Economics for Engineers

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Production Function

Shows maximum output possible for any combination of inputs (technical efficiency):

$$q = F(K, L, E, M, \dots).$$

Marginal products:

$$MP_K = \frac{\partial q}{\partial K}, \qquad MP_L = \frac{\partial q}{\partial L}, \qquad etc.$$

Average Products:

$$AP_K = \frac{q}{K}, \qquad AP_L = \frac{q}{L}, \qquad etc.$$

Relationship Between AP and MP

The marginal product curve must cut the average product curve at its maximum.

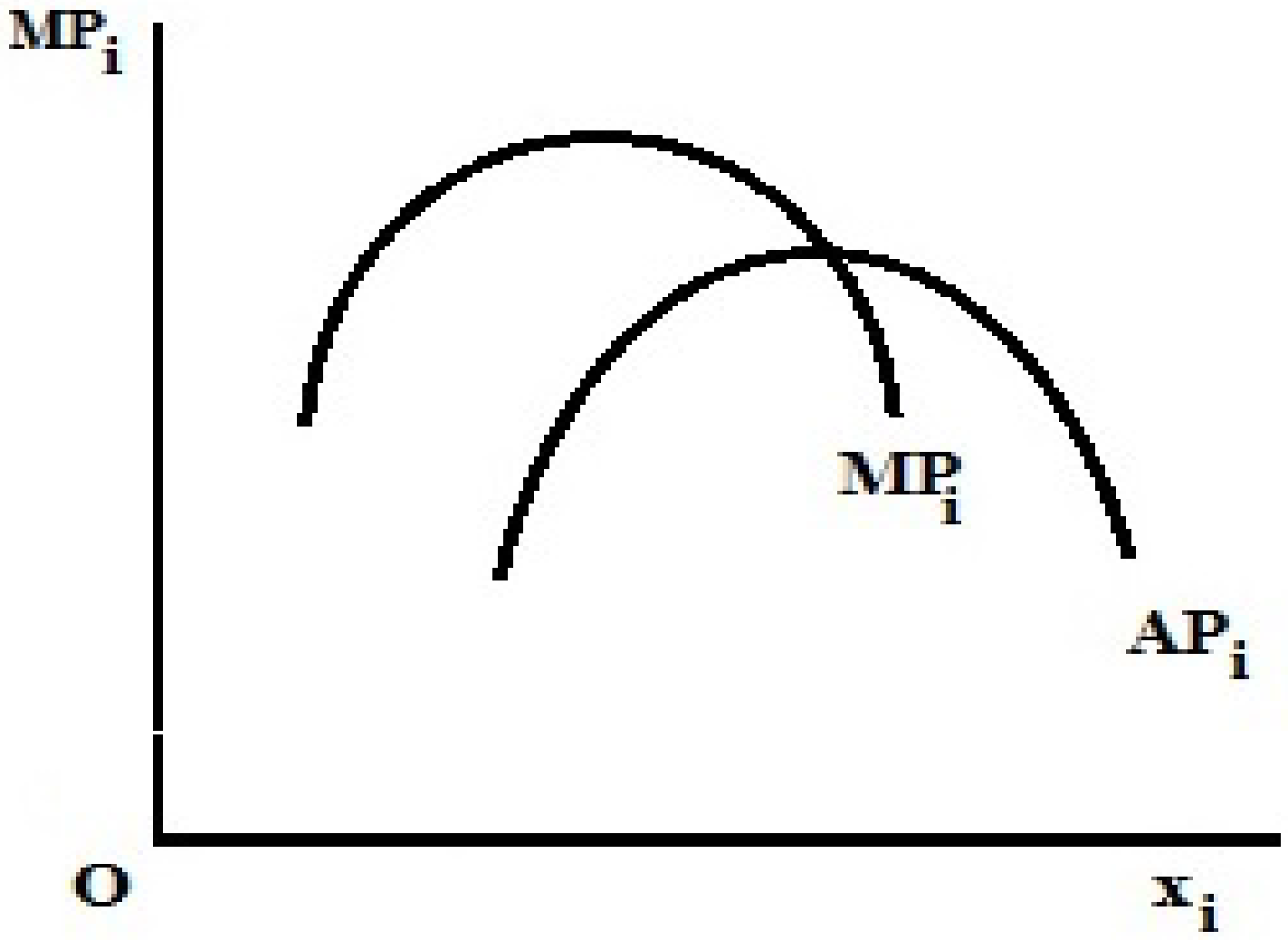
$$AP_L = \frac{q}{L}$$

$$\frac{\partial AP_L}{\partial L} = \frac{L\frac{\partial q}{\partial L} - q}{L^2} = \frac{\frac{\partial q}{\partial L} - \frac{q}{L}}{L}$$

$$= \frac{MP_L - AP_L}{L}.$$

So $MP_L = AP_L$ where AP_L is at a maximum.

Relationship Between AP and MP



Isoquants

Analagous to indifference curves.

$$F(K, L) = \bar{q}.$$

Total derivative:

$$F_K dK + F_L dL = d\bar{q} = 0.$$

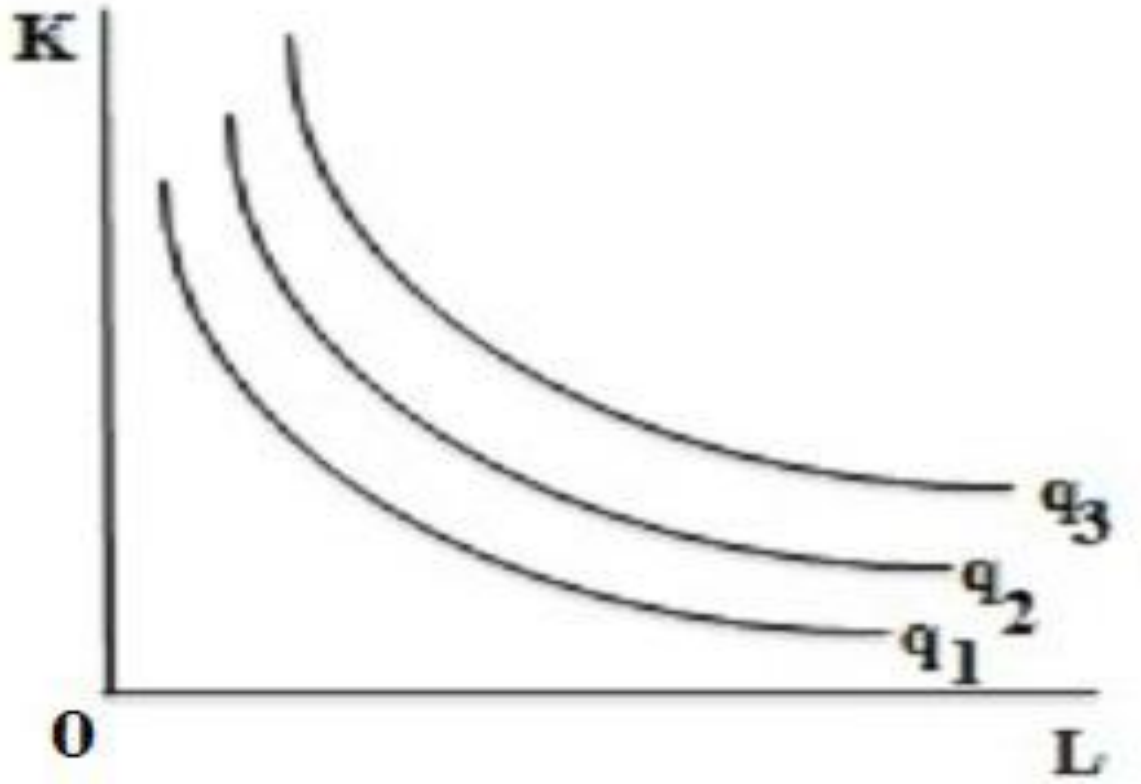
So slope of isoquant is:

$$\frac{dK}{dL} = -\frac{F_L}{F_K} = -\frac{MP_L}{MP_K}.$$

The marginal rate of technical substitution is the absolute value of the slope of the isoquant:

$$MRTS = \frac{MP_L}{MP_K}.$$

Contd…

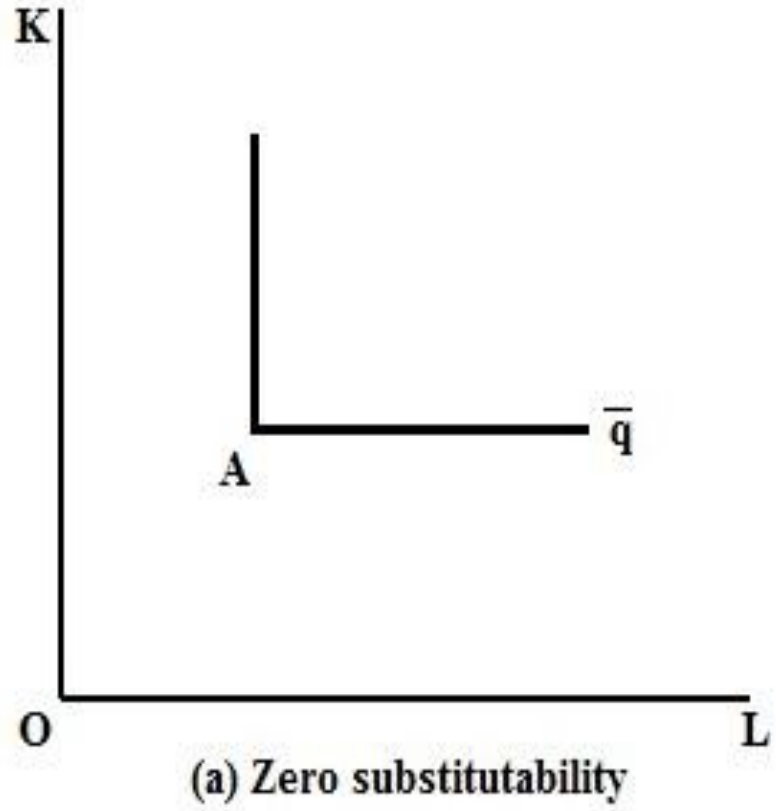


Elasticity of Substitution

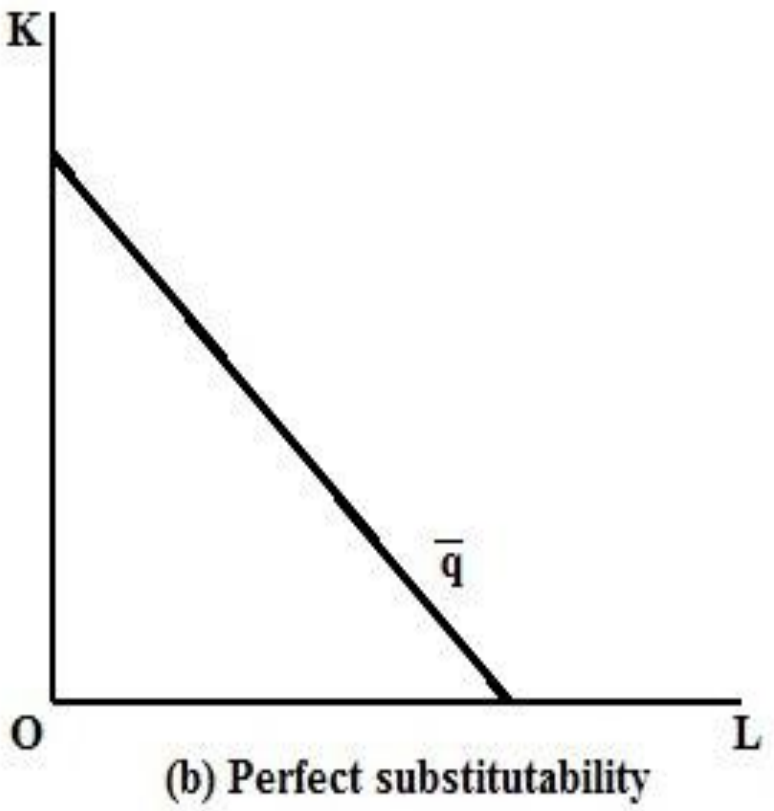
Measure of the degree to which inputs can be substituted for one another in production.

$$\sigma = \frac{\partial\left(K/L\right)}{\partial MRTS} \cdot \frac{MRTS}{K/L}$$

Contd…



(a) Zero substitutability



(b) Perfect substitutability

Elasticity of Substitution for Cobb–Douglas Production Function

$$q = AK^{\alpha}L^{\beta}$$

$$MP_K = A\alpha K^{\alpha-1}L^{\beta}$$

$$MP_L = AK^{\alpha}\beta L^{\beta-1}.$$

Then, $$MRTS = \frac{MP_L}{MP_K} = \frac{\beta K}{\alpha L}$$

or $$\frac{K}{L} = \frac{\alpha}{\beta} \cdot MRTS.$$

Therefore, $$\sigma = \frac{\partial(K/L)}{\partial MRTS} \cdot \frac{MRTS}{K/L} = 1.$$

Note this does not depend on values of α, β.


Thank
you!!