mind or art, and that nothing further can be known by any person which he is not also capable of knowing, provided only that he applies his mind to it as he ought. And although many things can often be proposed to him, the investigation of which are forbidden by this rule, he will nevertheless not think himself more ignorant for having clearly understood that they exceed the bounds of the human mind; but this knowledge itself, that no one can know the thing in question, will amply satisfy his curiosity if he is reasonable.

But that we may not always be uncertain what our mind is capable of, and that we may not labor wrongly and rashly, before we set ourselves to learn things in
397 detail; we ought to inquire carefully, once in our lives, of what knowledge human reason is capable. In order better to accomplish this task, among things that are equally simple we ought to investigate those which are more useful.

5. There is a gap in the original texts.

Indeed, this method resembles those of the mechanical arts which need no outside help, and which themselves teach us how to construct their instruments. Thus if one wished to practice one of them, the art of the blacksmith, for example, one would be forced at first to use as an anvil a hard stone or a rough lump of iron, to take a piece of rock in place of a hammer, to shape pieces of wood into tongs, and to collect other materials of this sort according to need. Thus equipped, one would not then at once try to forge swords or helmets or any object of iron for the use of others; but one would first of all manufacture hammers, an anvil, tongs, and the other things useful to oneself. This example teaches us that, if we have been able at the outset to find only some rough principles, which seem to be innate in our minds rather than prepared by art, we must not use them to try to settle immediately the controversies of the philosophers or to solve the puzzles of the mathematicians. We must rather use them first for seeking with the greatest care all that is more necessary for the examination of truth; since there is surely no reason why this should seem more difficult to discover than any of the questions usually propounded in geometry or physics or other disciplines.

Now nothing is more useful here than to inquire what human knowledge is and how far it extends. That is why we now embrace these problems in a single question, which we believe should be examined first in accordance with the rules pre- *398*
viously started. This must be done once in his life by anyone who has even the faintest love for truth, since this inquiry contains the true instruments of knowledge and the whole of method. Nothing seems to me more absurd, on the other hand, than to argue boldly about the mysteries of nature, the influence of the heavens on our earth, the prediction of the future, and the like, as many do, and yet never to have inquired whether human reason is adequate for the discovery of these things. Nor should it seem arduous or difficult to determine the limits of the mind, which we feel within ourselves, since often we do not hesitate to make judgments on things outside us and quite foreign to us. Nor is it an immense task to attempt to embrace in thought all the things contained in this universe, in order that we may recognize how each one is subjected to the examination of our minds; for nothing can be so complex or so scattered that, by means of the enumeration with which we have been dealing, it cannot be circumscribed within definite limits and arranged under a certain number of headings. In order to have experience of this in the question at hand, we first divide everything that pertains to it into two parts; for it ought to be referred to either us, who are capable of knowledge, or to those things which can be known; and we discuss these two parts separately.

Now we notice in ourselves that the intellect alone is capable of scientific knowledge; but that it can be helped or hindered by three other faculties, namely by imagination, sense, and memory. We must therefore see, in order, in what respect each of these faculties can be a hindrance so that we may be on our guard; *399*
or in what respect each can be of use so that we may use all its resources. Thus this part shall be treated by a sufficient enumeration, as the following rule will make clear.

We must then proceed to the things themselves, which are to be examined only insofar as they are touched by the intellect. In this respect we divide them into

maximally simple natures and natures that are complex or composite. Simple natures must be either spiritual or corporeal, or related to both. Then among the composites the intellect experiences some to be complex before it judges that it can determine anything about them; but others it puts itself together. All this will be expounded at greater length in the twelfth rule, where it will be proved that there can be no falsity except in these last natures, which are put together by the intellect. That is why we distinguish them again into two kinds: those which are deduced from natures that are of the greatest simplicity and known through themselves, of which we shall treat in the following book; and those which likewise presuppose others which the facts themselves show us to be composite, for the exposition of which we intend the whole of the third book.

And, indeed, in all of this treatise we shall try to follow through with so much care and to make so easy all the paths that are open to humans for the knowledge of truth that anyone who has learned perfectly the whole of this method, however
400 mediocre his mind, may yet see that none of these paths is more closed to him than to others, and that he is no longer ignorant of anything through a defect of mind or art. But as often as he applies his mind to the knowledge of anything, either he will reach it entirely; or he will clearly understand that it depends on some experience not in his power, and then he will not blame his own mind, although he is forced to stop at that place; or, finally, he will demonstrate that what he is seeking exceeds the bounds of the human mind, and consequently he will not think himself more ignorant, because it is not a lesser thing to know this knowledge than any other thing.

9. We ought to turn the whole force of our minds to the smallest and simplest things, and to stop there for a long time, until we become accustomed to intuiting the truth clearly and distinctly.

We have now expounded the two operations of our intellect: intuition and deduction, which we have said are alone to be employed in learning the sciences. We continue in this and the next proposition to explain by what procedure we can become more skilled in using them and at the same time in developing the two principal faculties of our mind, perspicacity, in having a distinct intuition of each thing, and sagacity, in easily deducing certain facts from others.

In fact, we learn the manner in which mental intuition should be used by comparing it with vision. For whoever wishes to look at many objects at one time
401 with a single glance, sees none of them distinctly; and similarly whoever is used to attending to many objects at the same time in a single act of thought, is confused in mind. But those artisans who practice delicate operations, and are accustomed to direct the force of their eyes attentively to single points, acquire by use the ability to distinguish perfectly things as tiny and subtle as may be. In the same way, likewise, those who never disperse their thought among different objects at one time, but always occupy all its attention in considering the simplest and easiest matters, become perspicacious.

But it is a failing common to mortals to consider difficult things as more attractive. And most people think they know nothing when they find a cause for

something that is really clear and simple, while they admire certain sublime and profound theories of the philosophers, although these rest for the most part on foundations never adequately examined by anyone. Poor fools, indeed, who prefer darkness to light! It should be noted, however, that those who really know discern the truth with equal facility whether they have drawn it from a simple or from an obscure subject. For they comprehend each truth by an act that is similar, single, and distinct, once they have arrived at it, but the whole difference is in the road, which should certainly be longer if it leads to a truth remote from the first and most absolute principles.

Thus we should all accustom ourselves to including in our thought, at one and the same time, matters so few and so simple that we cannot think we know anything at all unless we intuit it no less distinctly than we do those things that we *402*
know most distinctly of all. For this, indeed, some people are born much more capable than others; but method and practice can also make minds much better at it. And if there is one point that must be stressed here, it seems to me, with more insistence than all the others, it is that the sciences, however hidden, can be deduced, not from great and obscure matters, but only from those that are easiest and most obvious.

So, for example, let us suppose that I want to inquire whether some natural power exists than can pass in the same instant to a distant place, while traversing all the intervening space. I shall not at once turn my mind to the power of the magnet, or to the influence of the stars, or even to the speed of light, in order to inquire whether perchance such actions take place in an instant. For to investigate this would be more difficult than the question I am asking. But I would rather reflect on the local motion of bodies, since nothing in this whole area is more accessible to the senses. And I would notice that a stone cannot pass from one place to another in an instant, since it is a body; but that a power similar to the one that moves the stone can pass in its bare state from one subject to another For instance, if I move one of the extremities of a stick, of any length whatever, I easily conceive that the power that sets that part of the stick in motion necessarily moves all the other parts as well in one and the same instant, since it is then communicated as a bare power, and does not reside in some body by which it is carried.

In the same way, if I should wish to know how contrary effects can be produced by the same cause, I shall not seek help from physicians whose drugs *403*
expel certain humors and replace others. I shall not talk nonsense about the moon: that it heats by its light, and cools off by some occult quality. But instead I shall examine a scale, in which the same weight lifts one side at one and the same instant at which it depresses the other, and other examples of this kind.

10. In order that the mind may acquire sagacity, it is necessary to give it practice in investigating what has already been discovered by others; and it ought to traverse methodically even the most trifling inventions of men, but especially those which best explain or presuppose order.

I confess that I was born with a mind such that I have always found the greatest pleasure of study, not in hearing the explanations of others, but in finding them by my own efforts. This alone attracted me, when I was young, to the study of the sciences. So whenever a book promised a new discovery by its title, before going farther I tried if by chance I could not succeed in finding something analogous by natural sagacity; and I took good care not to deprive myself of this innocent pleasure by a hasty reading. I succeeded in this so often that I finally noticed that I was no longer arriving at the truth of things, as others usually do, by vague and blind disquisitions, by the help of fortune rather than art, but that by long experience I had perceived certain rules, which are of great help in this
404 study and which I afterward used to think out many others. And so I have diligently elaborated this whole method, and have become convinced from the start that I had followed the most useful mode of studying.

But since not all minds are equally inclined by their nature to discover things of their own power, this proposition teaches that we should not occupy ourselves immediately with the more difficult and arduous matters, but should first discuss those disciplines which are easiest and simplest, and those above all in which order most prevails. Such are the arts of the craftsmen who make cloth and tapestries, those of women who embroider or make lace, as well as all the games with numbers, and all that relates to arithmetic, and the like. All these arts give the mind excellent practice, provided we do not learn them from others, but discover them ourselves. For since nothing in them remains hidden, and they are entirely adjusted to the capacity of human knowledge, they show us very distinctly innumerable arrangements, all different from one another and yet regular, in the scrupulous observation of which the whole of human sagacity consists.

That is why we have warned that studies must be conducted with method. And method, in the more trivial cases, is usually nothing but the constant observation of order, whether existing in the thing itself or ingeniously thought out. For example, if we want to read something written in unknown characters, no order at all appears here, but nevertheless we invent one, in order to examine all the presumptions that can be held about each sign, each word or each phrase, so as
405 to order these presumptions in such a way as to recognize by enumeration everything that can be deduced from them. And we must take the greatest care not to waste time in trying to guess at random and without method the solution of problems of this kind. For even if it often happens that we can solve them without method, and sometimes even more rapidly than with method, if we are lucky, in this way we would weaken the light of the mind and would accustom ourselves so thoroughly to childish vanities that we would be constantly held on the surface of things, without being able to penetrate more deeply. At the same time we should not fall into the error of those who occupy their minds wholly with serious and deeper things, of which after much labor they have acquired only confused knowledge, while they were wishing for profound insight. We must therefore first practice those easier matters, but with method, so that we may become accustomed, through simple and known paths, and as if in a game, to penetrating always to the inner core of things. For in this way, by a continuous progress, and more rapidly than we could have hoped, we shall find that we our-

selves can just as easily deduce from evident principles many progressions that appeared very difficult and complicated.

Perhaps, however, some may wonder that here, where we are looking for the means to make us more skillful in deducing one truth from another, we should omit all the precepts by which the dialecticians think to govern human reason. They prescribe to it certain forms of argument which conclude with such necessity that reason, if confined to them, although it does not take the trouble to consider the inference itself in an attentive and evident manner, can nevertheless *406* sometimes arrive, by virtue of the form, at a sure conclusion. The thing is that, as a matter of fact, we are aware that truth often escapes these fetters, while those, meanwhile, who have used them remain entangled. That does not happen so frequently to other men; and experience shows that ordinarily the subtlest sophisms hardly ever refute those who use only pure reason, but lead astray the sophists themselves.

That is why here, fearing above all things that our reason should take a vacation while we are examining the truth of some matter, we reject these forms of reasoning as contrary to our end, and search rather for all the aids by which our thought may be kept attentive, as we shall show in what follows. But that it may appear with even greater evidence that this method of argument is of no use for knowledge of the truth, it must be noted that the dialecticians can find by their art no syllogism that yields a true conclusion unless they first have the material for it, that is, unless they have already learned the truth itself which they are deducing in their syllogism. Hence it is clear that they themselves learn nothing new from such a form, and that vulgar dialectic is therefore entirely useless for those who wish to investigate the truth of things. On the contrary, its only use is that now and then it can expound more easily to others arguments already known; hence it should be transferred from philosophy to rhetoric.

11. After we have grasped by intuition a certain number of simple propositions, if we wish to infer some other proposition from them, it is useful to run over them in a continuous and uninterrupted movement of thought in order to reflect on their relations to one another, and as far as possible to conceive distinctly several at a time. For it is in this way that our knowledge becomes much more certain and the power of our mind is greatly increased. *407*

This is the occasion to expound more clearly what has already been said of intuition in rules three and seven. For in one place we have contrasted it with deduction, and in another only with enumeration, which we have defined as an inference drawn from many and diverse things. But we said in the same place that the simple deduction of one thing from another is executed by intuition.

It was necessary to proceed in this way, because we demand two conditions of intuition: that the proposition be clearly and distinctly understood, and, further, that it be understood in its entirety at one time and not successively. Deduction, on the other hand, if we are thinking of its execution, as in rule three, does not seem to occur all at one time, but involves a certain movement of our mind,

which infers one thing from another. So we were right in distinguishing it from
408 intuition. But if we consider deduction as already accomplished, as in what we said in rule seven, then it no longer designates any movement, but rather the end of a movement. Therefore we suppose that it is seen by intuition when it is simple and clear, but not when it is complex and obscure. To the latter situation we give the name of enumeration, or induction, because it cannot then be comprehended by the intellect all at one time, but its certainty depends to some extent on memory, in which our judgments about the individual points enumerated must be retained if some one single judgment is to be drawn from all of them.

All these distinctions were necessary for the interpretation of this rule. For after the ninth rule had dealt with intuition alone, and the tenth with enumeration alone, this one explains how these two operations mutually assist and complete one another, to the point of seeming to merge into one by a certain movement of thought which perceives each fact attentively by intuition and at the same time passes to the others.

To this [cooperation] we assign a double advantage: namely, it promotes a more certain knowledge of the conclusion with which we are concerned, and it renders the mind more skillful in other discoveries. The fact is that memory (on which, we have said, depends the certainty of conclusions that embrace more than we can grasp in one intuition), though unstable and infirm, can be renewed and strengthened by this continuous and repeated movement of thought. Thus if by several operations I have first discovered the relation that exists between a first and a second magnitude, then between the second and a third, then between
409 the third and a fourth, and finally between the fourth and a fifth, I do not therefore see what relation exists between the first and fifth, and I cannot deduce it from the relations already known if I do not remember them all. That is why it is necessary for me to run through them repeatedly in thought, until I have passed so rapidly from the first to the last that practically no parts of the process are left to memory, and I seem to grasp the whole thing at once by intuition.

Everyone must see that the sluggishness of the mind is corrected by this scheme and its comprehension likewise enlarged. But it must be noted, further, that the greatest utility of this rule consists in the fact that, in reflecting on the mutual dependence of simple propositions, we get into the habit of distinguishing immediately what is more or less relative and by what degrees it is reduced to the absolute. For example, if I run through several magnitudes that are in continuous proportion, I shall reflect on all the following facts: that it is by a similar mental act—neither more nor less easy—that I recognize the relation that exists between the first magnitude and the second, the second and the third, the third and the fourth, and so on; but that I cannot grasp so easily what is the dependence of the second on the first and third at the same time; and that it is still more difficult to grasp the dependence of the second on the first and fourth, and so on. Hence I understand why I can easily find the third and fourth if only the first and second are given, and so on: it is because this is accomplished by particular and
410 distinct conceptions. But if only the first and third are given, I do not so easily learn the intermediate magnitude, because that can be done only by an effort of thought which simultaneously embraces the two given magnitudes. If only the

first and the fourth are given, I shall have still more trouble in getting an intuitive grasp of the two intermediates, because here three concepts are simultaneously involved. Thus it would seem, in consequence, even more difficult to find the three intermediates between the first and fifth. But there is another scheme by which this can be achieved in a different way. Although four concepts are conjoined here, they can nevertheless be separated, since four can be divided by another number. Thus, I can look for the third by itself from the first and fifth, then the second from the first and third, and so on. Whoever accustoms himself to reflect on these and similar matters, every time he examines a new question, he immediately discovers the source of the difficulty, and what of all ways is the very simplest one for solving it, and this is a very great aid to knowledge of the truth.

12. Finally we ought to use all the aids of intellect, imagination, sense, and memory, partly in order to have a distinct intuition of simple propositions; partly to compare correctly what we seek with what we know so that we may recognize it; partly in order to discover those things which should be so compared with one another so that no human resources may be neglected.

This rule gives the conclusion of all that has been said above, and teaches in *411*
general the points that had to be explained in particular, as follows.

In what concerns the knowledge of things, only two matters have to be considered: namely, ourselves who know and the objects themselves that are to be known. In us there are only four faculties that we can use for this purpose, namely, intellect, imagination, sense, and memory. To be sure, the intellect alone is capable of perceiving truth; but it must nevertheless be assisted by imagination, sense, and memory, if we are not to omit anything that lies in our power. On the side of the objects it is enough to examine three things: firstly, what presents itself spontaneously; secondly, how we learn one thing from another; and thirdly, what deductions we can make from each. This enumeration seems to me to be complete, and to omit nothing to which human powers can extend. [. . .]

13. If we understand a question perfectly, we must abstract it from every superfluous concept, simplify it as much as possible, and divide it by enumeration into the smallest possible parts. [. . .] *430*

14. The same question must be applied to the real extension of bodies, and represented in its entirety to the imagination by means of bare figures; for in this way it will be much more distinctly perceived by the understanding. *438*

If we wish also to use the aid of the imagination, we must notice that whenever we deduce something unknown from something else already known, we do not for all that discover a new genus of being; but it only happens that the knowledge we have is extended to the point of making us see that the thing sought after participates in one way or another in the nature of those things that are given in

the question. For example, if someone is blind from birth, we need not hope to bring it about by any argument that he should perceive true ideas of colors such as we have received from the senses. On the other hand, if someone has already seen the fundamental colors, but does not know the intermediate and mixed colors, it is possible for him by a sort of deduction to invent for himself the images
439 even of those he has not seen, according to their similarity with the others. In the same way, if there exists in the magnet some genus of being, to which our intellect has so far seen nothing similar, we need not hope ever to know it by reasoning. For that we should need either some new sense or a divine mind. All that the human mind can do in this matter, we shall think we have done if we see very distinctly the mixture of beings or of natures already known which produces the same effects that appear in the magnet.

In fact, whatever is the difference of subjects, it is by the same idea that we recognize all those beings already known, such as extension, figure, motion, and the like, which it is not the place to list here; and we do not imagine the shape of a crown differently, whether it is of silver or of gold. This common idea passes from one subject to another only by means of a simple comparison, through which we affirm that the thing sought after is, in one respect or another, similar, identical, or equal to the thing given, in such a way that in all ratiocination it is only by comparison that we know the truth with precision. For example in this: all A is B, all B is C, therefore all A is C: we compare with one another the thing sought after and the thing given, that is to say, A and C, with respect to the question whether either one is B, etc. But since, as we have often warned, the forms of the syllogisms are
440 of no help in perceiving the truth of things, it will be of advantage to the reader, if, after he has completely rejected them, he grasps the fact that every cognition whatsoever which is not gotten by a simple and pure intuition of one isolated object, is gotten by the comparison of two or more objects with one another. Indeed almost all the labor of human reason consists in preparing this operation; for, when it is open and simple, there is no need for any aid of art, but only of the light of nature alone, for the intuition of the truth that is gotten through it.

It must be noted that comparisons are not called simple and open except whenever the thing sought and the thing given participate equally in a certain nature; that all other comparisons, on the other hand, need preparation only because this common nature is not equally present in the one and the other, but with respect to other relations or proportions in which it is involved; and that the principal part of human contriving consists only in reducing these proportions in such a way as to see clearly an equality between what is sought and something known.

It must be noted, further, that nothing can be reduced to this equality except what admits of more and less, and that all this is comprised under the name of magnitude. Thus when the terms of the difficulty have been abstracted from every subject, according to the preceding rule, we understand that we have nothing further to occupy us except magnitudes in general.

But if we wish to imagine something more here, and to make use, not of the
441 pure intellect, but of the intellect aided by images depicted on the imagination, we must note, finally, that nothing is said about magnitudes in general which cannot also be referred to someone in particular.

Hence it is easy to conclude that there will be great advantage in transferring what we understand to be said about magnitudes in general to that species of magnitude which among all will be depicted most easily and most distinctly in our imagination. But, that this magnitude is the real extension of a body, abstracted from everything else but its figure, results from what has been said in rule twelve, where we have seen that imagination itself, with the ideas which exist in it, is only a true, real, extended, and figured body. This is also self-evident, because all the differences in proportion are not exhibited more distinctly in any other subject. For although one thing can be called more or less white than another, or again one sound more or less acute, and so of other things, still we cannot define exactly whether this more or less is in double or triple proportion, except by a certain analogy with the extension of a figured body. It remains sure and certain, therefore, that perfectly determined questions contain scarcely any difficulty beyond that which consists in resolving proportions into equalities; and that everything in which just this difficulty is discovered can and should be easily separated from every other subject, and then transferred to extension and figures, of which, for this reason, we shall later treat exclusively up to the twenty-fifth rule. [. . .]

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15. It is also useful in many cases to describe these figures and to exhibit them to the external senses, in order that by this device our thought should more easily be kept attentive. [. . .]

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16. As for the things which do not demand the immediate attention of the mind, although they are necessary for the conclusion it is better to designate them by very brief signs rather than by complete figures; for thus the memory cannot err, and meanwhile the thought will not be distracted for the purpose of retaining them, while it is applying itself to deducing other things. [. . .]

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17. A given difficulty should be run through directly, in abstraction from the fact that some of its terms are known and others unknown, and with the intuition, obtained by taking the right road, of the mutual dependence of each term on the others. [. . .]

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18. For this only four operations are required, addition, subtraction, multiplication, and division, among which the last two often do not need to be carried out here, as much to keep from complicating things needlessly as because they can be executed more easily later. [. . .]

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19. By this method of ratiocination we should seek out as many magnitudes expressed in two different modes, as we suppose unknown terms directly bearing on the difficulty in place of known ones: for thus we shall have as many comparisons between two equals.

469 ***20. When the equations have been found, we must finish the operations which we have left aside, never making use of multiplication whenever there is room for division.***

21. If there are several equations of this sort, we should reduce them all to a single one, that is to say, to the one whose terms will occupy the least number of degrees in the sequence of magnitudes in continuous proportion, according to which they are to be ordered.